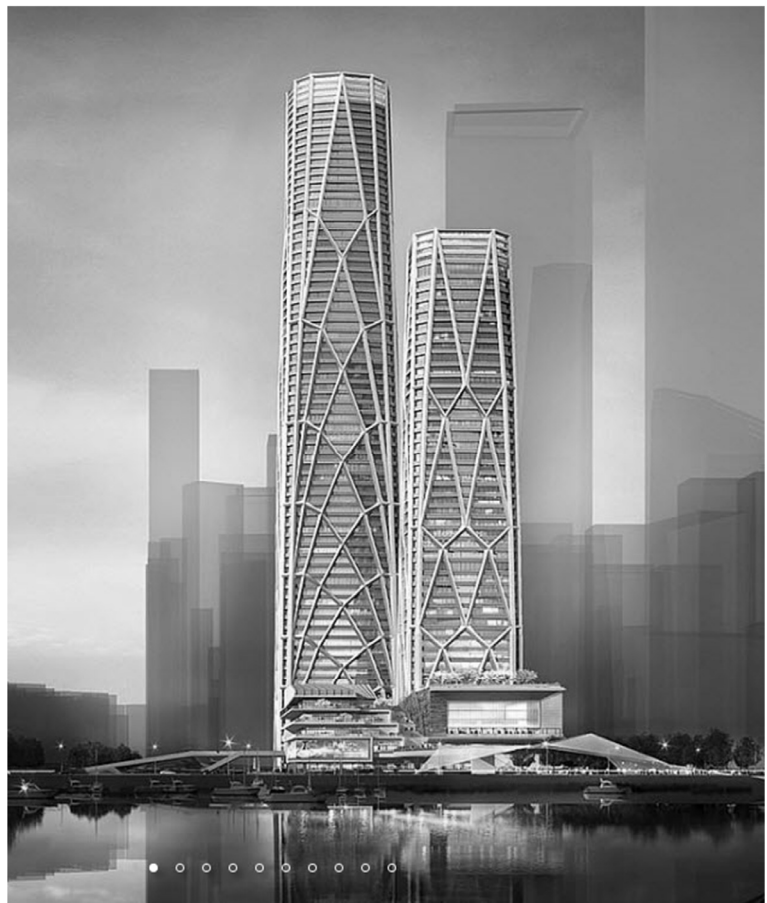


# Graphic Statics

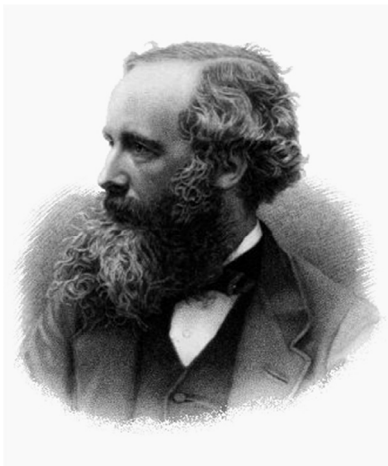
Analysis  
Design

CITIC Financial Center  
William Baker - SOM  
Shenzhen Bay, China

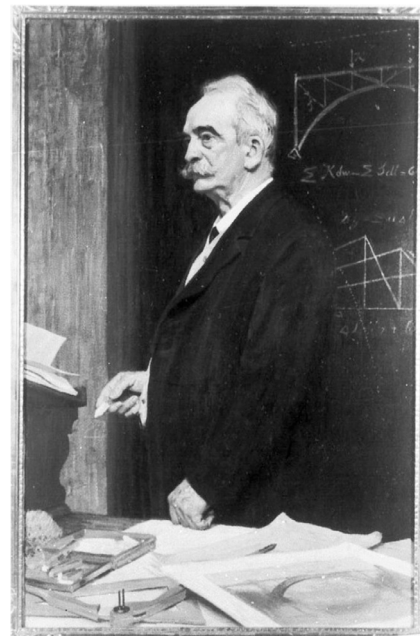


## Development of Graphic Statics

- James Clerk Maxwell 1869
- M. Williot 1877
- Otto Mohr 1887
- Luigi Cremona 1890
- Heinrich Müller-Breslau 1904



James Clerk Maxwell



Otto Mohr

# Development of Graphic Statics

Waclaw Zalewski & Edward Allen, 1998. *Shaping Structures: Statics*

Philippe Block, Matt DeJong and John Ochsendorf, 2006. *As Hangs the Flexible Line: Equilibrium of Masonry Arches.*

William Baker, 2015. *Combining Classic Theories with New Technologies.*

Caitlin Mueller, Corentin Fivet and John Ochsendorf, 2015. *Graphic Statics and Interactive Optimization for Engineering Education.*



William Baker



Caitlin Mueller



John Ochsendorf

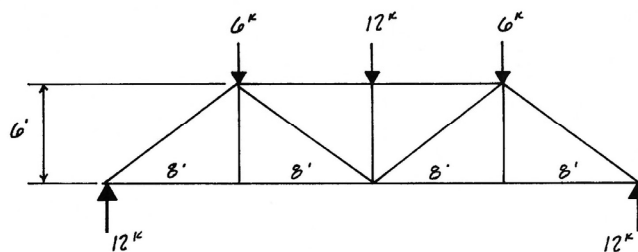
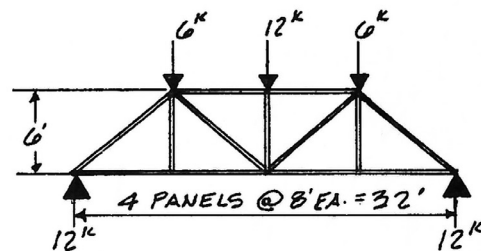


Philippe Block and Matt DeJong

## Graphic Statics - Step 1

Draw the truss to scale

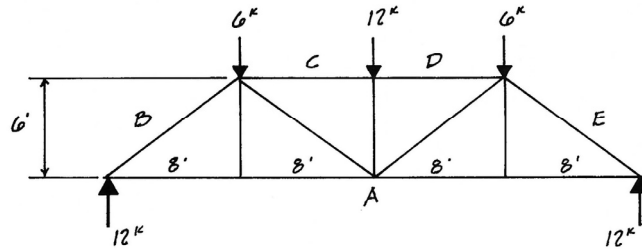
- Solve for all external forces on the truss.
- Make a simple line drawing of your truss.
- Draw it to scale.



# Graphic Statics - Step 2

## Letter external spaces

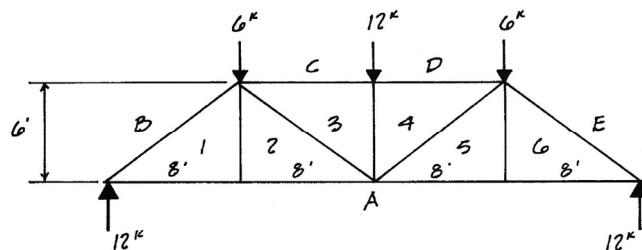
- Label each space outside the truss with a letter.
- Start at bottom center (by convention).
- Continue in a clockwise direction around the outside of the truss.



# Graphic Statics - Step 3

## Number internal spaces

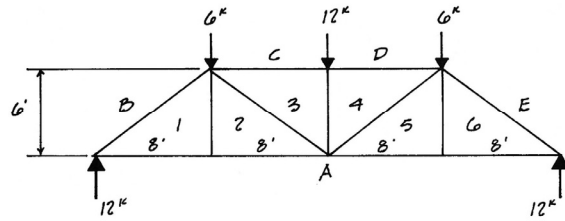
- Number each space inside the truss.
- Number from left to right.



# Graphic Statics - Step 4

## Draw the force diagram

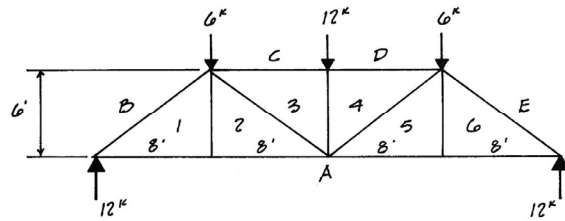
- Start in the external space (A) and move clockwise to the next space (B).
- Note the direction and magnitude of the external force you cross (12k up).
- Draw this force to scale.
- The force starts with the tail (A) and continues to the head (B).



# Graphic Statics - Step 4

## Draw the force diagram

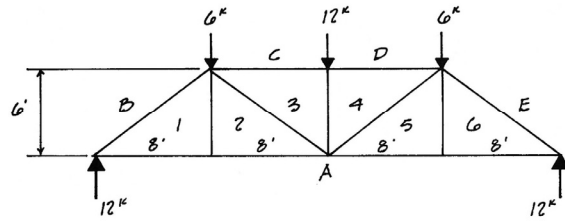
- Continue clockwise around the outside of the truss.
- Draw to scale the next force (6k downward) which is crossed as you move to the next space (C).
- Draw the tail at the first letter (B) and the head at the second (C).
- Draw the force in its actual direction and to scale.



# Graphic Statics - Step 4

## Draw the force diagram

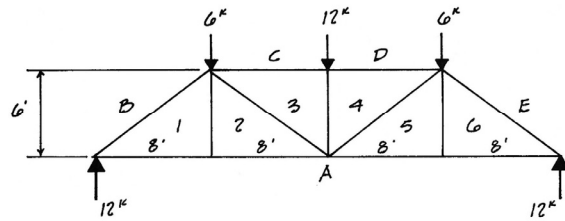
- Continue clockwise around the outside of the truss.
- Draw to scale the next force (12k downward) which is crossed as you move to the next space (D).
- Draw the tail at the first letter (C) and the head at the second (D).
- Draw the force in its actual direction and to scale.



# Graphic Statics - Step 4

## Draw the force diagram

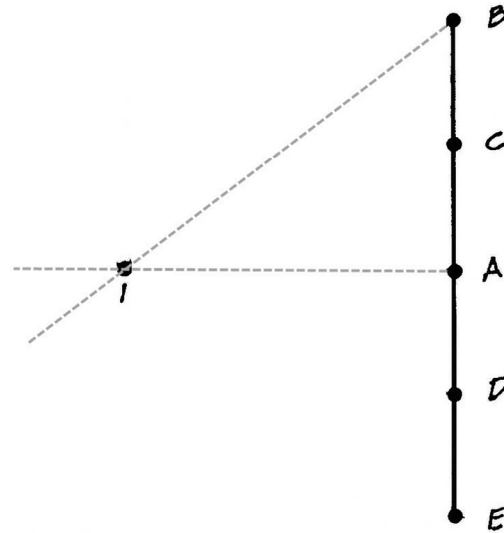
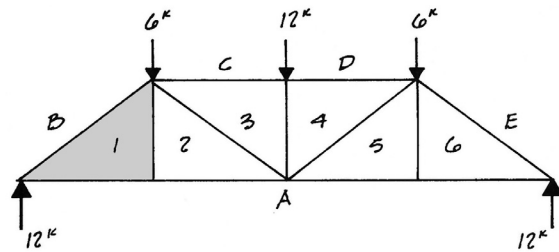
- Continue clockwise around the outside of the truss.
- Draw to scale the next force (6k downward) which is crossed as you move to the next space (E).
- Draw the tail at the first letter (D) and the head at the second (E).
- Draw the force in its actual direction and to scale.
- Draw the last force, 12k, starting at E and ending at A
- The head of the last force (A) should land on the tail of the first force (A). The external forces close to zero. If not, then there was some error in the forces (like the reactions may be wrong)



# Graphic Statics - Step 5

## Draw the member forces

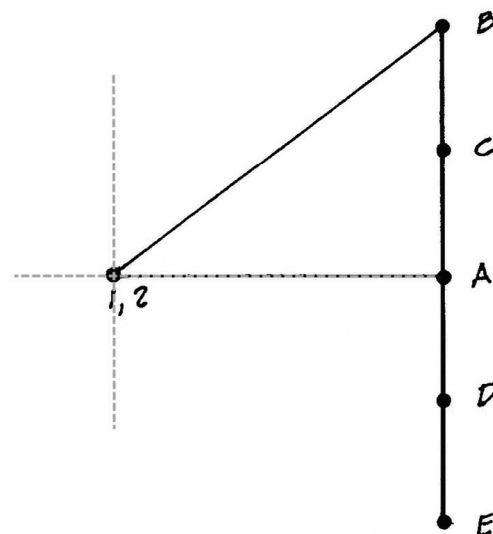
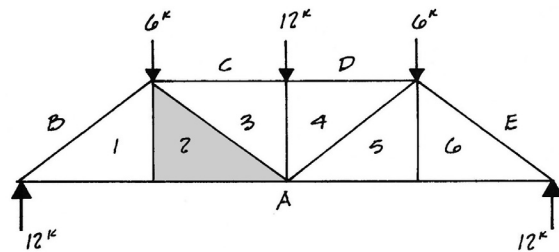
- Start in the external space (A) and cross to the first adjacent internal space (1).
- Draw the line you cross starting at the external letter (A).
- Now move to the next external space (B) adjacent to the same internal space (1).
- Draw a line with the same slope as the member crossed starting at the letter of the external space (B).
- These two lines cross at a point (1).
- Note that you needed 2 lines to define the location of the point (1).



# Graphic Statics - Step 5

## Draw the member forces

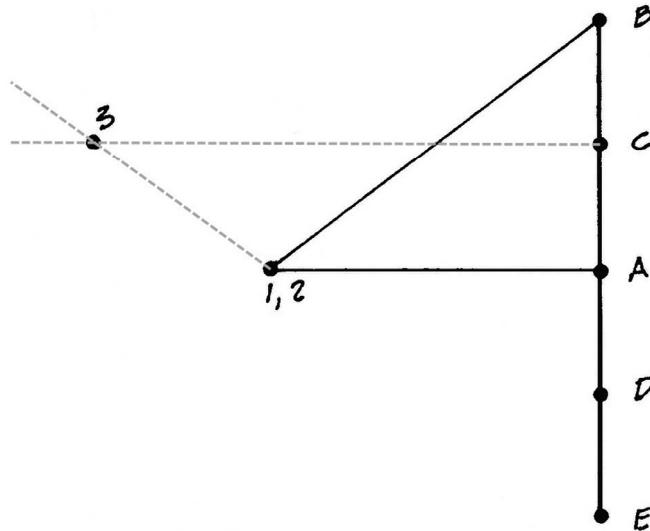
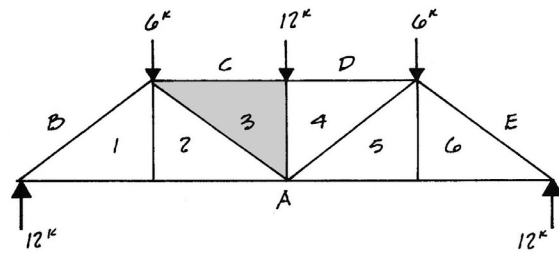
- Now find the next point (2).
- Choose two spaces that are adjacent to space this next space (2).
- Now move to the next external space (A) adjacent to this internal space (2).
- Draw a line with the same slope as the member crossed starting at the letter of the external space (A).
- Now observe the member which is between the next two spaces (1 & 2).
- Draw a line with the same slope as the member through the location of the point which labels the adjacent space (1).
- The scaled length of the line equals the force. If a line has zero length (like 1-2) then the force in the member is zero.



# Graphic Statics - Step 5

## Draw the member forces

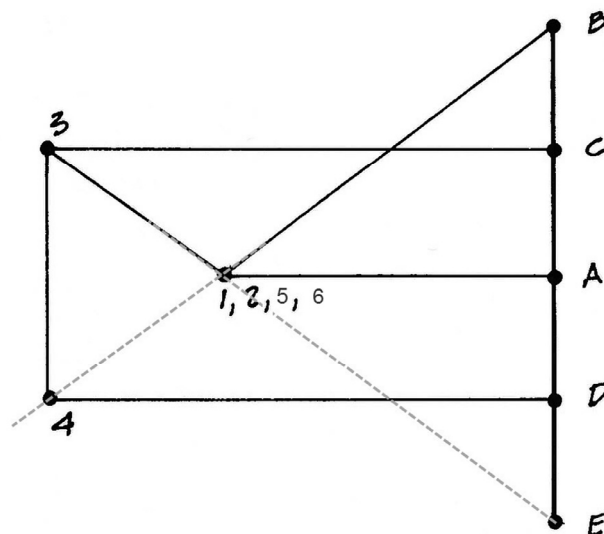
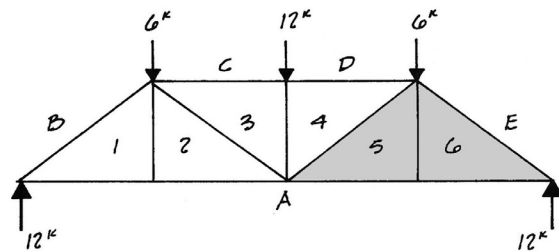
- Now find the next point (3).
- Choose two spaces that are adjacent to space this next space (3) which have points on the graph. (C&2)
- Find the member between the first pair of spaces (C&3). Draw a line with the same slope passing through the known point (C).
- Draw a second line with the same slope as the member between the first pair (2&3) passing through the known point of the pair (2).
- Again, where these lines cross the unknown point can be found (3)



# Graphic Statics - Step 5

## Draw the member forces

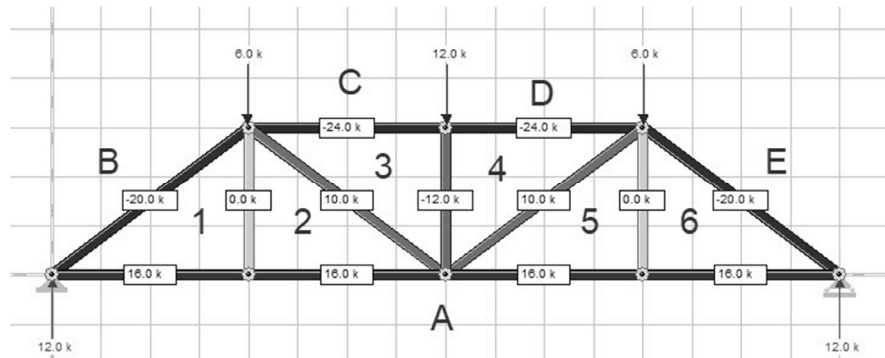
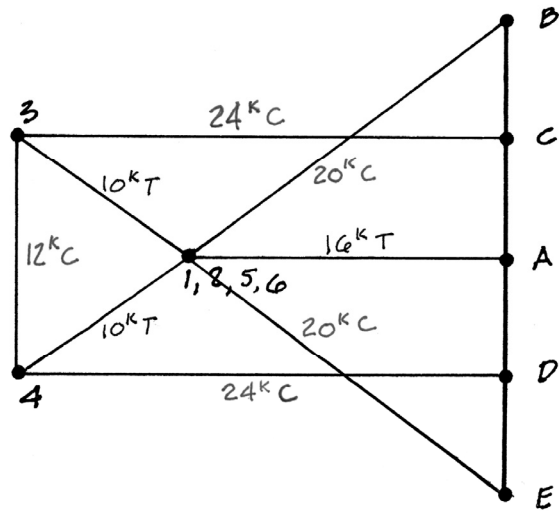
- Continue moving across the truss until all points corresponding to the interior spaces are determined
- Each line represents the scaled magnitude and direction of the force within corresponding member.



# Graphic Statics - Step 6

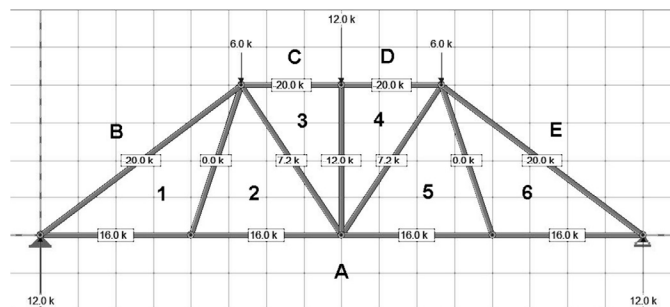
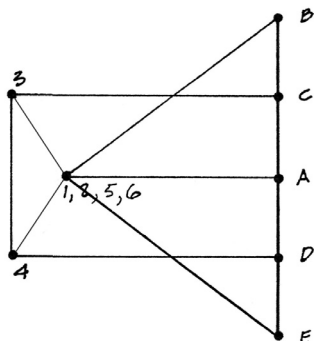
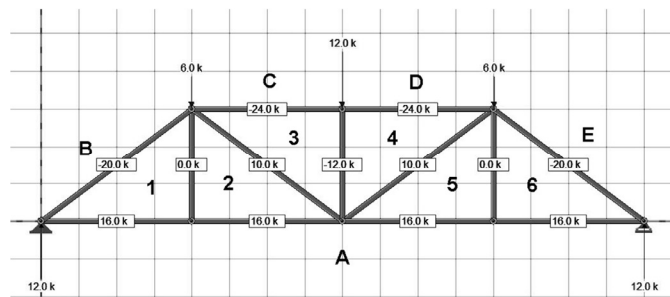
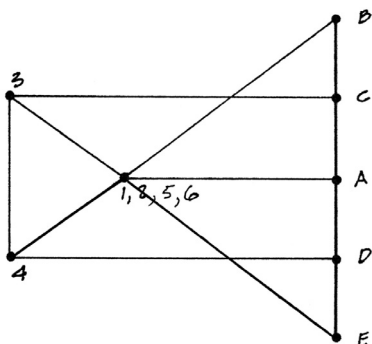
## Reading the forces

- Each line on the graph represents the scaled magnitude and direction of the force within it.
- to determine the sign of the force: Choose a member on the truss. Then choose a joint connected to that member. Moving about the joint in a clockwise direction note the space before and after crossing the member. This gives the direction of the force on the graphic diagram. Applying this direction to the FBD of the joint shows the sign: compression pushing and tension pulling.



# Graphic Statics - Design

The geometry and force diagrams are duals. Changing one will result in changes in the other and vice versa. Design a truss to achieve desired forces by modifying the force diagram, and then modifying the geometry diagram to match.

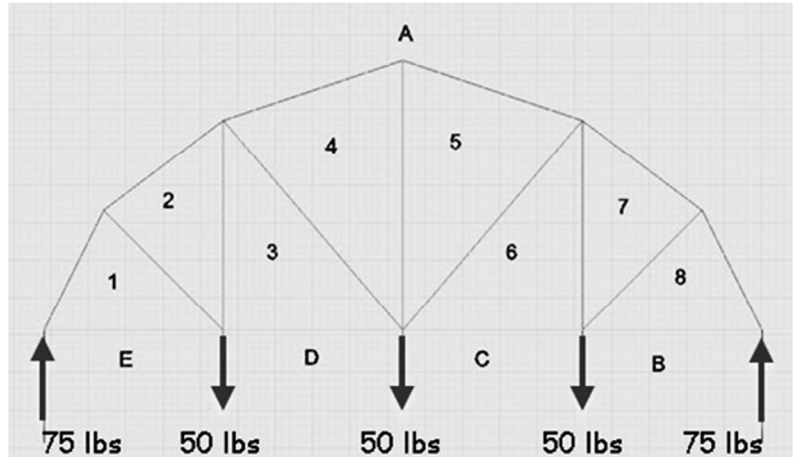
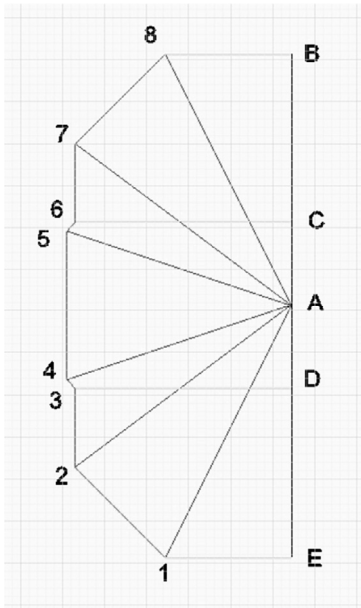




# Truss Design

## Start with a force diagram

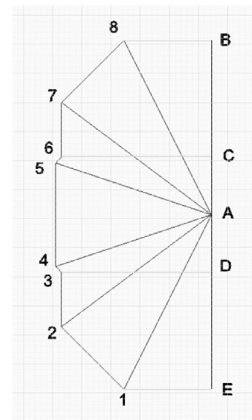
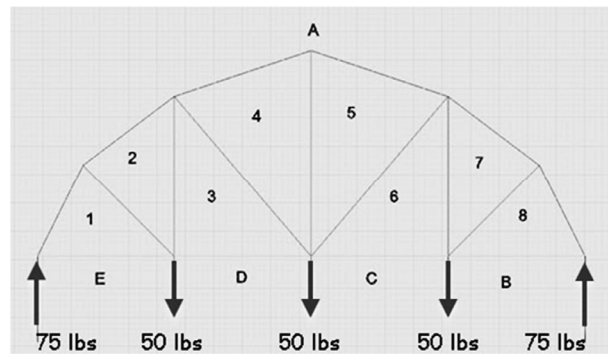
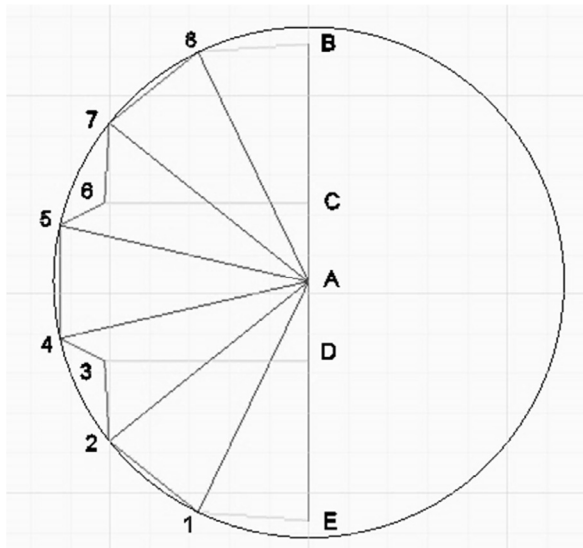
- Draw the starting geometry
- Construct the force diagram as described above



# Truss Design

## Modify the force diagram

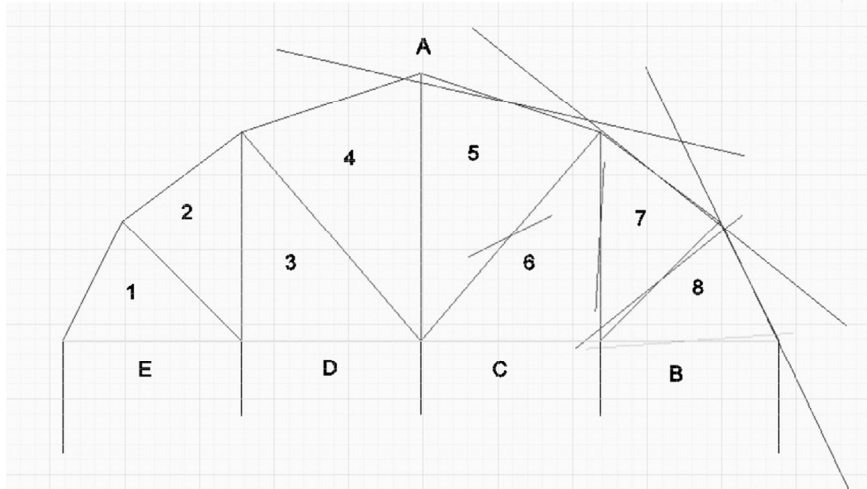
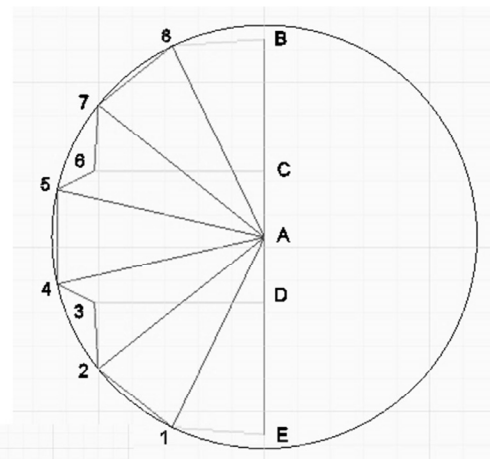
- Adjust the lengths of the members to alter the forces
- Retain the same topology (connectivity)



# Truss Design

## Change the slopes of the geometry

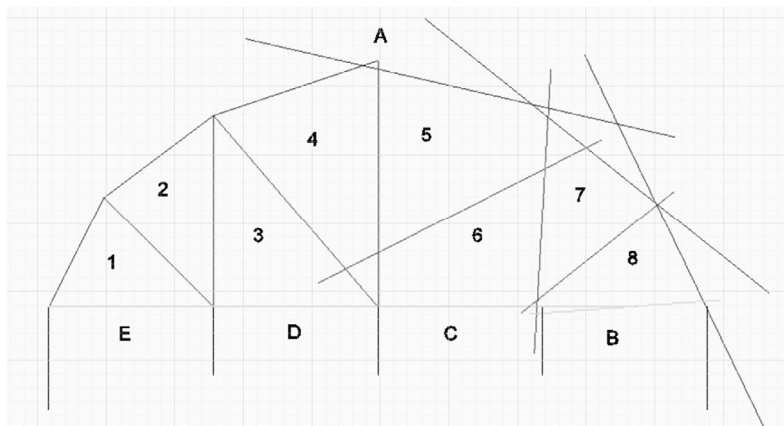
- Copy the members from the force diagram over the old members.
- Placing them at the mid-points of the old members is a good start.



# Truss Design

## Extend the new members

- Delete the old members.
- Extend the lengths of the new members so that they intersect.

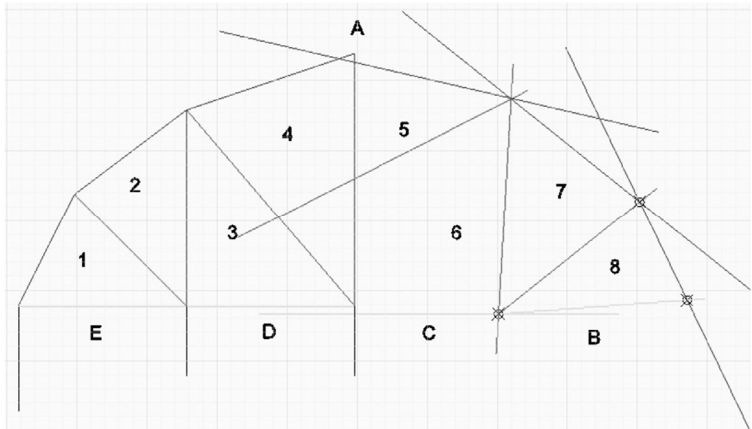
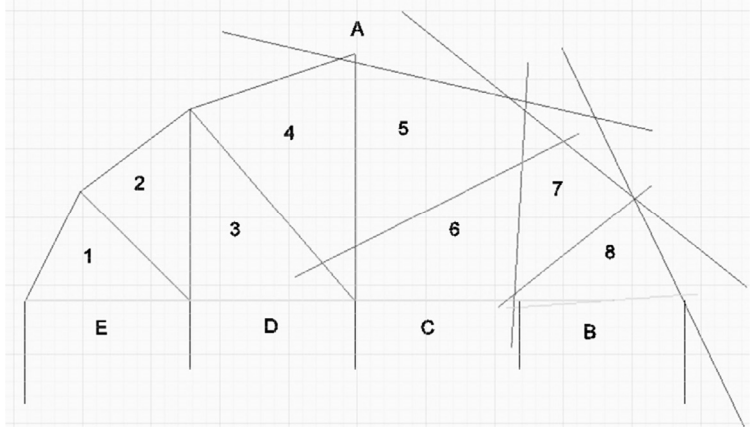


- Since this truss is symmetric about the center line, only half of the geometry need be adjusted and then mirrored.

# Truss Design

## Form the first cell

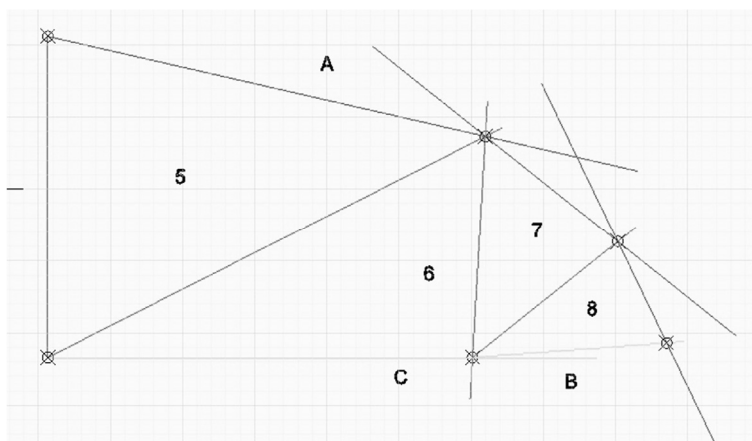
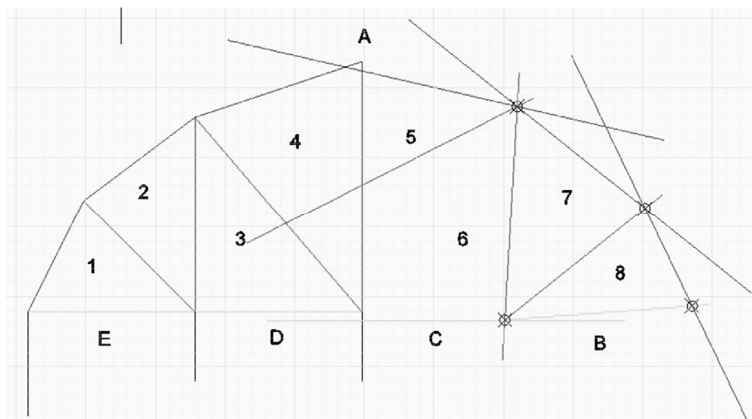
- Move the members in the first cell so that all members pass through the three corner points of the cell
- Be careful not to move single points. Move whole members so that the slope does not change.
- Extend members as necessary to intersect members.
- When all members are concurrent at a node, mark the node with a point.
- Try to work across the truss one cell at a time.



# Truss Design

## Form the next cells

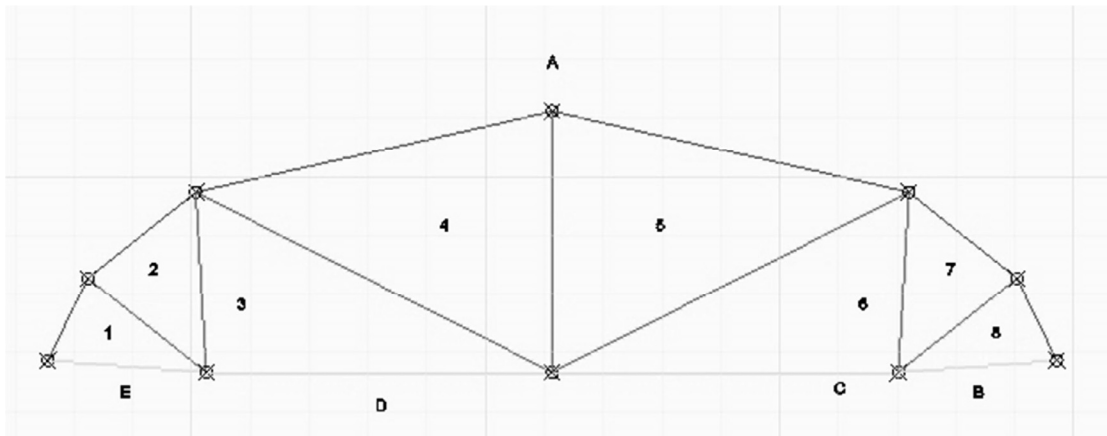
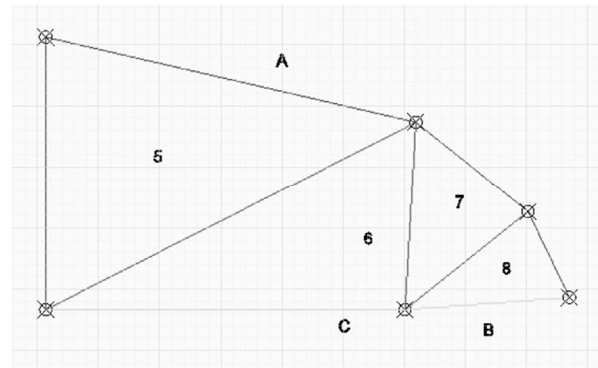
- Continue to move across the truss one node at a time.
- In this case only half of the truss needed to be solved due to symmetry.



# Truss Design

## Form the final truss

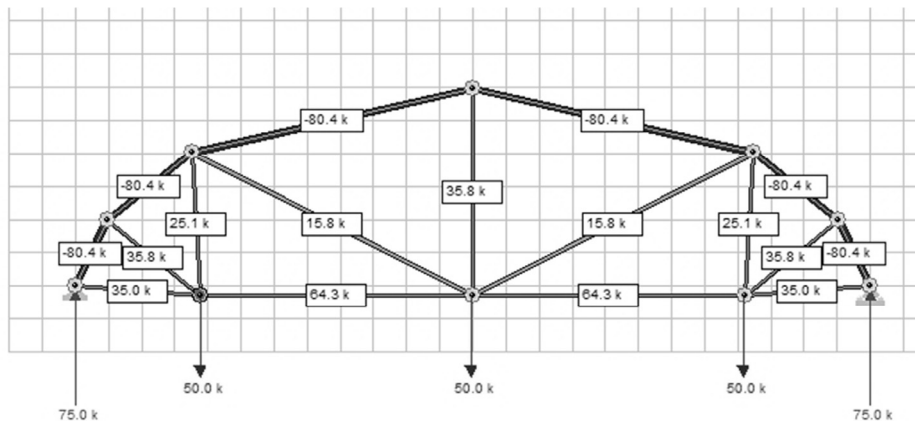
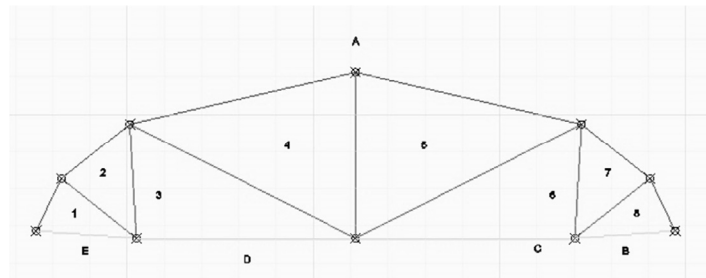
- Clean up each node by trimming members
- Mirror the half solution to obtain the full truss.
- Scale the width to the length of the original span.



# Truss Design

## Check the final solution in Dr Frame

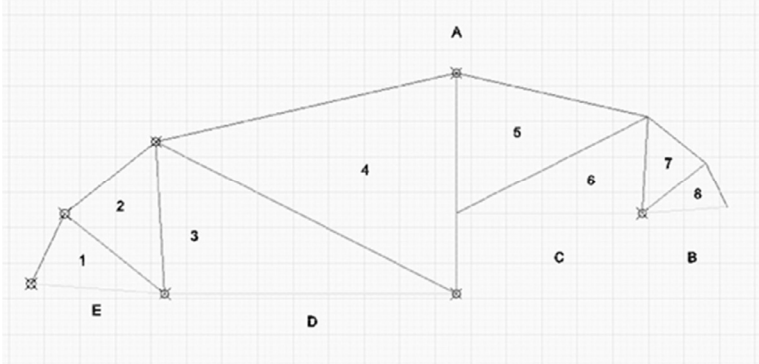
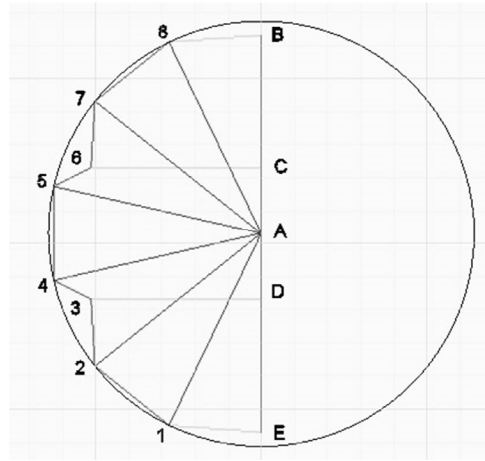
- Draw the geometry node by node in Dr Frame. Click a node and enter new coordinates.
- Check that the forces are as expected. Size the members.



# Truss Design

## Other solutions

- Taking a different starting point should yield the same geometry in the end.



# Truss Design

## Other solutions

- Additional constraints can be added to the geometry by altering the force diagram. For example a straight bottom chord.

