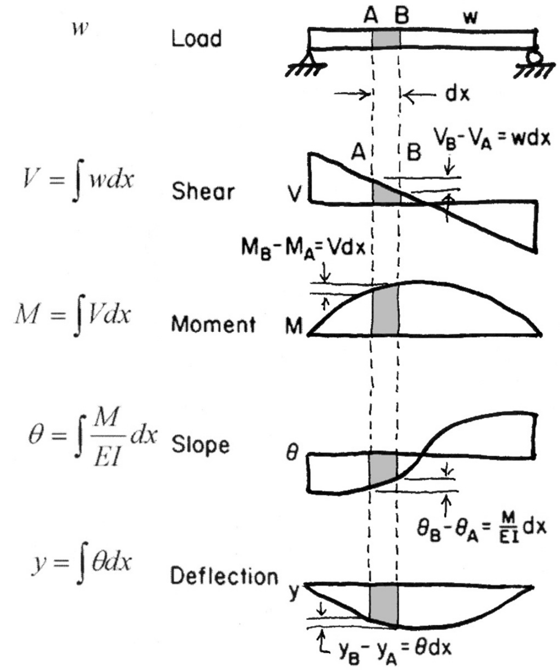
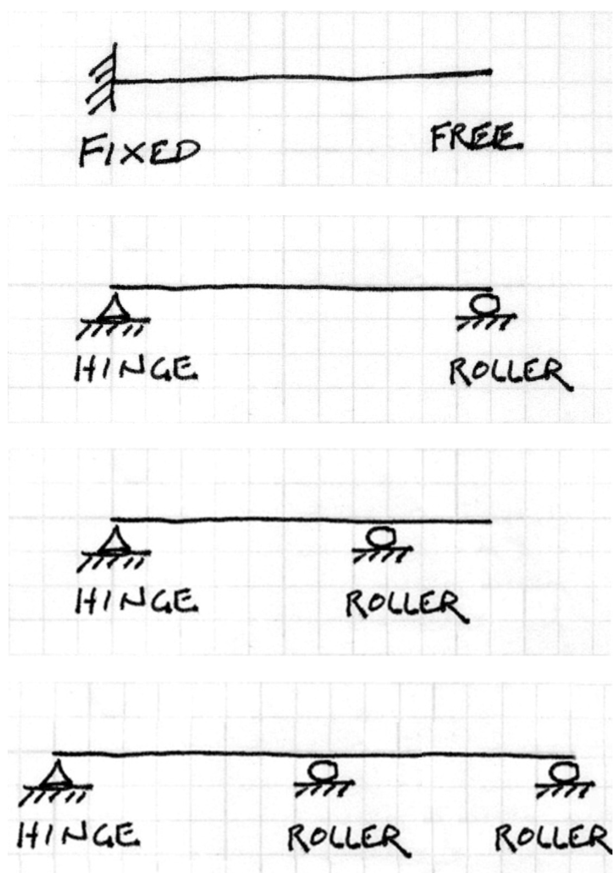
Bending and Shear in Simple Beams



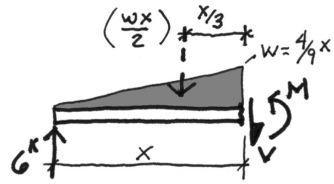
- Free Body Diagrams of Shear and Moment in Beams
- Sign Conventions for Plotting V & M Diagrams
- Diagrams by Equilibrium (FBD)
- Diagrams by Integration
- Diagrams by Areas (Semi-graphical)
- Diagrams by Equations
- Examples in Form (catenary curves)

Beam Types

- Cantilever
- Simple
- Simple with Cantilever
- Continuous (multi-span)

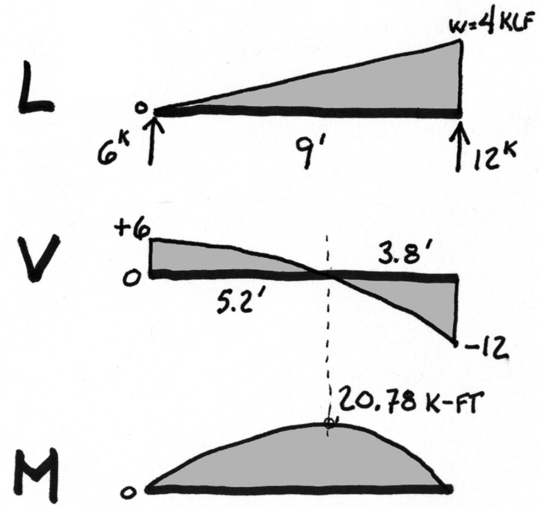


1. Equilibrium Method - example



Tabulated Results of FBD Calculations

Cut Location From R_L (ft)	Shear **V** (k)	Moment **M** (k-ft)
0-	0	0
0+	6	0
1	5.78	6.9
2	5.11	11.4
3	4.00	16.0
4	2.44	19.3
5	0.44	20.74
5.2	0	20.78
6	-2.00	20.0
7	-4.90	16.6
8	-8.24	10.0
9-	-12.00	0
9+	0	0



Relationships of Forces and Deformations - procedure

There are a series of relationships among forces and deformations in a beam, which can be useful in analysis. Using either the deflection or load as a starting point, the following characteristics can be discovered by taking successive derivatives or integrals of the beam equations.

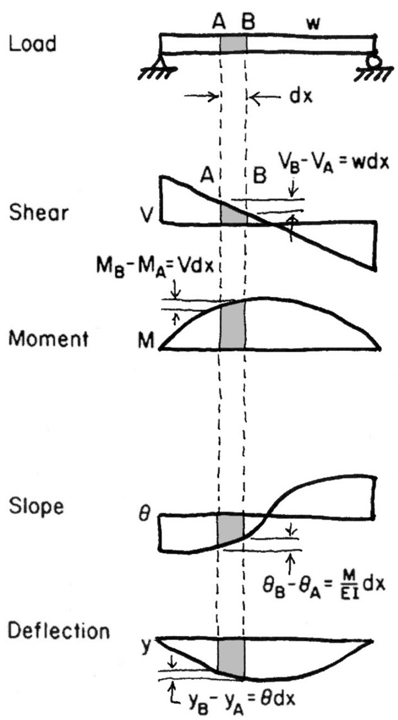
w

$V = \int w dx$

$M = \int V dx$

$\theta = \int \frac{M}{EI} dx$

$y = \int \theta dx$



load $= \frac{d^4y}{dx^4} = \frac{dV}{dx}\frac{1}{EI} = \frac{w}{EI}$

shear $= \frac{d^3y}{dx^3} = \frac{dM}{dx}\frac{1}{EI} = \frac{V}{EI}$

moment $= \frac{d^2y}{dx^2} = \frac{M}{EI}$

slope $= \frac{dy}{dx}$

deflection $= y$

2. Shear and Moment by Integration - example

One method of solving shear and moment forces is to write the loading equation and solve the integration equations for the shear and moment. One problem using this method can be finding the constant of integration, particularly with discontinuous load functions.

$-w$

$$V = \int w dx$$

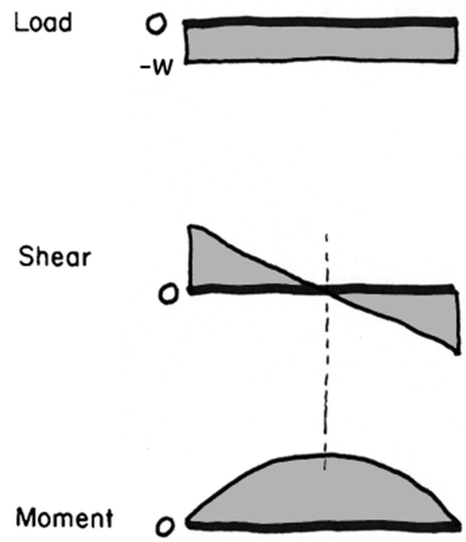
$$V = -wx + C$$

$$V = -wx + \frac{wl}{2}$$

$$M = \int V dx$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x + C$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x$$

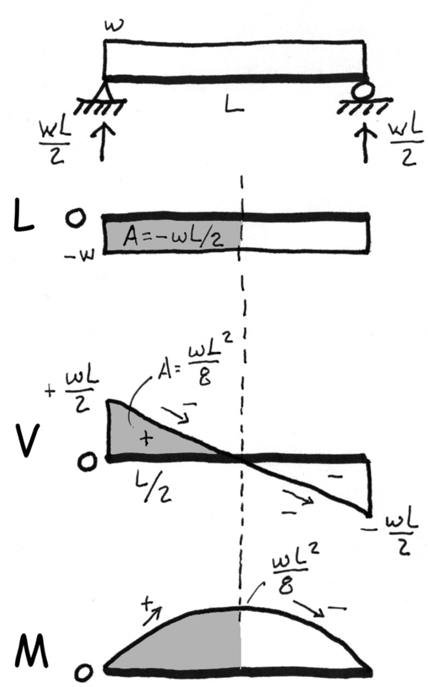


3. Shear and Moment by Semi-graphical Method – diagram relationships

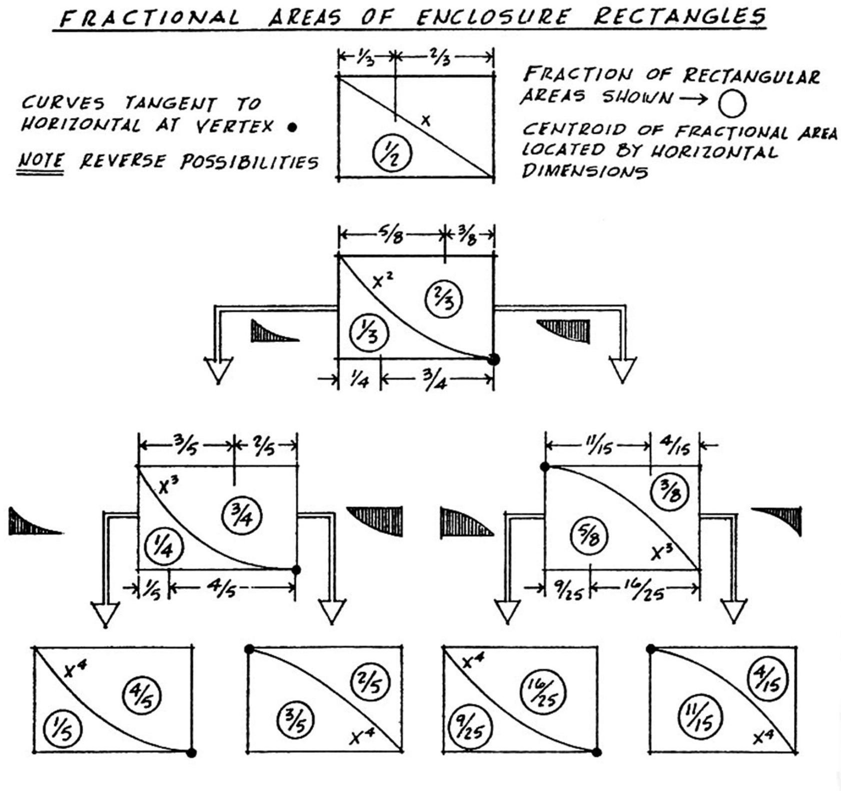
By recognizing the diagrammatic relationships between curves and their derivatives and integrals, shear and moment diagrams can be constructed based on areas and slopes of those curves.

Moving from Upper to Lower Diagrams:

- The area between any two points on the upper diagram is equal to the change in value between same points on the lower diagram.
- The degree of the curve increases by one for each diagram.
- The value on the upper diagram is equal to the slope of the lower diagram.
- Where the upper diagram crosses the 0 axis, the lower diagram is at a maximum or minimum.
- Points of inflection or "contraflexure" (between + and – curvature) on the elastic curve (deflected shape) are points of zero moment.



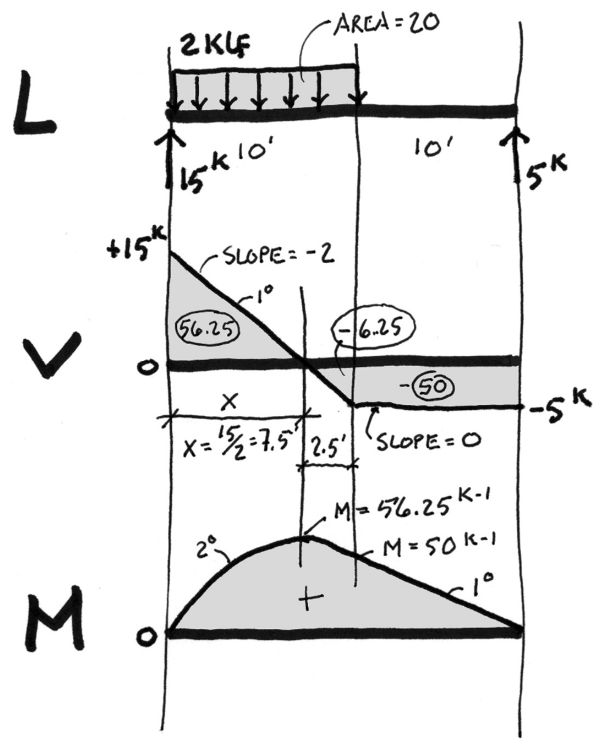
3. Semi-graphical Method



3. Semi-graphical Method

Procedure:

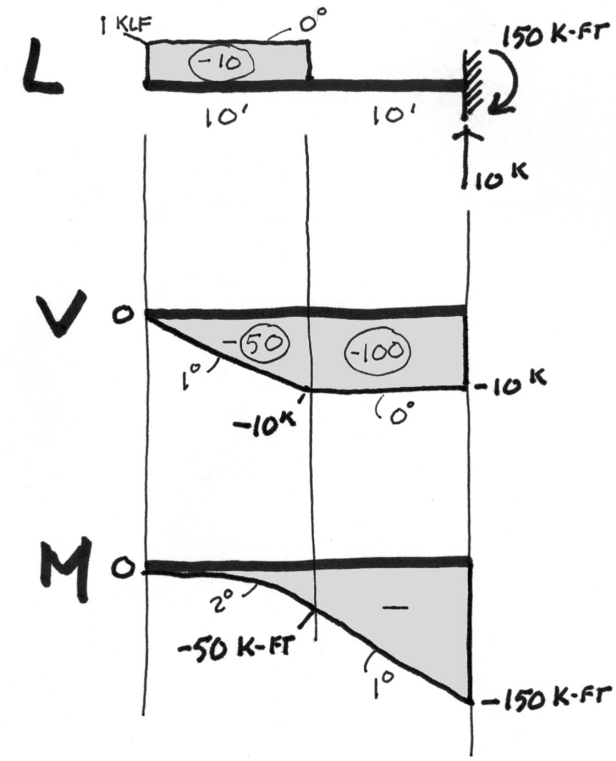
1. Find end reactions
2. Start at left end of V-Diagram and "apply" load from left to right
3. Calculate areas of V-Diagram
4. Find max. and min. values on M-Diagram using V-Diagram areas between axis crossings.
5. Check slope and + or - values



3. Semi-graphical Method

example

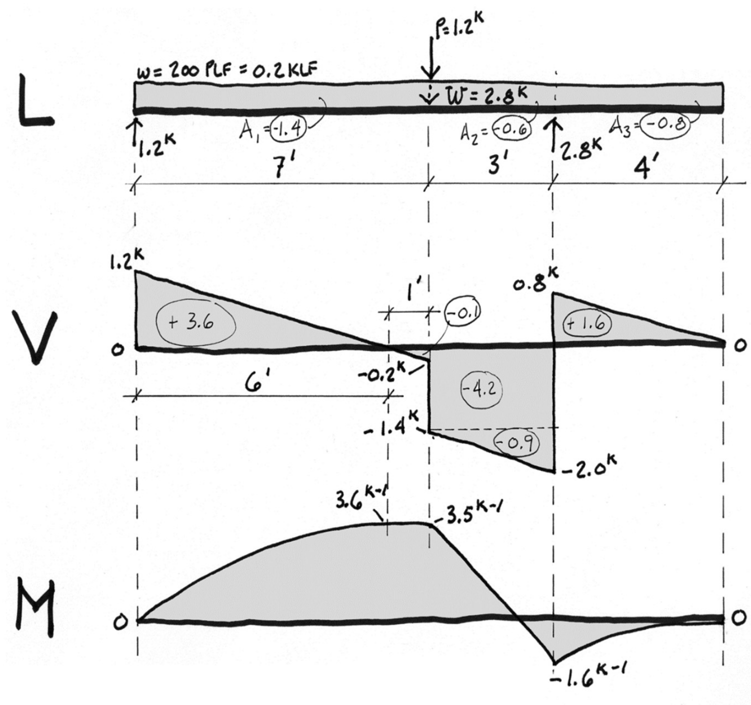
Cantilever Beam



3. Semi-graphical Method

example

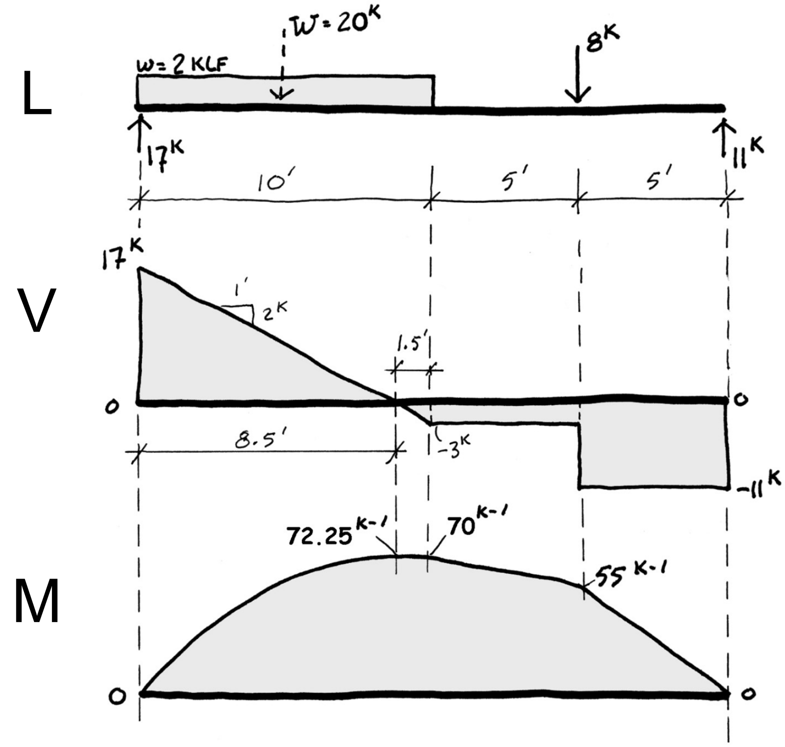
Beam with cantilever



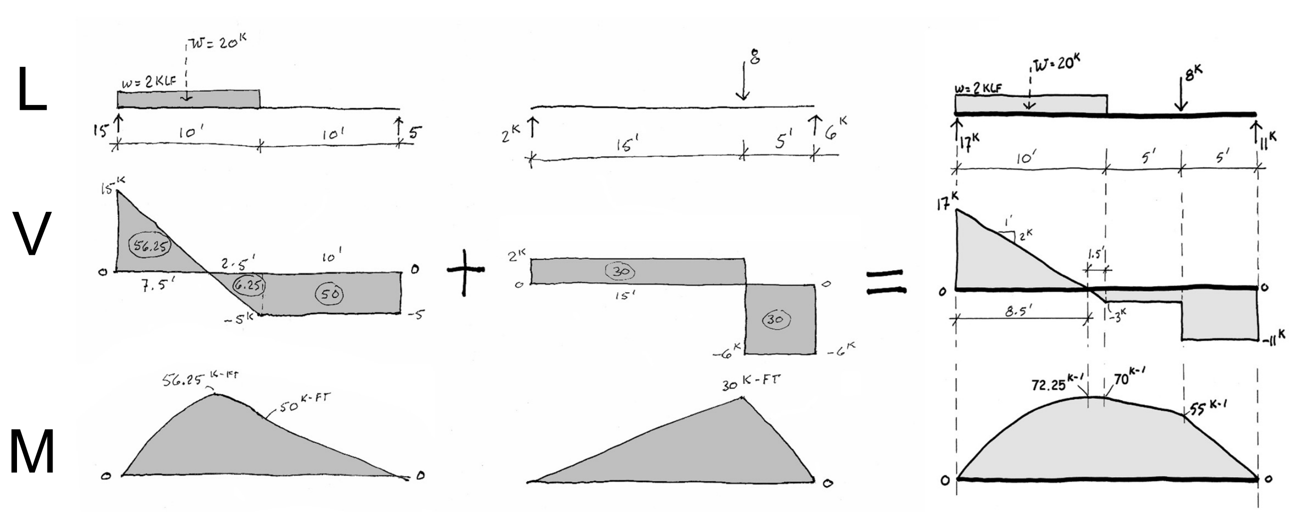
3. Semi-graphical Method

example

Simple beam



3. Semi-graphical Method - Superposition



Equations Method

For simple spans:

V_{max} is the larger reaction

For symmetric loadings:

M_{max} is at C.L.

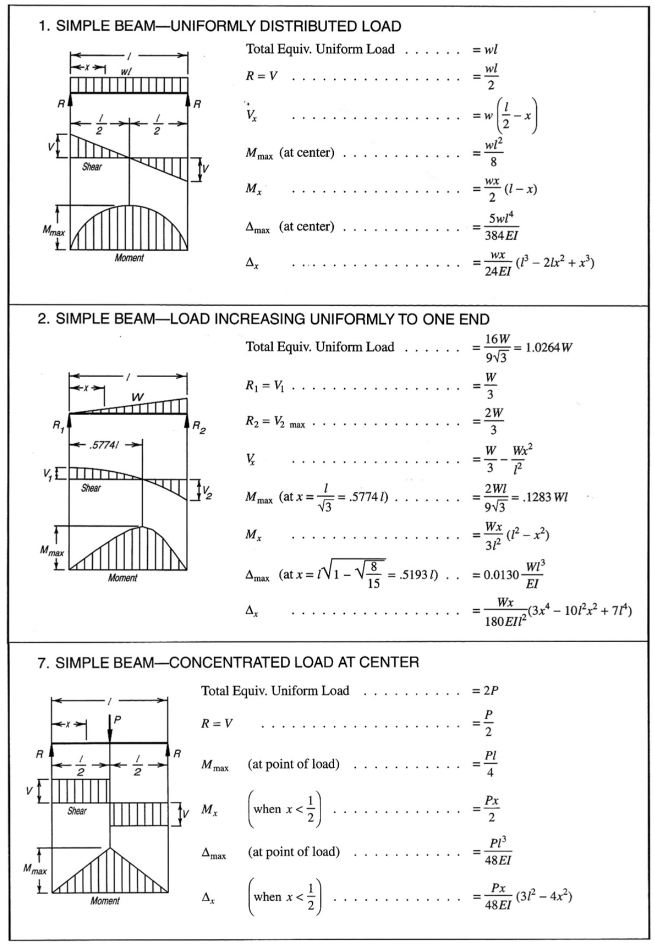
For cantilevers:

Both V_{max} and M_{max} are at the support

In these equations:

w = load per unit length (PLF or KLF)

W = the total load (LB or KIP)



1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load . . . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) . . . . $= \frac{5wl^4}{384EI}$

Δ_x . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

Total Equiv. Uniform Load . . . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_{2\ max}$ . . . . $= \frac{2W}{3}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

M_{max} (at $x = \frac{l}{\sqrt{3}} = .5774\,l$) . . . . $= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$

M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δ_{max} (at $x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193\,l$) . . $= 0.0130\frac{Wl^3}{EI}$

Δ_x . . . . $= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load . . . . $= 2P$

$R = V$ . . . . $= \frac{P}{2}$

M_{max} (at point of load) . . . . $= \frac{Pl}{4}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Px}{2}$

Δ_{max} (at point of load) . . . . $= \frac{Pl^3}{48EI}$

Δ_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Px}{48EI}(3l^2 - 4x^2)$

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4. Superposition of Equations

Equations of shear or moment may be combined (superimposed) for any number of cases.

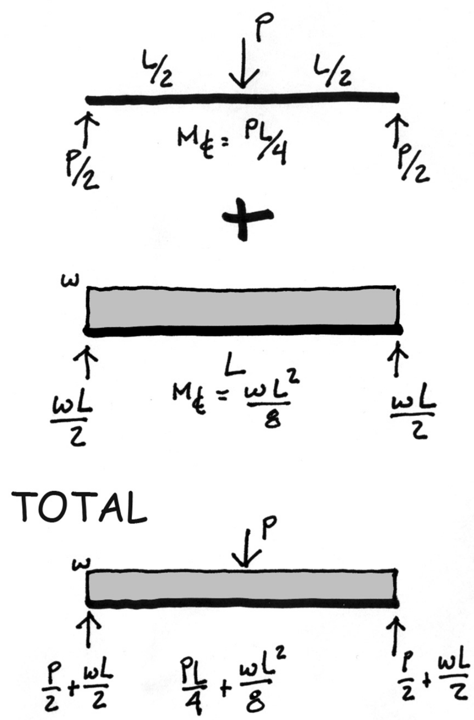
BUT

The appropriate location along the beam for which the equation is valid must be maintained

Thus

At the reaction, V = P/2 + wL/2

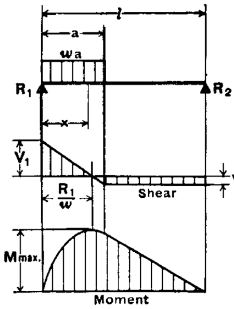
And at the C.L. M = PL/4 + wL^2/8



Non-symmetric

For more complex loads, care must be taken to combine equations at the same location or point on the beam (x).

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$

V_x (when $x < a$) . . . . . $= R_1 - wx$

M max. (at $x = \frac{R_1}{w}$) . . . . . . $= \frac{R_1^2}{2w}$

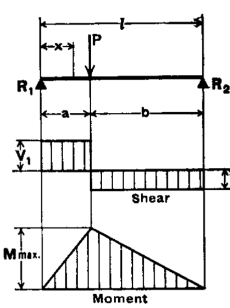
M_x (when $x < a$) . . . . . $= R_1 x - \frac{wx^2}{2}$

M_x (when $x > a$) . . . . . $= R_2(l - x)$

Δx (when $x < a$) . . . . . . $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δx (when $x > a$) . . . . . . $= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . . $= \frac{Pab}{l}$

M_x (when $x < a$) . . . . $= \frac{Pbx}{l}$

Δmax. (at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$) $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27\,EI\,l}$

Δa (at point of load) . . . . $= \frac{Pa^2b^2}{3EI\,l}$

Δx (when $x < a$) . . . . $= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

4. Superposition of Equations - example

find combined x for asymmetric cases

END REACTIONS (NOTE: VARIABLES DIFFER IN DIFFERENT EQUATIONS)

$$R_L = \frac{wa}{2l}(2l - a) + \frac{Pb}{l}$$

$$R_L = \frac{2(10)}{2(20)}(2(20) - 10) + \frac{8(5)}{20}$$

$$R_L = 15 + 2 = 17^K$$

MOMENT EQUATIONS AT x

$$M_x = R_1 x - \frac{wx^2}{2} + \frac{Pbx}{l}$$

TO FIND x: DIFFERENTIATE AND SOLVE AT 0 (M max)

$$R_1 - wx + \frac{Pb}{l}$$

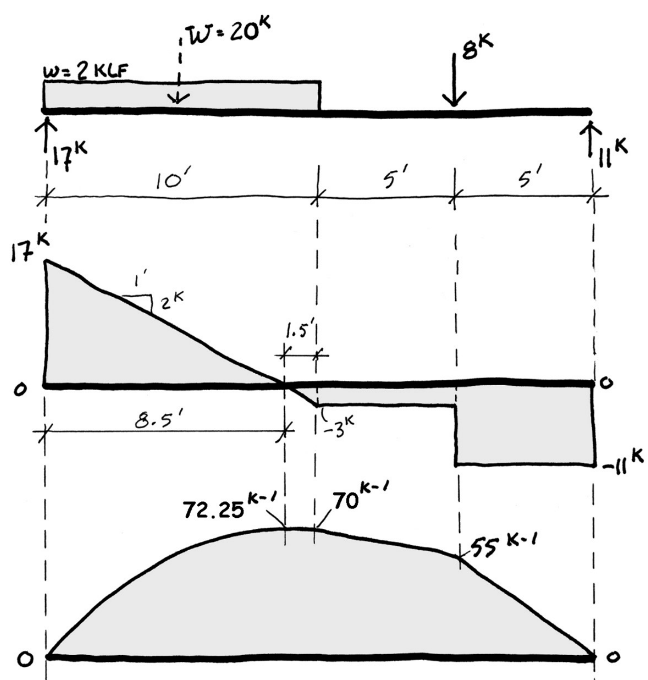
$$15 - 2x + 2 = 0$$

$$x = 8.5'$$

AND

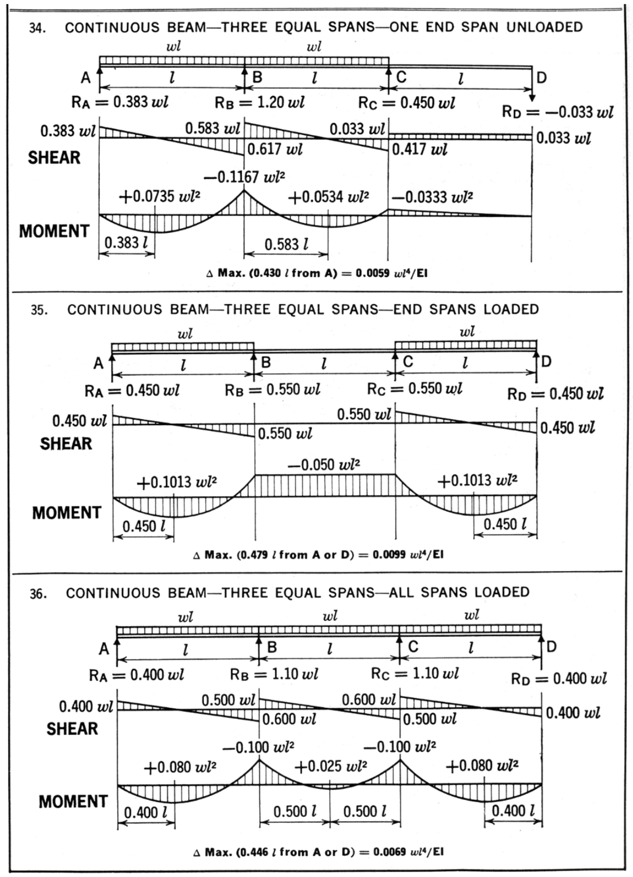
$$M_{max} = 15(8.5) - \frac{2(8.5)^2}{2} + \frac{8(5)(8.5)}{20}$$

$$M_{max} = 55.25 + 17 = 72.25^{K-'}$$



Simple vs. Continuous Beams

- Simple Beam
  - End moments = 0
  - Mmax at C.L = $wL^2/8 = 0.125wL^2$
- Continuous Beam
  - Exterior end moments = 0
  - Interior support moments are usually negative
  - Mid-span moments are usually positive
  - End + Mid = $0.125wL^2$



Note: moments shown reversed

Moment Diagram vs. Catenary Curve

For a gravity loaded simple span beam the shape of the of the moment diagram is the inverse of the catenary curve.