

Stress and Strain

- Stress
- Strain
- Analysis – ASD vs. LRFD
- Modes of Failure

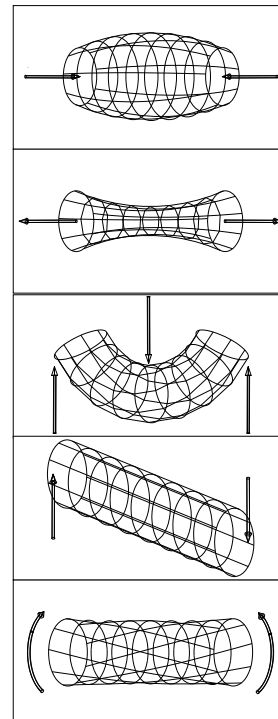
$$\sigma = \frac{P}{A}$$

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$$\sigma = \frac{M c}{I}$$

$$\tau = \frac{P}{A} \text{ or } \frac{VQ}{Ib}$$

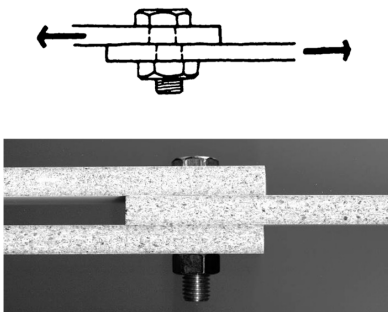
$$\tau = \frac{T r}{J}$$



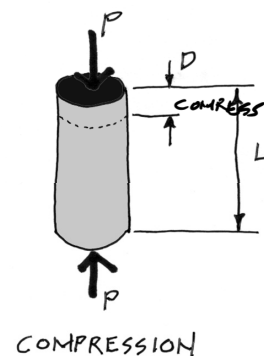
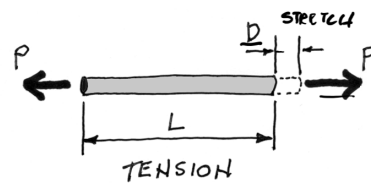
Stress

Stress is the result of some force being applied to an area of some material.

$$\underline{\sigma} = \frac{P}{A} \quad \begin{array}{l} \text{FORCE} \\ \hline \text{AREA} \end{array}$$



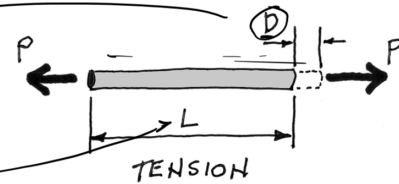
Shear Stress



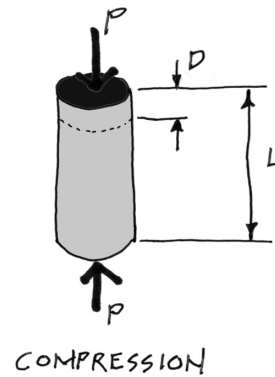
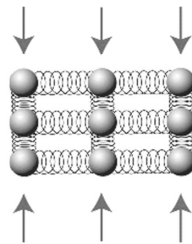
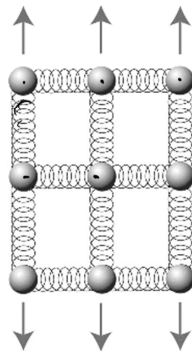
Strain

Strain is the amount of deformation in the material, per unit length.

$$\epsilon = \frac{D}{L}$$



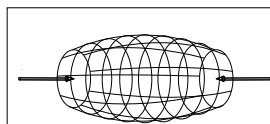
Deformation occurs either in stretching (tension) or in compressing (compression) but not always at the same rate.



Types of Stress

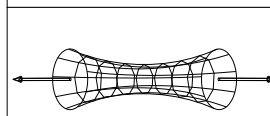
- Compression

$$\sigma = \frac{P}{A}$$



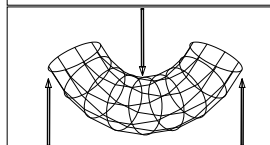
- Tension

$$\sigma = \frac{P}{A}$$



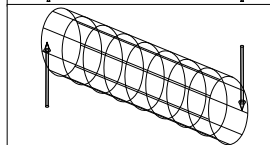
- Flexure

$$\sigma = \frac{M c}{I}$$



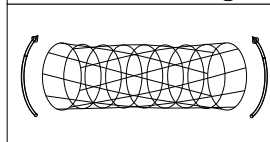
- Shear

$$\tau = \frac{P}{A} \text{ or } \frac{VQ}{Ib}$$



- Torsion

$$\tau = \frac{T r}{J}$$



Stress Analysis

Allowable Stress Design (ASD)

- use design loads (no F.S. on loads)
- reduce stress by a Factor of Safety F.S.

$$\underline{f_{actual}} = \frac{P_{LOAD}}{A_{AREA}}$$

$$\underline{f_{actual}} \leq \underline{F_{allowable}} \quad \text{—————} \quad \underline{F_{allowable}} = \underline{F.S.} \cdot \underline{f_{yield}}$$

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use factor on ultimate strength ϕ

$$\underline{P_{load}} = \underline{\gamma} \cdot P_{applied}$$

$$\underline{P_{load}} \leq \underline{P_{resisting}}$$

$$\underline{P_{resisting}} = \underline{\phi} \cdot P_{material}$$

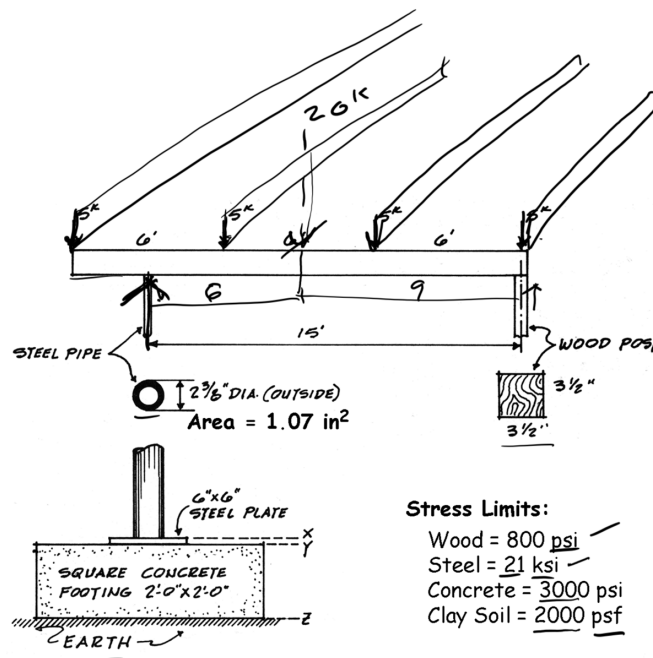
Stress Calculations

Find the stress in each material

Axial Compression

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$

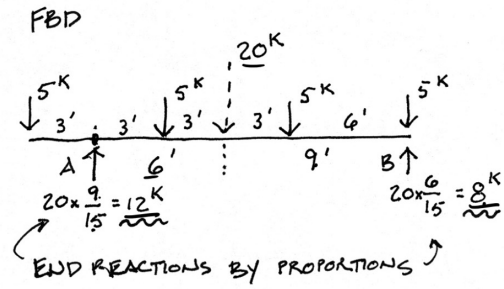
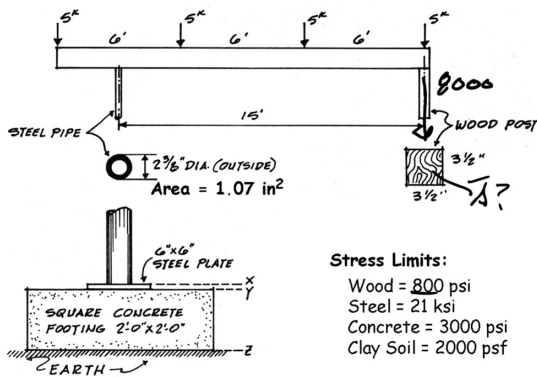


Stress Limits:

Wood = 800 psi
 Steel = 21 ksi
 Concrete = 3000 psi
 Clay Soil = 2000 psf

Stress Calculations

FBD – reactions



OR

ΣM AT EACH REACTION

$$\Sigma M @ A = -5k(3') + 5k(3') + 5k(9') + 5k(15') - B(15')$$

$$B = 8k$$

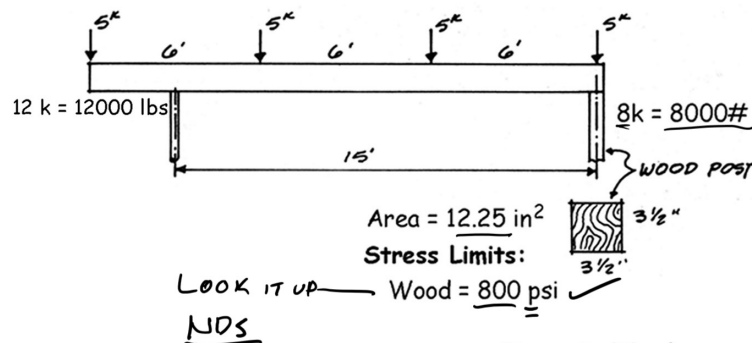
$$\Sigma F_v = 0 = -20 + A + B = 0$$

$$A = 12k$$

Stress Calculations

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



Stress in Wood:

$$f = P/A$$

$$f = 8000\text{lbs}/12.25\text{in}^2 = 653\text{psi}$$

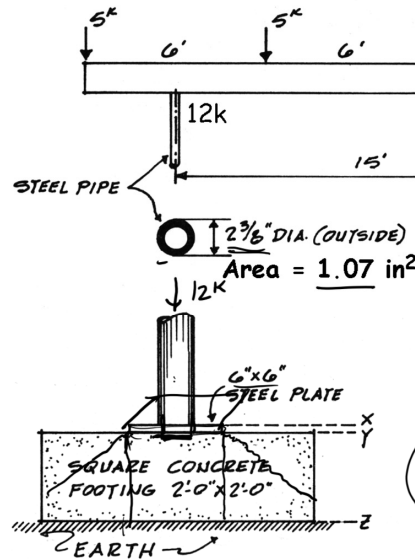
$$F = 800\text{psi}$$

$$f < F \text{ ok}$$

Stress Calculations

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



Stress Limits:

Steel = 21 ksi $f_y = 36$
Concrete = 3000 psi

Stress in Steel:

$f = P/A = 12k / 1.07 \text{ in}^2 \text{ ksi}$
 $f = 11.2 \text{ ksi} < F = 21 \text{ ksi} \text{ ok } \checkmark$

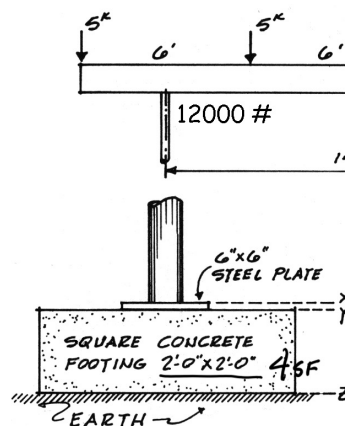
Stress in Concrete:

$f = 12000 \text{ lbs} / 1.07 \text{ in}^2$
 $f = 11200 \text{ psi} > 3000 \text{ psi} \text{ FAILS!}$
 $f = 12000^k / 36 = 333 \text{ psi}$
 $333 \text{ psi} < 3000 \text{ psi} \text{ ok}$

Stress Calculations

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



Stress in Soil:

$f = 12000 \# / 4 \text{ sf}$
 $f = 3000 \text{ psf} > 2000 \text{ psf} \text{ FAILS!}$

$$\frac{P}{A} = 2000$$

$$A = \frac{P}{2000}$$

Stress Limits:

Clay Soil = 2000 psf

Stress Calculations

Axial Tension

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



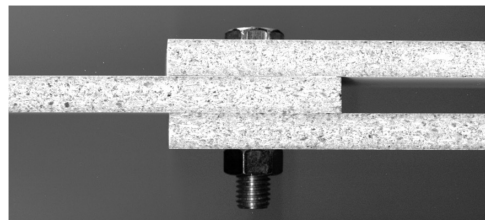
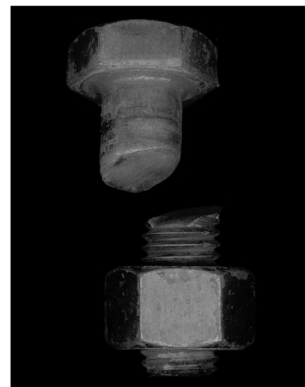
Santiago Calatrava - Serreria Bridge - Valencia 2008

Stress Calculations

Shear

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



Stress Calculations

Bending

Flexure Stress

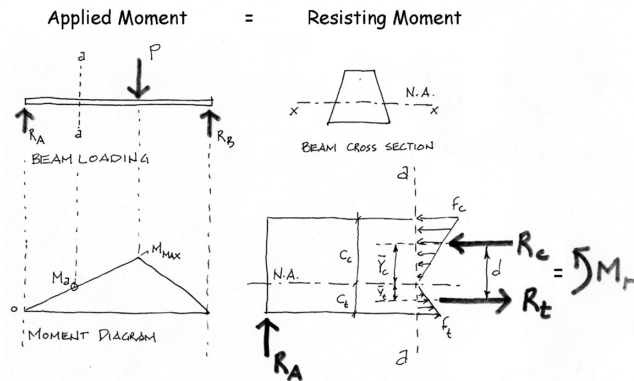
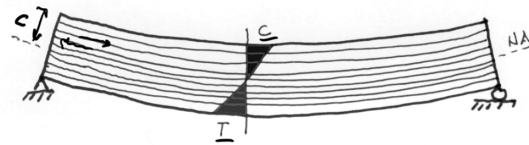
The stress is on the “fibers” or longitudinal layers

$$\sigma = \frac{M c}{I}$$

Shear Stress

The stress is between the longitudinal layers.

$$\tau = \frac{VQ}{Ib}$$



Modes of Failure

Strength

- Tension rupture
- Compression crushing

Stability

- Column buckling
- Beam lateral torsional buckling

Serviceability

- Beam deflection
- Building story drift
- cracking

