

Plane Trusses Method of Joints

Definition and Assumptions
Nomenclature
Stability and Determinacy
Analysis by joints



Phaeodaria – Ernst Haeckel

Definitions and Assumptions of Truss Systems

2 Force Members

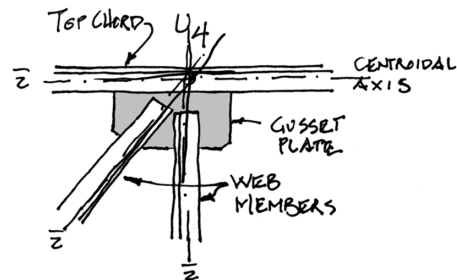
Pinned Joints

Concurrent Member Centroids at Joints

Joint Loaded

Straight Members

Small Deflections



Bullring Covering, Xàtiva, Spain
Kawaguchi and Engineers, 2007

Nomenclature

Panels

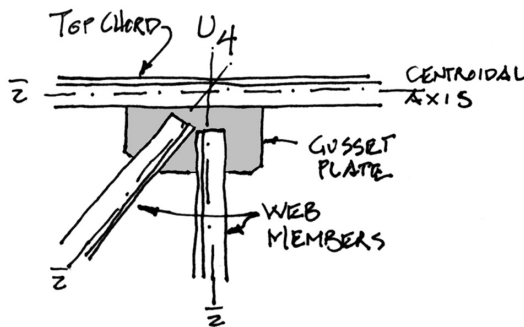
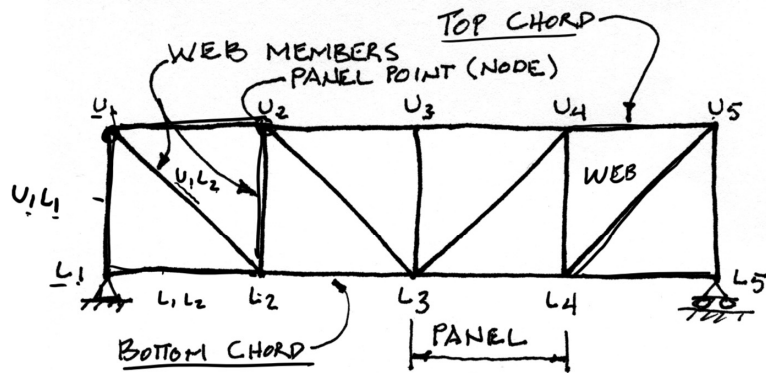
- Segments: left to right

Joints

- Upper: $U_1, U_2, U_3, \dots$
- Lower: $L_1, L_2, L_3, \dots$

Members

- Chords ✓
- Web



Trussed Force Systems

2D Trusses

- Concurrent Coplanar

3D Trusses

- Concurrent Non-Coplanar



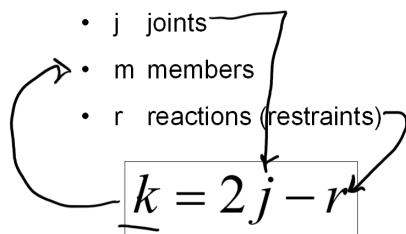
Foster Bridge, 1889
Ann Arbor, Michigan



University of Michigan Architectural Research Lab
Unistrut System, Charles W. Attwood

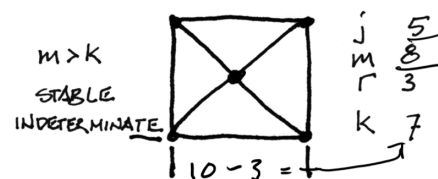
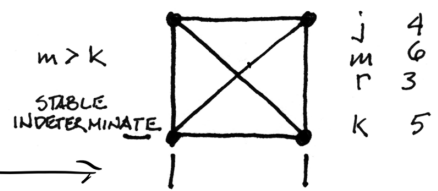
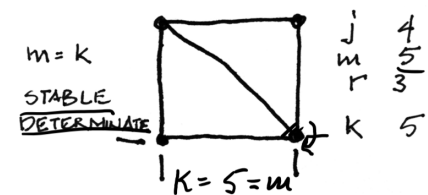
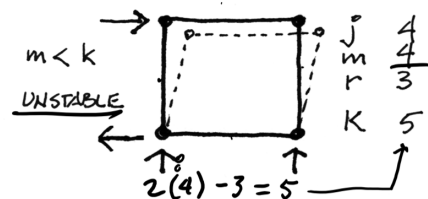
Stability and Determinacy of 2D Trusses

For:



Three conditions

- $m < k$ unstable
- $m = k$ stable and determinate
- $m > k$ stable and indeterminate



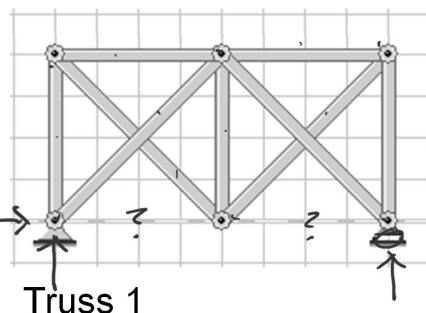
SIMPLE SUPPORTS

Quiz

For each of the following trusses, determine whether they are:

- A) Stable
- B) Unstable

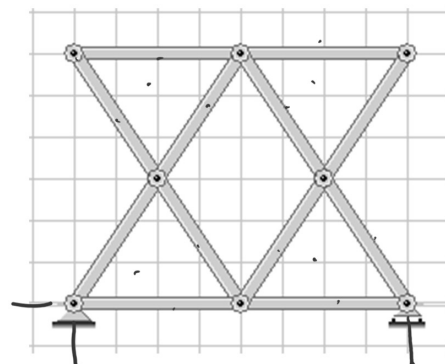
$$2(6) - 3 = 9$$



$$k = 2j - r$$

- $m < k$ unstable
 - $m = k$ stable and determinate
 - $m > k$ stable and indeterminate
- $12 < 13$

$$2(8) - 3 = 13$$

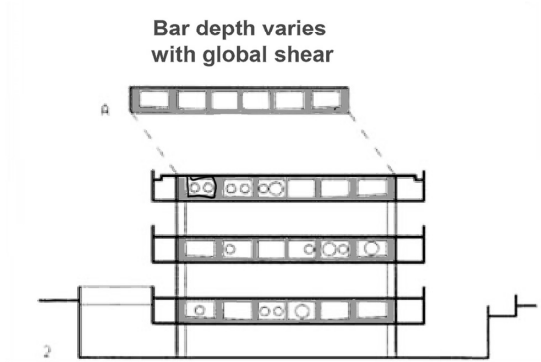


$$2(6) - 3 = 9 = k$$

Vierendeel “Truss”

Not a true truss

Moment frame structure
Rigid joints as moment connections
Flexure in members



Salk Institute, La Jolla.
Architect: Louis Kahn
Engineer: Komendant and Dubin



Vierendeel bridge
at Grammene, Belgium
Photo by Karel Roose

Truss Analysis

Method of Joints ✓

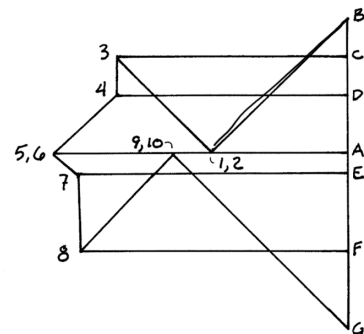
Method of Sections ✓

Graphic Methods

James Clerk Maxwell 1869
M. Williot 1877
Otto Mohr 1887
Heinrich Müller-Breslau 1904
William Baker, SOM

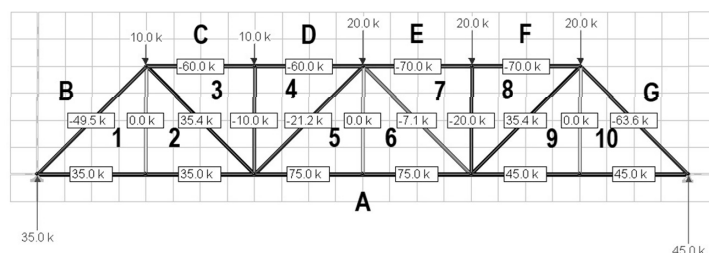


James Clerk Maxwell



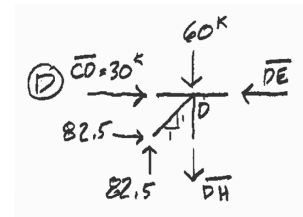
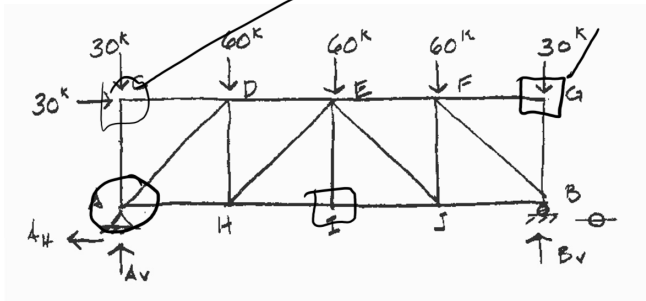
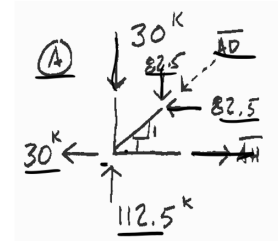
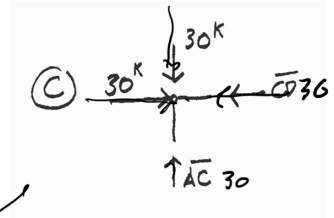
Computer Programs

Dr. Frame (2D) ✓
STAAD Pro (2D or 3D) ✓
West Point Bridge Designer



Method of Joints – procedure

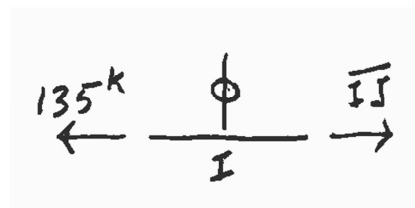
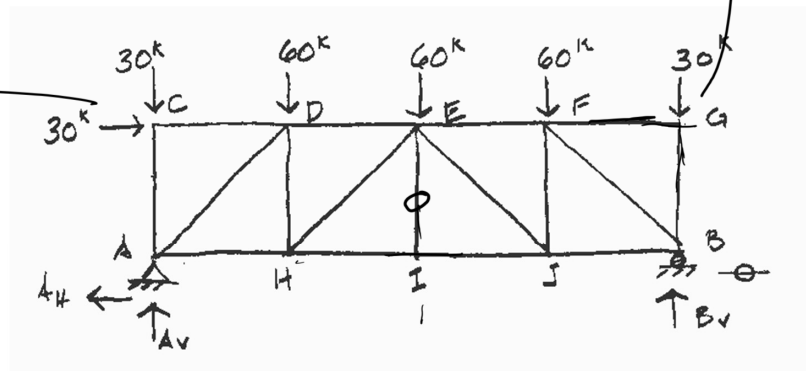
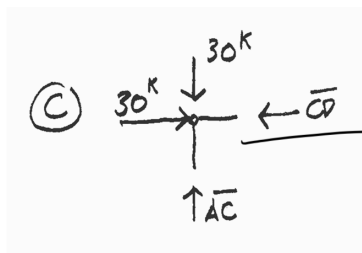
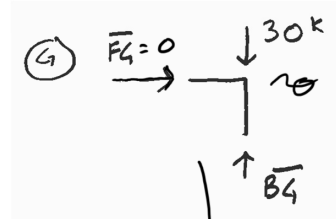
1. Solve reactions (all external forces)
2. T & L Joints by Inspection
3. Cut FBD of one joint
4. Show forces as orthogonal components
5. Solve with ΣF_H and ΣF_V (no ΣM)
6. Find resultant member forces (Pythagorean Formula)



T & L Joints by Inspection

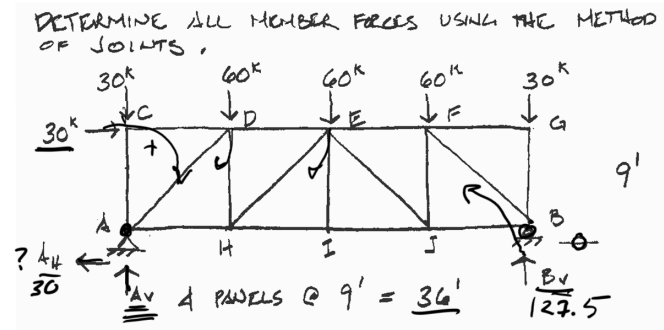
T – joints

L – joints



Method of Joints - example

1. Solve the external reactions for the whole truss.



REACTIONS :

$$\sum F_H = 0 = 30 - A_H \quad A_H = 30^k \leftarrow$$

$$\underline{\Sigma McA} = 0 = 30(9) + \underline{60(9)} + \underline{60(18)} + \underline{60(27)}$$

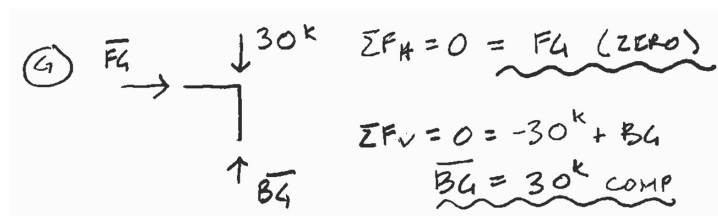
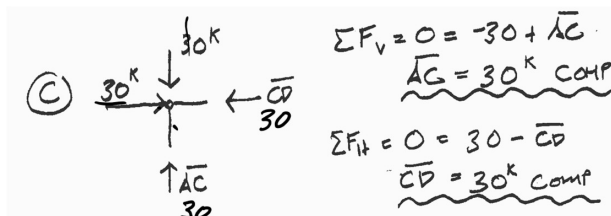
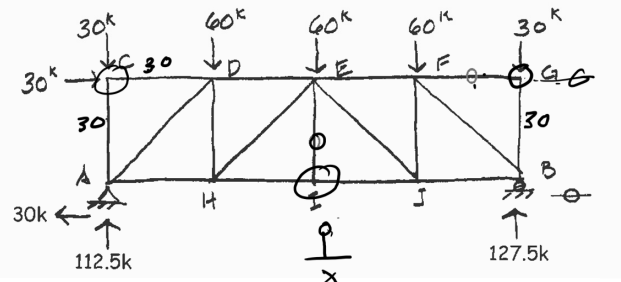
$$\begin{aligned} B_v(36) &= 4590 \\ B_v &= 127.5^{\text{K}} \uparrow \end{aligned}$$

$$\sum F_v = 0 \Rightarrow A_v + 127.5 - 2(30) - 3(60) = 0$$

$$A_v = 112.5 \text{ k} \uparrow$$

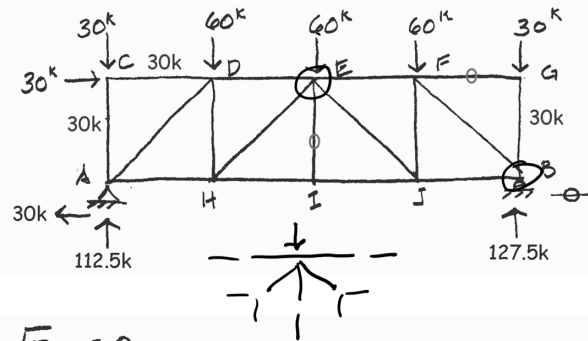
Method of Joints - example

2. T or L joints by inspection.
3. Cut FBD of joint
4. Show orthogonal components
5. Solve by ΣF horz. and vert.



Method of Joints - example

Continue with joints having only one unknown in either horizontal or vertical direction. Generally, work starting at the reactions and move inwards.



①

$$\sum F_v = -30 + 112.5 - \overline{AD}_v = 0$$

$$\overline{AD}_v = 82.5 \text{ k}$$

$$\overline{AD}_h = \overline{AD}_v = 82.5 \text{ k}$$

$$\overline{AD} = \sqrt{82.5^2 + 82.5^2} = 116.67 \text{ k comp}$$

$$\sum F_h = 0 = -30 - 82.5 + \overline{AH}$$

$$\overline{AH} = 112.5 \text{ k TENSION}$$

②

$$\sum F_v = 0 = -30 + 127.5 - \overline{BF}_v$$

$$\overline{BF}_v = 97.5 \text{ k}$$

$$\overline{BF}_h = \overline{BF}_v = 97.5 \text{ k}$$

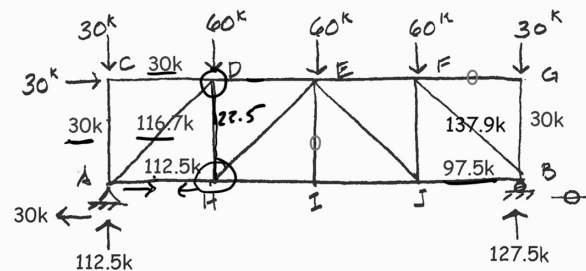
$$\overline{BF} = \sqrt{97.5^2 + 97.5^2} = 137.9 \text{ k comp}$$

$$\sum F_h = 0 = 97.5 - \overline{BJ}$$

$$\overline{BJ} = 97.5 \text{ k TENSION}$$

Method of Joints - example

Continue moving across the truss, joint by joint. Solve by $\sum F_h$ and $\sum F_v$.



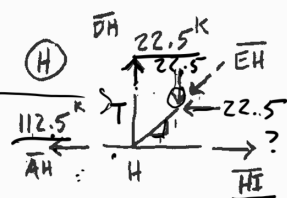
③

$$\sum F_h = 30 + 82.5 - \overline{DE} = 0$$

$$\overline{DE} = 112.5 \text{ k comp}$$

$$\sum F_v = 82.5 - 60 - \overline{DH} = 0$$

$$\overline{DH} = 22.5 \text{ k TENSION}$$



$$\sum F_v = 0 = 22.5 - \overline{EH}_v$$

$$\overline{EH}_v = 22.5$$

$$\overline{EH}_h = \overline{EH}_v = 22.5$$

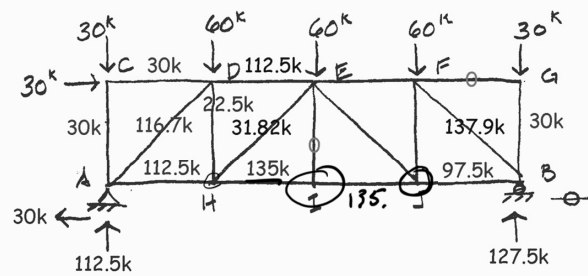
$$\overline{EH} = \sqrt{22.5^2 + 22.5^2} = 31.82 \text{ k comp}$$

$$\sum F_h = 0 = -112.5 - 22.5 + \overline{HI}$$

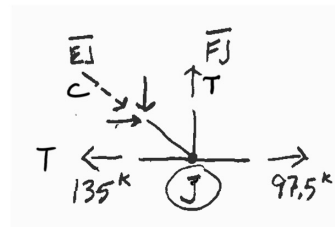
$$\overline{HI} = 135 \text{ k TENSION}$$

Method of Joints - example

Continue moving across the truss, joint by joint. Choose joints that have only one unknown in each direction, horizontal or vertical.



$\sum F_H = 0 = -135 + \overline{IJ}$
 $\overline{IJ} = 135^k$ TENSION
 $\sum F_V = 0 = \overline{EI}$ (ZERO)

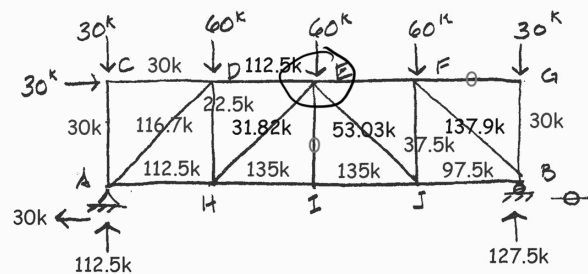


$\sum F_H = -135 + 97.5 + \overline{EJ}_H$
 $\overline{EJ}_H = 37.5^k$
 $\overline{EJ}_V = \overline{EJ}_H = 37.5^k$
 $\overline{EJ} = \sqrt{37.5^2 + 37.5^2} = 53.03^k$ COMP
 $\sum F_V = 0 = \overline{FJ} - 37.5 = 0$
 $\overline{FJ} = 37.5^k$ TENSION

Method of Joints - example

Solve the joints with the most members last.

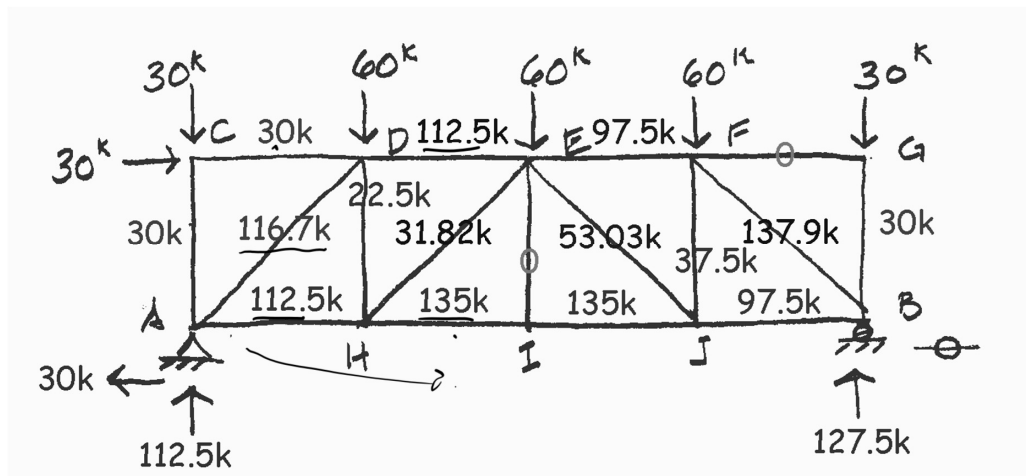
Check that all forces balance.



$\sum F_H = 0 = 112.5 + 22.5 - 37.5 - \overline{EF}$
 $\overline{EF} = 97.5^k$ COMP
 CHECK
 $\sum F_V = 0 = -60 + 22.5 + 37.5 = 0$

Method of Joints - example

Inspect the final solution to see that it seems to make sense.



Qualitative T or C

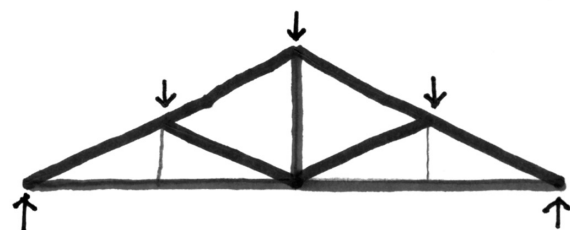
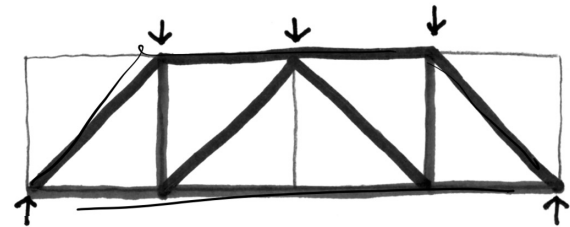
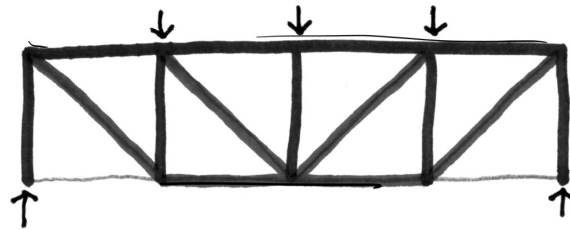
For typical gravity loading:
(tension=red compression=blue)

Top chords are in compression

Bottom chords are in tension

Diagonals down toward center are in tension (usually)

Diagonals up toward center are in compression (usually)



Qualitative Force

For spanning trusses with uniform loading:
(tension=blue compression=red)

Top and bottom chords greatest at center when flat (at maximum curvature or moment)

Diagonals greatest at ends (near reactions, i.e. greatest shear)

