

Cross-Sectional Properties of Structural Members

Resultant of Parallel Forces

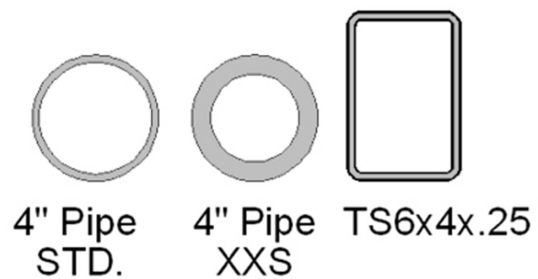
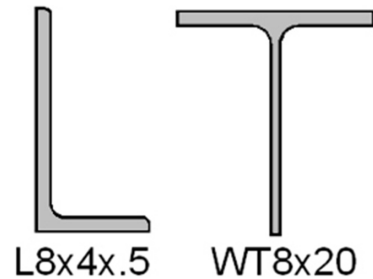
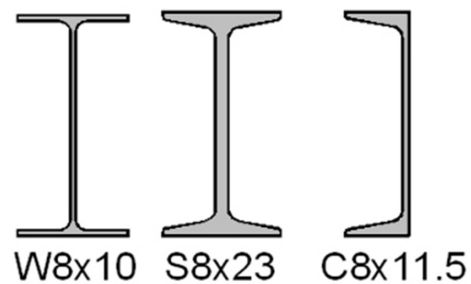
Center of Gravity

Centroid of Area

First Moment of Area

Second Moment of Area
(Moment of Inertia)

Radius of Gyration



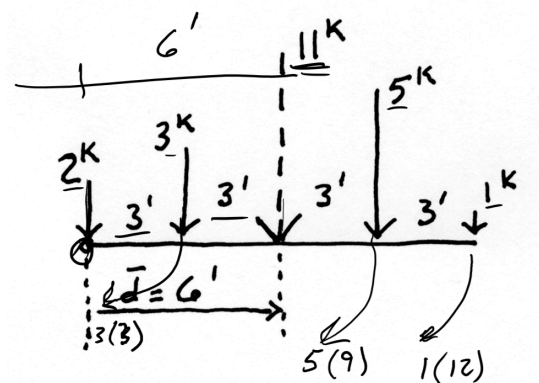
Parallel Force Resultant

The resultant is the single force that has the same effect as the group of forces.

$$\sum \underline{M} = \sum (\underline{F} \times \underline{d}) = \underline{R} \times \underline{\bar{d}}$$

$$9 + 45 + 12 = 11 \bar{d}$$

$$\sum \underline{F} = \underline{R}$$



$$\bar{d} = \frac{\sum (\underline{F} \times \underline{d})}{\sum \underline{F}} = \frac{66}{11} = 6$$

Centers

The point about which a body may be balanced.

This is the point of application of the resultant weight.

Center of Gravity

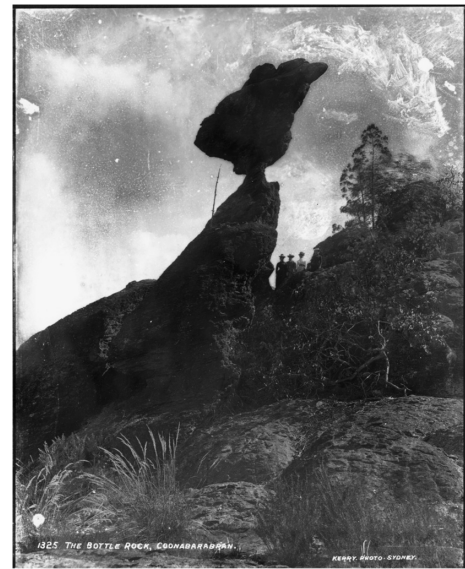
$$\bar{x} = \frac{\sum \underline{W} \times d_x}{\sum \underline{W}}$$

Center of Volume

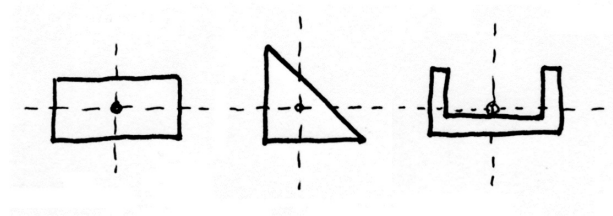
$$\bar{x} = \frac{\sum \underline{V} \times d_x}{\sum \underline{V}}$$

Center of Area (centroid)

$$\bar{x} = \frac{\sum \underline{A} \times d_x}{\sum \underline{A}}$$



Tyrrell Photographic Collection, Powerhouse Museum

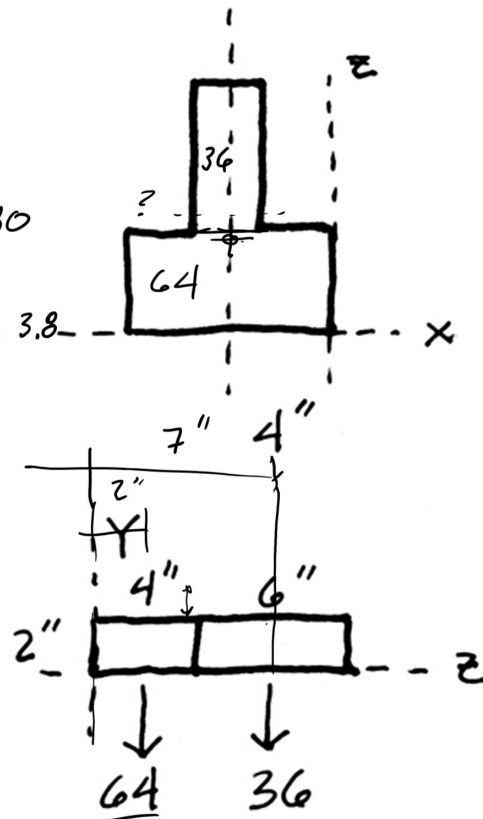
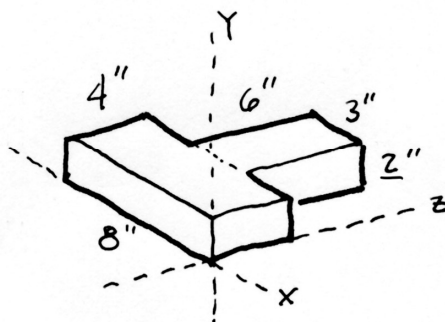


Center of Gravity (or Volume)

The Center of Gravity is located at the point defined by:

$$(\bar{x}, \bar{y}, \bar{z})$$

$$\bar{z} = \frac{\sum \underline{W} \times d_z}{\sum \underline{W}} = \frac{128 + 252 = 380}{64 + 36 = 100} = 3.8$$



Center of Area - the Centroid

The “center of area” for a cross section.

$$\bar{x} = \frac{\sum (\text{Area} \times d_x)}{\sum \text{Area}} = \frac{A x_A + B x_B + C x_c}{A + B + C}$$

$$\text{Area A} = 2 \times 7 = 14$$

$$\text{Area B} = 3 \times 2 = 6$$

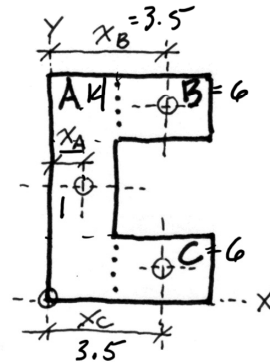
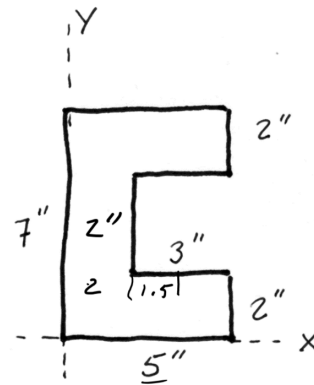
$$\text{Area C} = 3 \times 2 = 6$$

$$\text{sum} = 26$$

$$x_A = 1$$

$$x_B = 3.5$$

$$x_C = 3.5$$



Centroid Example 1 cont.

$$\text{Area A} = 2 \times 7 = 14$$

$$x_A = 1$$

$$\text{Area B} = 3 \times 2 = 6$$

$$x_B = 3.5$$

$$\text{Area C} = 3 \times 2 = 6$$

$$x_C = 3.5$$

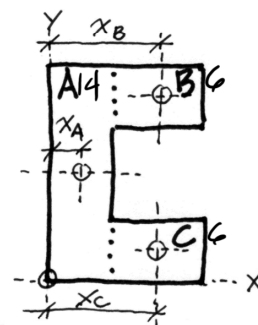
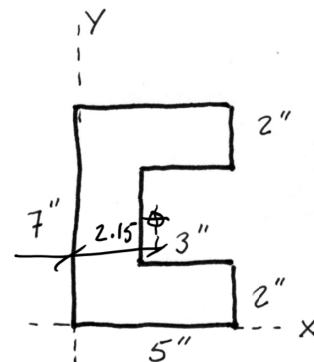
$$\text{sum} = 26$$

Calculation.

$$\bar{x} = \frac{\sum \text{Area} \times d_x}{\sum \text{Area}} = \frac{A x_A + B x_B + C x_c}{A + B + C}$$

$$\bar{x} = \frac{(14 \times 1) + (6 \times 3.5) + (6 \times 3.5)}{14 + 6 + 6}$$

$$\bar{x} = \frac{56}{26} = \underline{\underline{2.15''}}$$



Centroid Example 1 cont.

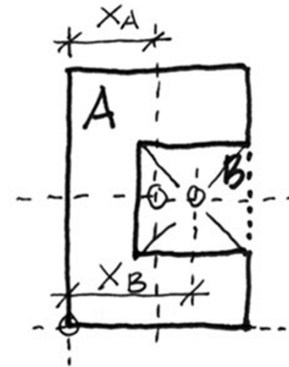
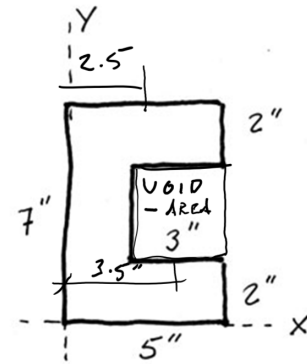
Calculation: by Solid - Void.



$$\bar{x} = \frac{\sum A \times d_x}{\sum A} = \frac{A x_A - B x_B}{A - B}$$

$$\bar{x} = \frac{\sum (35 \times 2.5) - (9 \times 3.5)}{\sum 35 - 9} = \frac{56}{26}$$

$$\bar{x} = 2.15"$$



Static Moment of Area

1st moment of area

USED FOR SHEAR

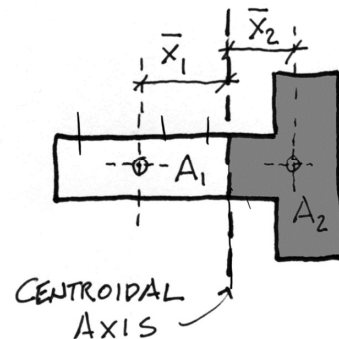
$$F_v = \frac{VQ}{Ib}$$

The tendency of an area alone to rotate about an axis in the plane of that area.

$$Q = A \bar{x}$$

At the Neutral Axis

$$A_1 \bar{x}_1 = A_2 \bar{x}_2$$

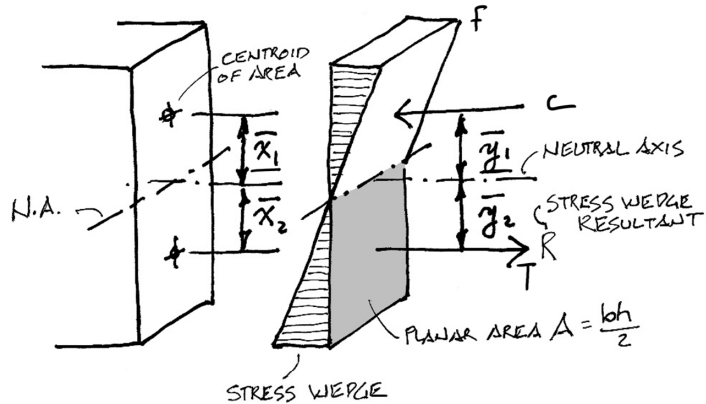


Moment of Inertia

2nd moment of area

By definition:

$$I_x = A \bar{x}_i \bar{y}_i$$



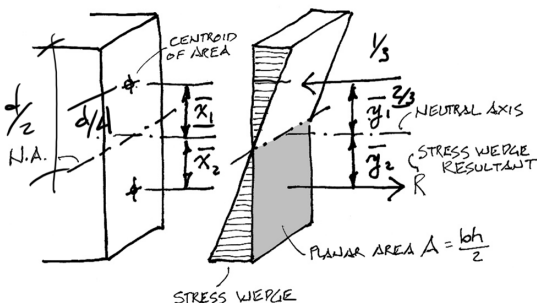
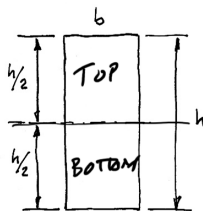
For a rectangle at the N.A.

STIFFNESS $\rightarrow I_x = \frac{bh^3}{12}$

Moment of Inertia

2nd moment of area

$$I_x = A \bar{x} \bar{y}$$



BY HALVES

FOR A RECTANGULAR SECTION:

$$A = b \left(\frac{h}{2} \right)$$

$$\bar{x} = \frac{h}{2} \div 2 = \frac{h}{4}$$

$$\bar{y} = \frac{2}{3} \frac{h}{2} = \frac{h}{3}$$

$$A \bar{x} \bar{y}_{(TOP)} = \frac{bh}{2} \times \frac{h}{4} \times \frac{h}{3} = \frac{bh^3}{24}$$

$$A \bar{x} \bar{y}_{(BOTTOM)} = \frac{bh}{2} \times \frac{h}{4} \times \frac{h}{3} = \frac{bh^3}{24}$$

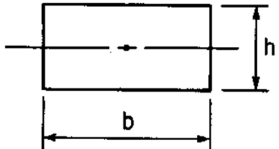
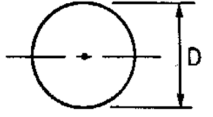
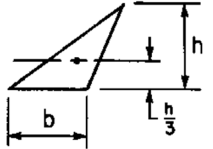
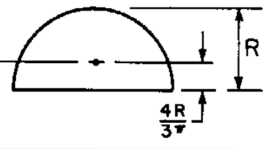
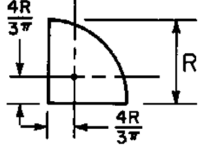
FOR TOTAL SECTION:

$$2 \times \frac{bh^3}{24} = \frac{bh^3}{12}$$

$$I_x = \frac{bh^3}{12}$$

Moment of Inertia

Solutions for basic shapes:

	Shape	Moment of inertia
Rectangle		$I = \frac{1}{12}bh^3$
Circle		$I = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$
Triangle		$I = \frac{1}{36}bh^3$
Semicircle		$I = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) R^4 = 0.11R^4$
Quarter circle		$I = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) \frac{R^4}{2} = 0.055R^4$

Moment of Inertia

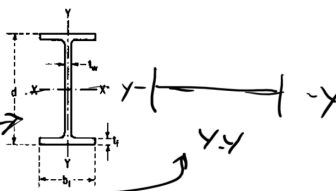
Solutions for basic shapes:

- Single Shapes
- Combination Shapes

COMBINATION SECTIONS S shapes and channels Properties of sections													
Beam	Channel	Total Wt. per Ft	Total Area	AXIS X-X					AXIS Y-Y				
				I	$S_x = I/y_1$	$S_y = I/y_2$	r	y_1	I	S	r	r_T	
				Lb.	in. ⁴	in. ³	in.	in.	in. ⁴	in. ³	in.	in.	
S 10 × 25.4	C 8 × 11.5	36.9	10.84	176	27.2	46.6	4.02	6.45	39.4	9.85	1.91	2.44	
× 25.4	C 10 × 15.3	40.7	11.95	186	27.6	52.9	3.94	6.73	74.2	14.8	2.49	3.16	
S 12 × 31.8	C 8 × 11.5	43.3	12.73	299	39.8	63.2	4.84	7.50	42.0	10.5	1.82	2.38	
× 31.8	C 10 × 15.3	47.1	13.84	316	40.4	71.4	4.78	7.82	76.8	15.4	2.36	3.06	
× 40.8	C 10 × 15.3	56.1	16.49	377	50.1	80.0	4.78	7.53	81.0	16.2	2.22	2.94	

WIDE FLANGE SHAPES

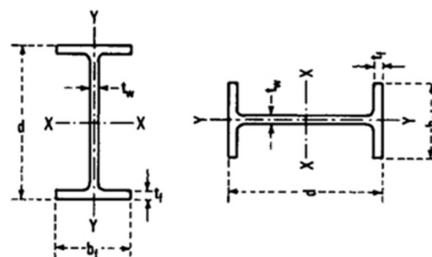
Theoretical Dimensions and Properties for **Designing**



Section Number	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X-X			Axis Y-Y			r_T
				Width	Thick-ness		I_x	S_x	r_x	I_y	S_y	r_y	
				b_f	t_f		in. ⁴	in. ³	in.	in. ⁴	in. ³	in.	
W27 x 178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72	
161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70	
146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68	
W27 x 114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58	
102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56	
94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53	
84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49	

Section Properties

WIDE FLANGE SHAPES



Theoretical Dimensions and Properties for Designing

Section Number	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X-X			Axis Y-Y			r _T
				Width	Thick-ness		I _x	S _x	r _x	I _y	S _y	r _y	
	lb	in. ²	in.	b _f	t _f	t _w	in. ⁴	in. ³	in.	in. ⁴	in. ³	in.	in.
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Section Properties

PROPERTIES OF SAWN LUMBER SECTIONS

Rectangular :

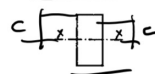
$$A = bd$$

$$I = db^3/12$$

$$S = I/c$$

$$c = d/2 \text{ (maximum)}$$

$$F_b = \frac{Mc}{I} = \frac{M}{S}$$



$\frac{1}{c}$

Nominal Size b × d	Actual Size b × d	Area in. ²	I_x in. ⁴	S_x in. ³
1 × 4	3/4 × 3 1/2	2.63	2.68	1.53
1 × 6	" × 5 1/2	4.13	10.40	3.78
1 × 8	" × 7 1/4	5.44	23.82	6.57
1 × 10	" × 9 1/4	6.94	49.47	10.70
1 × 12	" × 11 1/4	8.44	88.99	15.83
2 × 4	1 1/2 × 3 1/2	5.25	5.36	3.06
2 × 6	" × 5 1/2	8.25	20.80	7.56
2 × 8	" × 7 1/4	10.88	47.64	13.14
2 × 10	" × 9 1/4	13.88	98.93	21.39
2 × 12	" × 11 1/4	16.88	177.98	31.64
3 × 4	2 1/2 × 3 1/2	8.75	8.93	5.10
3 × 6	" × 5 1/2	13.75	34.66	12.60
3 × 8	" × 7 1/4	18.13	79.39	21.90
3 × 10	" × 9 1/4	23.13	164.89	35.65
3 × 12	" × 11 1/4	28.13	296.63	52.73
4 × 4	3 1/2 × 3 1/2	12.25	12.50	7.15
4 × 6	" × 5 1/2	19.25	48.53	17.65
4 × 8	" × 7 1/4	25.38	111.15	30.66
4 × 10	" × 9 1/4	32.38	230.84	49.91
4 × 12	" × 11 1/4	39.38	415.28	73.83

Moment of Inertia

Shapes with common centroidal axes

$$I_{\text{solid}} + I_{\text{solid}} = I_x$$

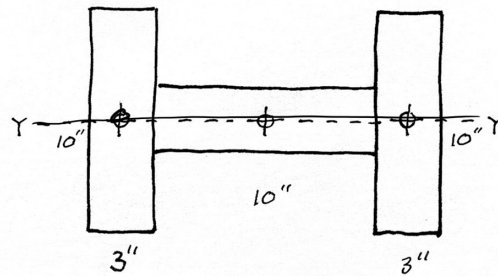


Diagram showing the decomposition of the C-shape into three rectangles:

- Left flange: height $h = 10''$, width $b = 3''$
- Web: height $h = 3''$, width $b = 10''$
- Right flange: height $h = 10''$, width $b = 3''$

Formula for the moment of inertia of a rectangle: $\frac{bh^3}{12}$

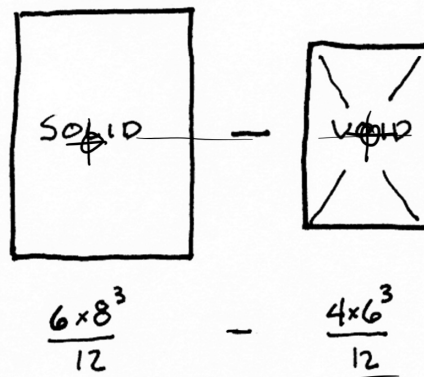
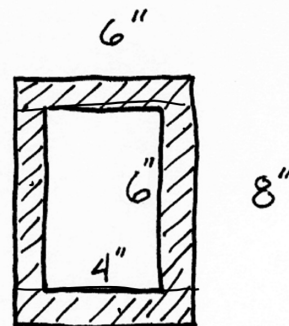
$$\frac{3(10)^3}{12} + \frac{10(3)^3}{12} + \frac{3(10)^3}{12}$$

$$250 \text{ in}^4 + 22.5 \text{ in}^4 + 250 \text{ in}^4 = 522.5 \text{ in}^4$$

Moment of Inertia

Shapes with common centroidal axes

$$I_{\text{solid}} - I_{\text{void}} = I_x$$



$$256 - 72 = 184 \text{ in}^4$$

Moment of Inertia

The **Transfer Equation** or **Parallel Axis Theorem**,

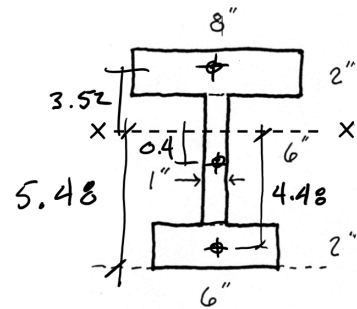
taken about the x-x axis:

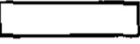
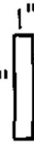
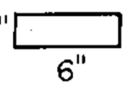
$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$y\text{-bar} = 186/34 = 5.48''$$

$$I_x = \sum \bar{I}_x + \sum Ad^2$$

$$I_x = 27.3 + 439.4 = 466.7 \text{ in}^4$$



Shape	$\underline{A}$	$\underline{y}$	$\underline{Ay}$	$\underline{\bar{I}_x}$	$\underline{d, \text{ in.}}$	$\underline{Ad^2}$
2" 	$(2)(8) = 16$	9	144	$(\frac{1}{12})(8)(2)^3 = \underline{5.3}$	<u>3.52</u>	$(16)(3.52)^2 = 198$
6" 	$(1)(6) = 6$	5	30	$(\frac{1}{12})(1)(6)^3 = \underline{18}$	<u>0.48</u>	$6(0.48)^2 = 1.4$
2" 	$(2)(6) = 12$	1	12	$(\frac{1}{12})(6)(2)^3 = \underline{4}$	<u>4.48</u>	$12(4.48)^2 = 240$
	$\sum A = 34$	$\sum Ay = 186$		$\sum \bar{I}_x = 27.3$		$\sum Ad^2 = \underline{439.4}$

$$y\text{-bar} = 186/34 = \underline{5.48''}$$

$$I_x = \underline{27.3 + 439.4 = 466.7 \text{ in}^4}$$

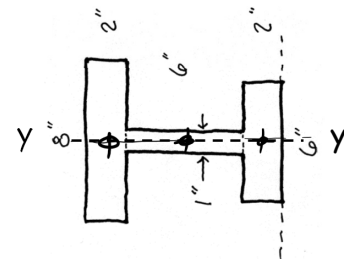
Moment of Inertia

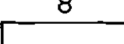
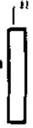
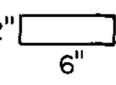
The Transfer Equation or Parallel Axis Theorem:

$$I_y = \sum \bar{I}_y + \sum Ad^2$$

Taken about the y-y axis:

$$I_y = 121.8 + 0 = 121.8$$



Shape	$\underline{A}$	$\underline{\bar{I}_y}$	$\underline{d}$	$\underline{Ad^2}$
2" 	16	$(\frac{1}{12})(2)(8)^3 = 85.3$	<u>0</u>	0
6" 	6	$(\frac{1}{12})(6)(1)^3 = 0.5$	<u>0</u>	0
2" 	12	$(\frac{1}{12})(2)(6)^3 = 36.0$	<u>0</u>	0
		$\sum \bar{I}_y = \underline{121.8}$		0

SUMMARY:

$$I_x = \underline{466.7 \text{ in}^4}$$

$$I_y = \underline{121.8 \text{ in}^4}$$

Radius of Gyration

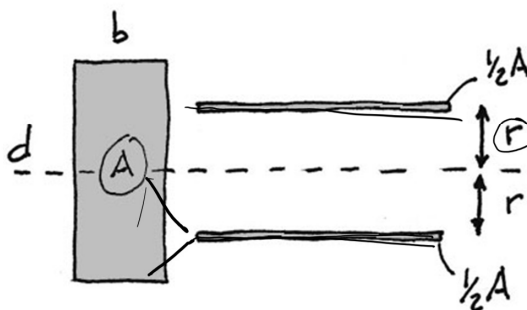
The distance from the centroid where all area could be collected to yield an equivalent Moment of Inertia.

$$I = A r^2 = \frac{b d^3}{12}$$

$$r = \sqrt{\frac{I}{A}}$$

$$r = 0.289 d$$

for a rectangle about the N.A

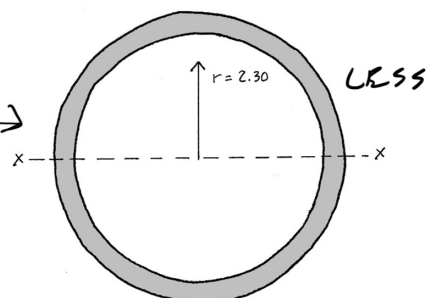
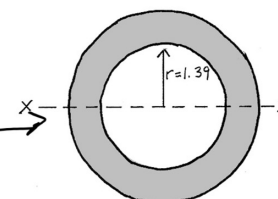
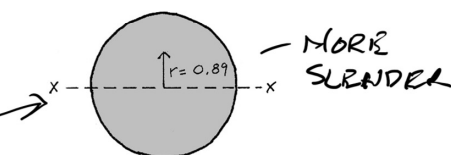


Radius of Gyration

The larger the radius of gyration, the more resistant the section is to buckling.

The areas in the table below are constant, while diameters increase.

OD	ID	t	A	r
3.57	0.00	1.78	10.00	0.89
3.71	1.00	1.35	10.00	0.96
4.09	2.00	1.05	10.00	1.14
4.66	3.00	0.83	10.00	1.39
5.36	4.00	0.68	10.00	1.67
6.14	5.00	0.57	10.00	1.98
6.98	6.00	0.49	10.00	2.30
7.86	7.00	0.43	10.00	2.63
8.76	8.00	0.38	10.00	2.97
9.68	9.00	0.34	10.00	3.30
10.62	10.00	0.31	10.00	3.65

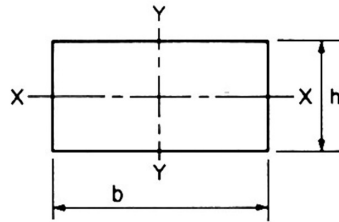


$$P_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

BUCKLING

← SLENDERNESS

Section Formulas



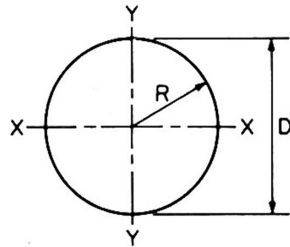
Rectangle.

Rectangle

$$A = bh,$$

$$I_x = \frac{1}{12}bh^3,$$

$$\underline{r_x} = \sqrt{I_x/A} = 0.288h.$$



Circle.

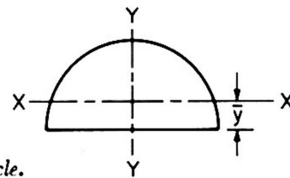
Circle

$$A = \frac{1}{4}\pi D^2 = \pi R^2,$$

$$I_x = \frac{\pi D^4}{64} = \frac{\pi R^4}{4},$$

$$\underline{r_x} = \sqrt{I_x/A} = \frac{D}{4} = \frac{R}{2},$$

$$\underline{J} = I_x + I_y = \frac{\pi D^4}{32} = \frac{\pi R^4}{2}.$$



Semicircle.

Semicircle

$$A = \frac{1}{8}\pi D^2 = \frac{1}{2}\pi R^2,$$

$$\bar{y} = \frac{4r}{3\pi},$$

$$I_x = 0.00682D^4 = 0.11R^4,$$

$$I_y = \frac{\pi D^4}{128} = \frac{\pi R^4}{8},$$

$$r_x = 0.264R.$$