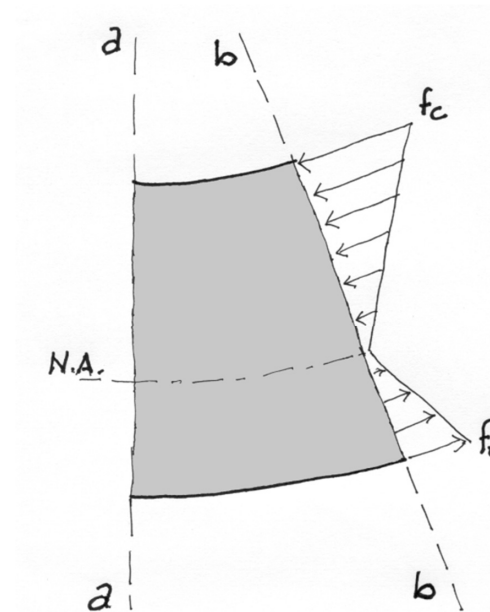


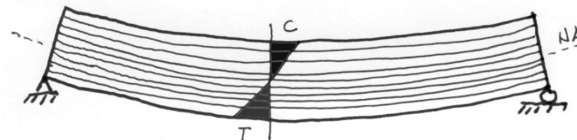
Bending Stresses in Beams

- Elastic Bending
- Stress Equation
- Section Modulus
- Flexure Capacity Analysis
- Flexure Beam Design

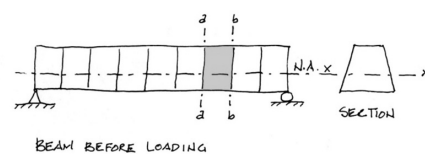


Elastic Bending

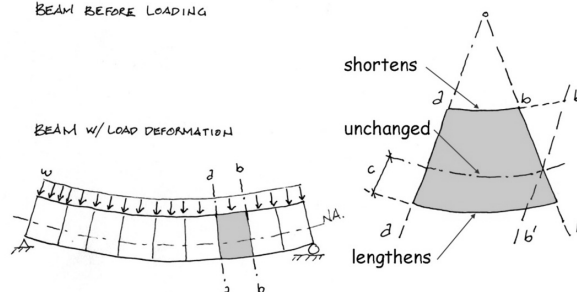
Flexure results in internal tension and compression forces, the resultants of which form a couple which resists the applied moment.



In the initial unloaded state, all transverse sections are parallel.



The application of load causes the member to bend in a curve. This means the initial parallel plane sections, while remaining plane, now follow the radii of the curves.

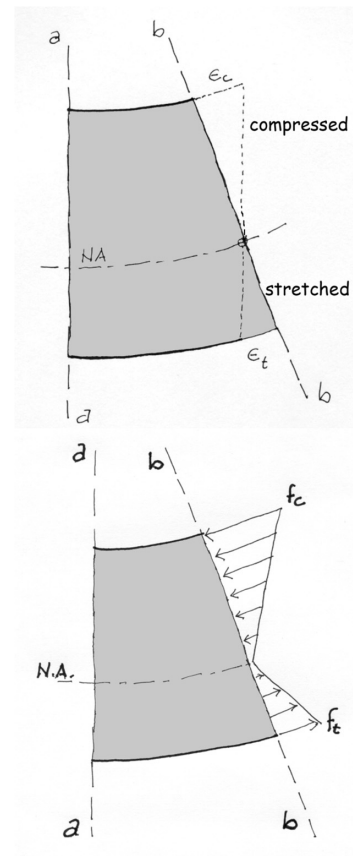


Notice that by the geometry of the curved member the top edge is shortened and the bottom edge is lengthened. Only the neutral axis remains its original length.

Elastic Bending

The change in lengths, top and bottom, results in the material straining. For a simple span with downward loading, the top is compressed and the bottom stretched. The change in length is linear and proportional to the distance from the Neutral Axis.

The material strains result in corresponding stresses. By **Hooke's Law**, these stresses are proportional to the strains which are proportional to the change in length of the radial arcs of the beam "fibers". This assumes that the Modulus of Elasticity is constant across the section.



Elastic Bending

The applied moment at any point on the beam is equal to the resisting moment which is formed by the internal force couple, R_c and R_t .

$$M_{\text{applied}} = M_{\text{resisting}}$$

Balance of the external and internal moments

$$R_{\text{comp.}} = R_{\text{tens.}}$$

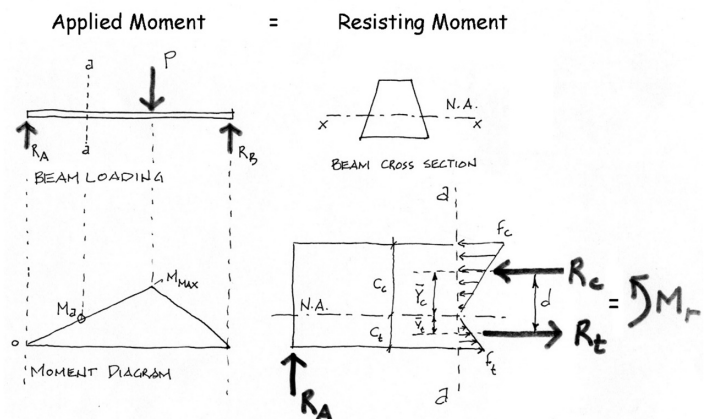
Balance of the internal force couple

$$M_r = R_c \cdot y_c + R_t \cdot y_t$$

$$M_r = R_c \cdot d$$

$$M_r = R_t \cdot d$$

Expressions of the internal resisting moment



Elastic Bending

The internal moment, M_r , can be expressed as the result of the couple R_c and R_t

$$M_r = R_c \cdot \bar{y}_1 + R_t \cdot \bar{y}_2$$

In turn, the forces R_c and R_t can be written as the resultants of the "stress volumes" acting through the centroids of those volumes. The stress volumes equal c times b times the average stress (s). The average unit stress, $s = fc/2$. The resultant R is the area times s :

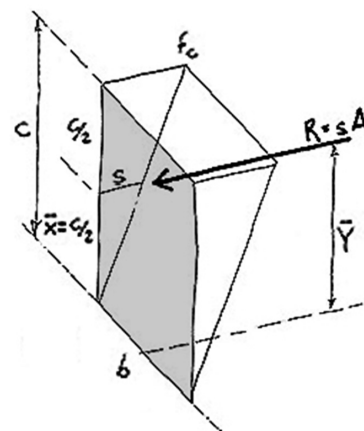
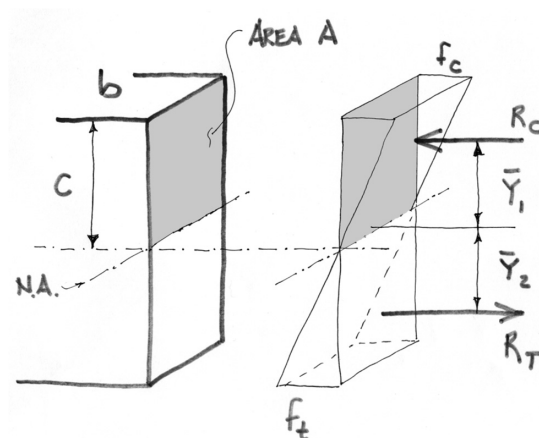
$$R = A \cdot s$$

Using similar triangles, s can be expressed as:

$$\frac{s}{f_c} = \frac{\bar{x}}{c} \quad \text{and} \quad s = \frac{f_c \cdot \bar{x}}{c}$$

Substituting these values back into the moment equation gives:

$$M_r = \frac{f_c A_c \bar{x}_1 \bar{y}_1}{c_c} + \frac{f_t A_t \bar{x}_2 \bar{y}_2}{c_t}$$



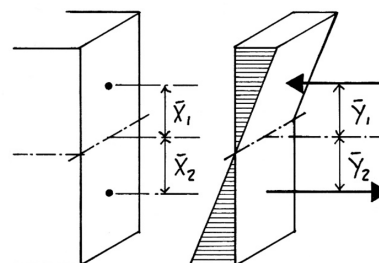
Elastic Bending

By definition:

$$I_x = A \bar{x} \bar{y}$$

And for homogeneous materials with $E_c = E_t$

$$M_r = \frac{f I_1}{c} + \frac{f I_2}{c} = \frac{f}{c} (I_1 + I_2)$$



Or using the I for the whole section:

$$M_r = \frac{f I}{c}$$

And so,

$$f = \frac{M c}{I}$$

The Section Modulus is:

$$S = \frac{I}{c}$$

With $c = h/2$ at extreme fibers of a symmetric section.

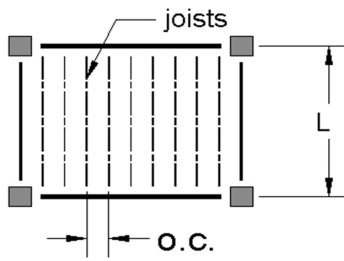
So, at extreme fibers:

$$M = f S$$

And:

$$f = \frac{M}{S}$$

Beam Analysis



Allowable Capacity (ASD):

$$M = F_b S$$

for steel: $F_b = (0.66 \text{ to } 0.6) F_y$ ksi

for wood: $F_b = 1000 \text{ to } 600$ psi

Applied Load:

$$M = \frac{wl^2}{8} \quad (\text{uniform load})$$

Pass

$$M = F_b S > M = \frac{wl^2}{8}$$

Fail

$$M = F_b S < M = \frac{wl^2}{8}$$

Capacity

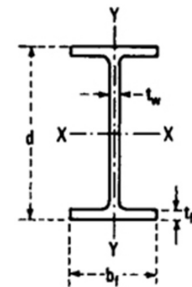
$$M = F_b S = M = \frac{wl^2}{8} \quad \text{solve for } w$$

Design

$$M = \frac{wl^2}{8} = M = F_b S \quad \text{solve for } S$$

Beam Capacity Analysis - procedure

1. Determine section properties. (from table)
2. Choose safe allowable stress. (depends on bracing)
3. Calculate allowable moment capacity. $M = F_b S$
4. Set equal to applied moment and find load. $M = \frac{wl^2}{8}$



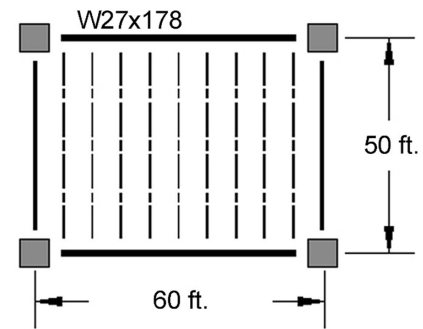
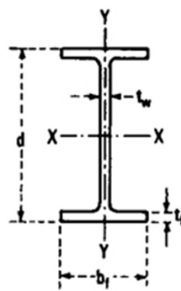
WIDE FLANGE SHAPES

Section Number	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thick-ness	Axis X-X			Axis Y-Y			r_T
				Width	Thick-ness		I_x	S_x	r_x	I_y	S_y	r_y	
lb	in. ²	in.	in.	in.	in.	in. ⁴	in. ³	in.	in. ⁴	in. ³	in.	in.	
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Beam Capacity Analysis - example

Given:

Beam = W27x178
 $S_x = 502 \text{ in}^3$
 $F_y = 50 \text{ ksi}$
 $F_b = .66F_y = 33 \text{ ksi}$ (braced by joists)



Find:

Floor capacity

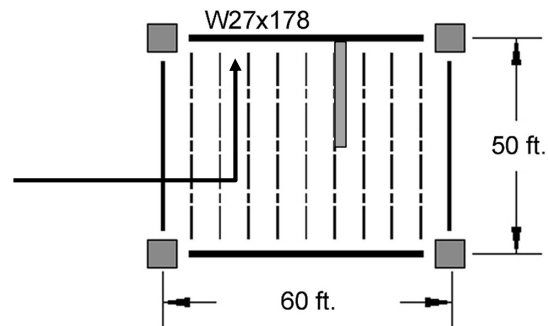
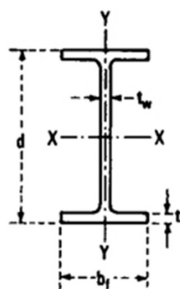
WIDE FLANGE SHAPES

Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange		Web Thickness t _w	Axis X-X			Axis Y-Y			r _T
				Width b _f	Thick-ness t _f		I _x	S _x	r _x	I _y	S _y	r _y	
	lb	in. ²	in.	in.	in.	in.	in. ⁴	in. ³	in.	in. ⁴	in. ³	in.	in.
W27 x 178	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
161		47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
146		42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x 114	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
102		30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
94		27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
84		24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Beam Capacity Analysis

Given:

Beam = W27x178
 $S_x = 502 \text{ in}^3$
 $F_y = 50 \text{ ksi}$
 $F_b = .66F_y = 33 \text{ ksi}$ (fully braced)



Find:

Floor capacity

$$M = F_b S_x$$

$$M = 33 \text{ ksi} \cdot 502 \text{ in}^3 = 16566 \text{ K-in} = 1380.5 \text{ K-ft}$$

$$M = 1380.5 \text{ K-ft}$$

$$M = \frac{w l^2}{8}$$

$$w = \frac{M \cdot 8}{l^2} = \frac{1380.5 (8)}{60^2} = 3.068 \text{ k/ft} = 3068 \text{ #/ft}$$

$$\text{PSF} = \frac{w}{l/2} = \frac{3068}{50/2} = \underline{\underline{123 \text{ PSF}}}$$

Quiz

Given:

Beam = W27x114

$F_y = 36$ ksi

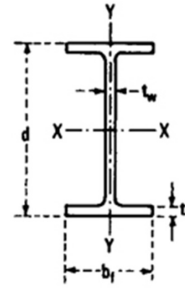
F.S. = 0.6

$S_x = \underline{\hspace{2cm}}$ in³

$F_b = .6F_y = \underline{\hspace{2cm}}$ ksi

Find:

Allowable Moment = ft-lbs



WIDE FLANGE SHAPES

Section Number	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X-X			Axis Y-Y			r_T
				Width	Thick-ness		I_x	S_x	r_x	I_y	S_y	r_y	
				b_f	t_f		in. ⁴	in. ³	in.	in. ⁴	in. ³	in.	
W27 x 178	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
161		47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
146		42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x 114	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
102		30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
94		27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
84		24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Section Properties

Section Modulus Table

Sorted by S_x for design selection

with:

$$S = I/c$$

f_b is actual stress

F_b is allowable stress

- for bracing $< L_c$, $F_b = 0.66F_y$
- for bracing $< L_u$, $F_b = 0.6F_y$

F_y is the yield stress

$$M_r = .66F_y S_x$$

So the design equations is:

$$S_x = M_{\text{applied}} / F_b$$

ALLOWABLE STRESS DESIGN SELECTION TABLE For shapes used as beams										S_x	
$F_y = 50$ ksi			S_x	Shape	Depth d	F_y	$F_y = 36$ ksi			L_c	L_u
L_c	L_u	M_R					L_c	L_u	M_R		
ft	ft	kip-ft	in. ³		in.	ksi	ft	ft	kip-ft		
10.6	11.2	2130	776	W 44x198	42%	—	12.5	15.5	1540		
14.1	15.2	2110	769	W 40x199	38%	—	16.6	20.0	1520		
11.8	45.7	2110	769	W 21x333	25	—	13.9	63.4	1520		
14.2	19.8	2080	757	W 33x221	33%	—	16.7	27.6	1500		
13.5	24.0	2050	746	W 30x235	31%	—	15.9	33.3	1480		
12.8	29.0	2040	742	W 27x258	29	—	15.1	40.3	1470		
10.9	15.1	1980	719	W 36x210	36%	—	12.9	20.9	1420		
11.9	34.7	1970	718	W 24x279	26%	—	14.0	48.2	1420		
12.8	16.7	1880	708	W 40x192	38%	37.1	17.8	19.7	1400		
11.6	42.7	1900	692	W 21x300	24%	—	13.7	59.4	1370		
14.1	17.9	1880	684	W 33x201	33%	—	16.6	24.9	1350		
10.6	12.3	1880	682	W 40x183	39	—	12.5	17.1	1350		
12.7	26.7	1850	674	W 27x235	28%	—	15.0	37.0	1330		
10.9	13.9	1830	664	W 36x194	36%	—	12.8	19.4	1310		
13.5	21.4	1820	663	W 30x211	31	—	15.9	29.7	1310		
11.8	31.4	1770	644	W 24x250	26%	—	13.9	43.7	1280		
11.5	39.2	1740	632	W 21x275	24%	—	13.6	54.5	1250		
12.6	24.9	1720	624	W 27x217	28%	—	14.9	34.5	1240		
10.8	49.0	1720	624	W 18x311	22%	—	12.7	68.1	1240		
10.8	13.1	1710	623	W 36x182	36%	—	12.7	18.2	1230		
10.4	11.0	1650	599	W 40x167	38%	—	12.5	14.5	1190		
13.5	19.4	1640	598	W 30x191	30%	—	15.9	26.9	1180		
11.7	29.0	1620	588	W 24x229	26	—	13.8	40.3	1160		
10.8	12.2	1600	580	W 36x170	36%	—	12.7	17.0	1150		
11.4	35.5	1560	569	W 21x248	23%	—	13.5	49.3	1130		
10.6	45.0	1550	564	W 18x283	21%	—	12.6	62.6	1120		
12.6	22.4	1530	556	W 27x194	28%	—	14.8	31.1	1100		
10.3	13.8	1510	549	W 33x169	33%	—	12.1	19.2	1090		
10.7	11.4	1490	542	W 36x160	36	—	12.7	15.7	1070		
13.4	17.5	1480	539	W 30x173	30%	—	15.8	24.2	1070		
11.7	26.5	1460	531	W 24x207	25%	—	13.7	36.7	1050		
10.5	42.2	1410	514	W 18x258	21%	—	12.4	58.6	1020		
8.5	10.7	1410	512	W 40x149	38%	—	11.9	12.8	1010		
11.4	32.7	1400	510	W 21x223	23%	—	13.4	45.4	1010		
10.5	11.3	1390	504	W 36x150	35%	—	12.6	14.6	998		
12.6	20.1	1380	502	W 27x178	27%	—	14.9	27.9	994		
11.6	24.7	1350	491	W 24x192	25%	—	13.7	34.3	972		
10.4	12.2	1340	487	W 33x152	33%	—	12.2	16.9	964		
10.4	38.8	1280	466	W 18x234	21	—	12.3	53.8	923		
11.3	29.8	1270	461	W 21x201	23	—	13.3	41.3	913		
12.6	18.3	1250	455	W 27x161	27%	—	14.8	25.4	901		
11.5	22.8	1240	450	W 24x176	25%	—	13.6	31.7	891		

Beam Design - procedure

1. Choose a steel grade and allowable stress.
2. Determine the applied moment (e.g. moment diagram)
3. Calculate the section modulus, S_x
4. Choose a safe section. (from S_x table)

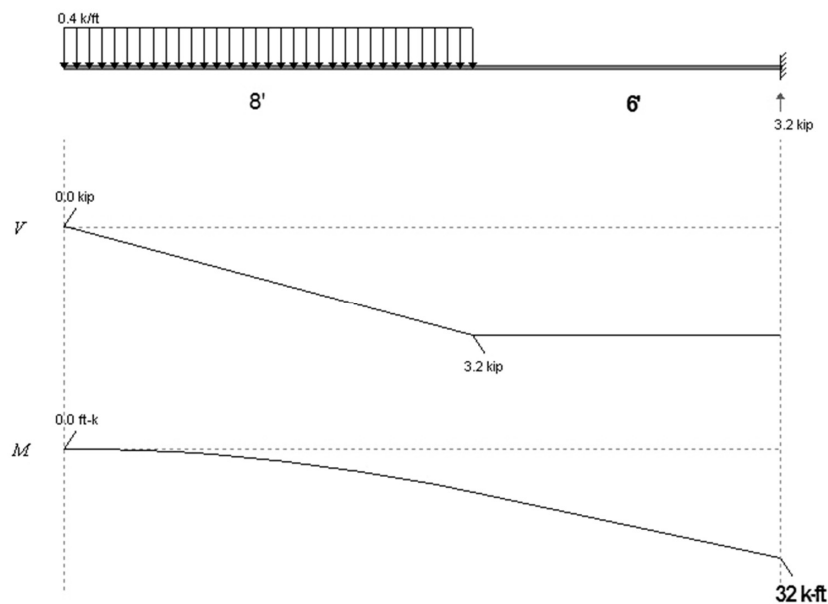
$$S_x = \frac{M}{F_b}$$

ALLOWABLE STRESS DESIGN SELECTION TABLE For shapes used as beams									
$F_y = 50$ ksi			S_x	Shape	Depth d	F_y	$F_y = 36$ ksi		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	In ³		In	Ksi	Ft	Ft	Kip-ft
2.9	3.6	47	17.1	W 12×16	12	—	4.1	4.3	34
5.4	14.4	46	16.7	W 6×25	6 ³ / ₈	—	6.4	20.0	33
3.6	4.4	45	16.2	W 10×17	10 ¹ / ₈	—	4.2	6.1	32
4.7	7.1	42	15.2	W 8×18	8 ¹ / ₈	—	5.5	9.9	30
2.5	3.6	41	14.9	W 12×14	11 ⁷ / ₈	54.3	3.5	4.2	30
3.6	3.7	38	13.8	W 10×15	10	—	4.2	5.0	27
5.4	11.8	37	13.4	W 6×20	6 ¹ / ₄	62.1	6.4	16.4	27
5.3	12.5	36	13.0	M 6×20	6	—	6.3	17.4	26
1.9	2.6	33	12.0	M 12×11.8	12	—	2.7	3.0	24
3.6	5.2	32	11.8	W 8×15	8 ¹ / ₈	—	4.2	7.2	23
2.8	3.6	30	10.9	W 10×12	9 ⁷ / ₈	47.5	3.9	4.3	22

Beam Design - steel

Using **Steel W section**:

1. Choose a steel grade: Using $F_y = 50$ ksi $F_b = 0.6 F_y$
2. Determine the applied moment



Beam Design – steel

Using Steel W section:

2. Calculate section modulus, S_x

$$S_x = \frac{M}{F_b}$$

$$S_x = \frac{M}{F_b} = \frac{32 \text{ k} \cdot (12)}{0.6 (50 \text{ ksi})}$$

$$S_x = 12.8 \text{ in}^3$$

Beam Design – steel

Using Steel W section:

3. Choose a safe section. (from S_x table)

$$S_x \geq 12.8 \text{ in}^3$$

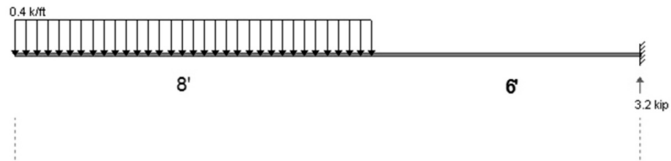
W12x14 is the lightest section with $S_x > 12.8$

ALLOWABLE STRESS DESIGN SELECTION TABLE For shapes used as beams									
$F_y = 50 \text{ ksi}$			S_x	Shape	Depth d	F_y	$F_y = 36 \text{ ksi}$		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	in ³		in	Ksi	Ft	Ft	Kip-ft
2.9	3.6	47	17.1	W 12×16	12	—	4.1	4.3	34
5.4	14.4	46	16.7	W 6×25	6¾	—	6.4	20.0	33
3.6	4.4	45	16.2	W 10×17	10½	—	4.2	6.1	32
4.7	7.1	42	15.2	W 8×18	8½	—	5.5	9.9	30
2.5	3.6	41	14.9	W 12×14	11¾	54.3	3.5	4.2	30
3.6	3.7	38	13.8	W 10×15	10	—	4.2	5.0	27
5.4	11.8	37	13.4	W 6×20	6¼	62.1	6.4	16.4	27
5.3	12.5	36	13.0	M 6×20	6	—	6.3	17.4	26
1.9	2.6	33	12.0	M 12×11.8	12	—	2.7	3.0	24
3.6	5.2	32	11.8	W 8×15	8½	—	4.2	7.2	23
2.8	3.6	30	10.9	W 10×12	9¾	47.5	3.9	4.3	22

Beam Design – Glulam

Using **Glulam Timber**:

$F_b = 1250$ psi (DF grade L3)



$$S_x = \frac{M}{F_b}$$

$$S_x = \frac{M_{APPLIED}}{F_b} = \frac{32000 \text{ ft-lb} (12)}{1250 \text{ psi}} = 307.2 \text{ in}^3$$

Table 5B Reference Design Values for Structural Glued Laminated Softwood Timber

(Members stressed primarily in axial tension or compression) (Tabulated design values are for normal load duration and dry service conditions. See NDS 5.3 for a comprehensive description of design value adjustment factors.)

Use with Table 5B Adjustment Factors

Combination Symbol	Species	Grade	All Loading			Axially Loaded			Bending about Y-Y Axis				Bending About X-X Axis		Fasteners
			Modulus of Elasticity		Compression Perpendicular to Grain	Tension Parallel to Grain	Compression Parallel to Grain		Loaded Parallel to Wide Faces of Laminations			Loaded Perpendicular to Wide Faces of Laminations			
			For Deflection Calculations	For Stability Calculations			Bending			Shear Parallel to Grain ⁽¹⁾⁽²⁾⁽³⁾	Bending	Shear Parallel to Grain ⁽³⁾			
							2 or More Lami- nations	4 or More Lami- nations	2 or 3 Lami- nations				4 or More Lami- nations	3 Lami- nations	2 Lami- nations
			E (10 ⁶ psi)	E _{min} (10 ⁶ psi)	F _{c⊥} (psi)	Specific Gravity for Fastener Design G									
Visually Graded Western Species															
1	DF	L3	1.5	0.79	560	950	1550	1250	1450	1250	1000	230	1250	265	0.50
2	DF	L2	1.6	0.85	560	1250	1950	1600	1800	1600	1300	230	1250	265	0.50
3	DF	L2D	1.9	1.00	650	1450	2300	1900	2100	1850	1550	230	2000	265	0.50
4	DF	L1CL	1.9	1.00	590	1400	2100	1950	2200	2000	1650	230	2100	265	0.50
5	DF	L1	2.0	1.06	650	1650	2400	2100	2400	2100	1800	230	2200	265	0.50

Section Properties

Using Glulam Timber:

Glulam Timbers – 8 3/4" wide

S_x required = 307.2 in³

Use 8 3/4" x 15" $S_x = 328.1 > 307.2$ in³

Table 1C Section Properties of Western Species Structural Glued Laminated Timber (Cont.)

Depth	Area	X-X Axis			Y-Y Axis	
d (in.)	A (in. ²)	I _x (in. ⁴)	S _x (in. ³)	r _x (in.)	I _y (in. ⁴)	S _y (in. ³)
8-3/4 in. Width					(r _y = 2.526 in.)	
9	78.75	531.6	118.1	2.598	502.4	114.8
10-1/2	91.88	844.1	160.8	3.031	586.2	134.0
12	105.0	1260	210.0	3.464	669.9	153.1
13-1/2	118.1	1794	265.8	3.897	753.7	172.3
15	131.3	2461	328.1	4.330	837.4	191.4
16-1/2	144.4	3276	397.0	4.763	921.1	210.5
18	157.5	4253	472.5	5.196	1005	229.7
19-1/2	170.6	5407	554.5	5.629	1089	248.8
21	183.8	6753	643.1	6.062	1172	268.0

Section Properties

PROPERTIES OF SAWN LUMBER SECTIONS



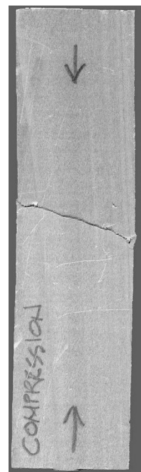
Sawn Lumber

Nominal Size b × d	Actual Size b × d	Area in. ²	I_x in. ⁴	S_x in. ³
1 × 4	3/4 × 3 1/2	2.63	2.68	1.53
1 × 6	" × 5 1/2	4.13	10.40	3.78
1 × 8	" × 7 1/4	5.44	23.82	6.57
1 × 10	" × 9 1/4	6.94	49.47	10.70
1 × 12	" × 11 1/4	8.44	88.99	15.83
2 × 4	1 1/2 × 3 1/2	5.25	5.36	3.06
2 × 6	" × 5 1/2	8.25	20.80	7.56
2 × 8	" × 7 1/4	10.88	47.64	13.14
2 × 10	" × 9 1/4	13.88	98.93	21.39
2 × 12	" × 11 1/4	16.88	177.98	31.64
3 × 4	2 1/2 × 3 1/2	8.75	8.93	5.10
3 × 6	" × 5 1/2	13.75	34.66	12.60
3 × 8	" × 7 1/4	18.13	79.39	21.90
3 × 10	" × 9 1/4	23.13	164.89	35.65
3 × 12	" × 11 1/4	28.13	296.63	52.73
4 × 4	3 1/2 × 3 1/2	12.25	12.50	7.15
4 × 6	" × 5 1/2	19.25	48.53	17.65
4 × 8	" × 7 1/4	25.38	111.15	30.66
4 × 10	" × 9 1/4	32.38	230.84	49.91
4 × 12	" × 11 1/4	39.38	415.28	73.83

Modes of Failure

Strength

- Tension rupture
- Compression crushing
- Flexure
- Shear



Stability

- Column buckling
- Beam lateral torsional buckling

Serviceability

- Beam deflection
- Building story drift
- cracking

