Architecture 314

Structures I

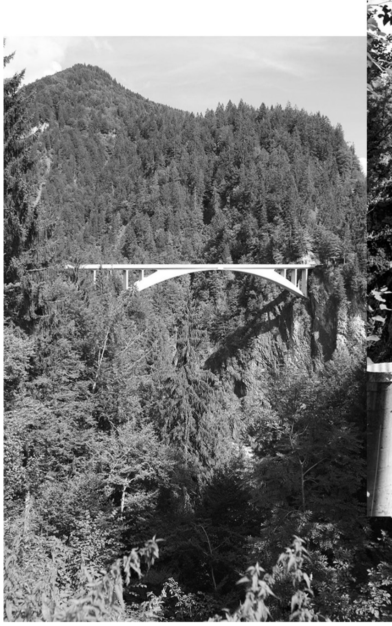
Three-Hinged Arches

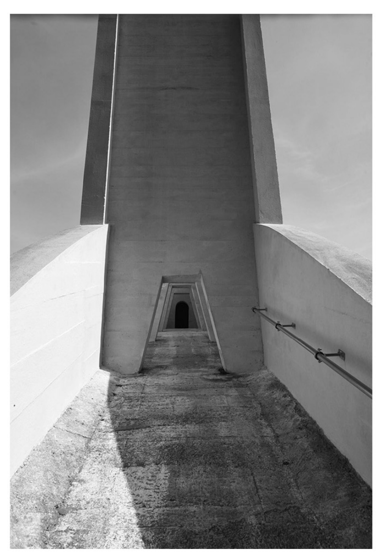
- Arches on even supports
- Arches on uneven supports

Salginatobel Bridge
Switzerland, 1930
Robert Maillart

Three Hinged Arches

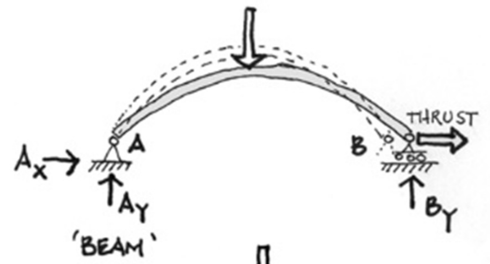
Salginatobel Bridge
Schiers, Switzerland, 1930
Robert Maillart



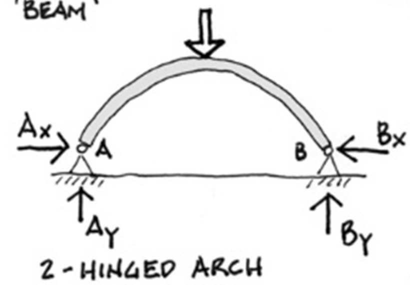


Arch Reactions

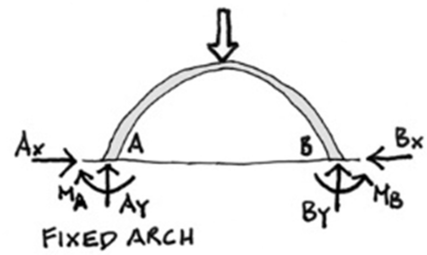
Unlike a beam, where reactions are principally vertical, reactions of an arch need to resist **horizontal thrust** as well.



If a reaction is **"pinned"** only two forces are needed to describe it. When both reactions are pinned, there are a total of 4 unknowns, and the structure is **indeterminate to the 1st degree**.

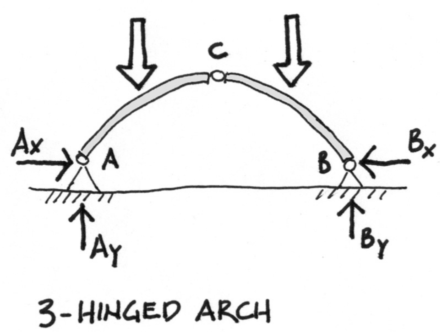


If a reaction is **"fixed"**, three forces are needed to describe it. When both reactions are fixed, there are a total of six unknown components, and the structure is **indeterminate to the 3rd degree**.



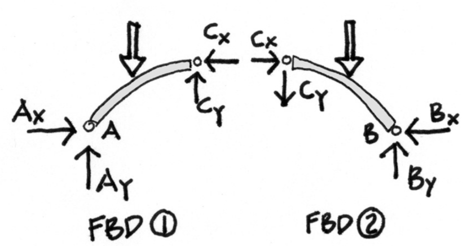
3-Hinged Arch

The 3-Hinged Arch has a "hinge" at each pinned support **plus one more internally**. The internal hinge provides one additional statics equation to be written since the moment at C is known (M_C = 0). This makes the system **statically determinate**.



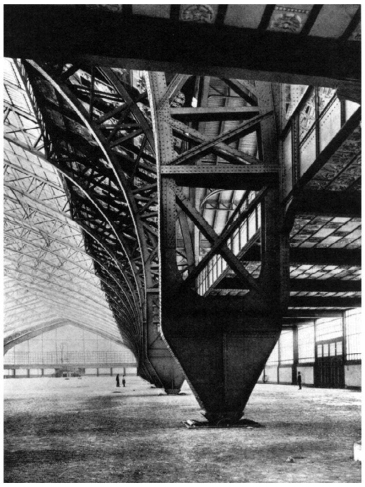
The solution of the end reactions can usually be obtained in **two steps**.

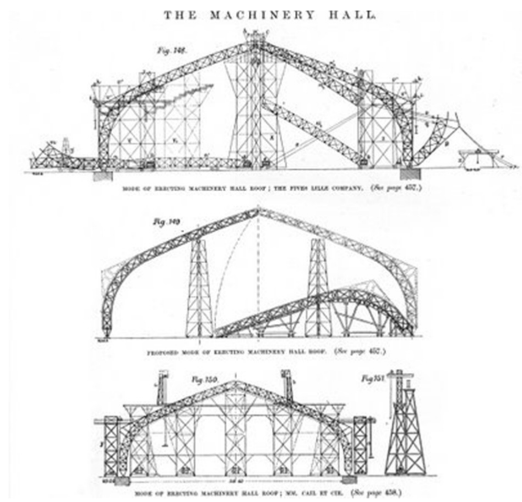
1. finding the **vertical reactions** by using the diagram of the whole structure
2. summing moments at the internal hinge on an FBD of half of the structure to find the **horizontal forces**



Characteristics of a 3-Hinged Arch

- Statically determinate – can be calculated with statics
- Movement or settling of foundations will not alter member stresses
- Small fabrication errors in length do not affect internal stresses
- Hinge placement can reduce internal stresses



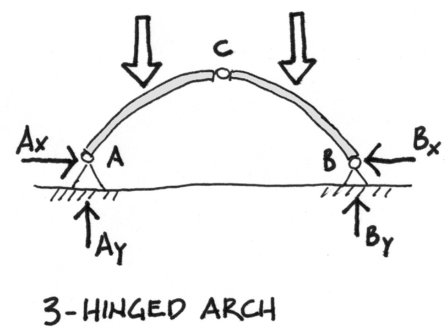


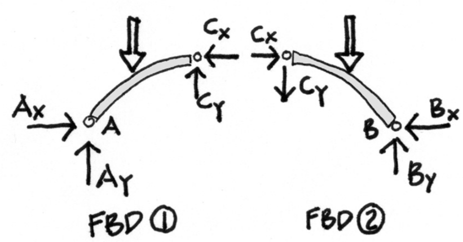
Gallery of the Machines, 1889 Paris
Architect: Ferdinand Dutert
Engineer: Victor Contamin

3-Hinged Arch analysis

procedure

1. Determine all external loads.
   - find resultants of distributed loads (e.g. wind, snow, dead load)
2. Calculate vertical end reactions.
   - sum moments at each reaction
3. Draw an FBD of each side of the arch.
   - split at the hinge
4. Find the horizontal reactions.
   - sum moments at hinge
5. Find internal moments.
   - cut additional FBDs (e.g., at the knees)





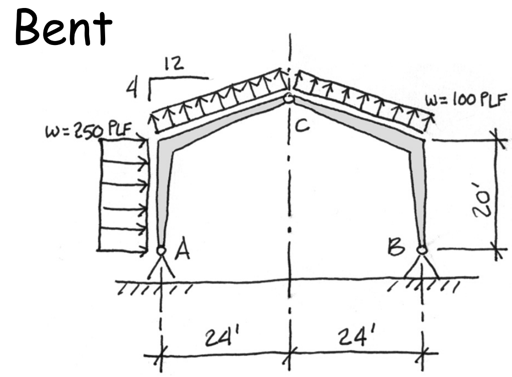
3-Hinged Arch even supports
example 1

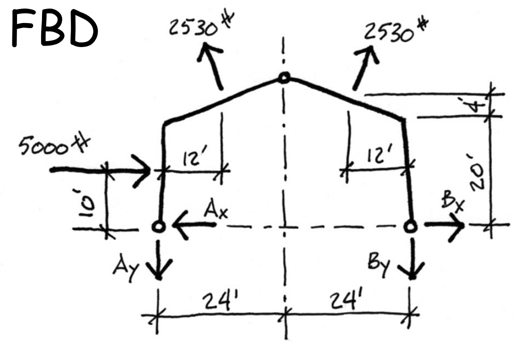
1. Determine all external loads – find resultants of distributed loads (e.g. wind, snow, dead load)
   - Wind causes a pressure load, normal to the surface of the structure.
   - Wind can be positive pressure or negative suction and the pressure varies depending on the slope of the surface.
   - The pressure is typically expressed in PSF which translates to a PLF load on the members.

   In this example with bents @ 10 ft. o.c.:
   - wind on the wall is 25 PSF = 250 PLF
   - wind on the roof is 10 PSF = 100 PLF

An FBD is drawn for a single bent (arch) of the structure.

The resultants of the uniform PLF loads are found in lbs.





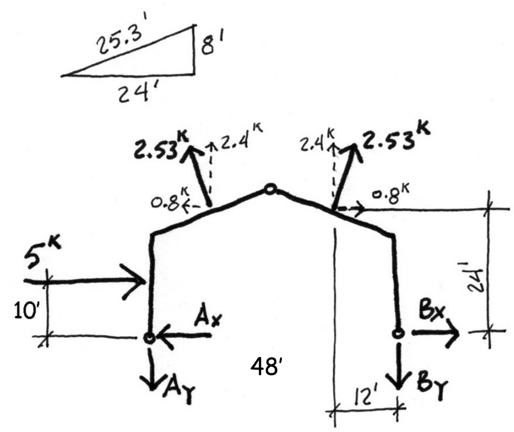
3-Hinged Arch even supports
example 1

2. Calculate the vertical end reactions.
   - sum moments at each reaction.

An FBD is drawn for a single bent of the structure.

The resultants of the uniform PLF loads are found and broken into horizontal and vertical components.

If the reactions are on the same horizontal, summing moments at either reaction will find the vertical component of the opposite reaction.



ΣM @ A to get By = 1.36 k ↓

ΣM @ B to get Ay = 3.44 k ↓

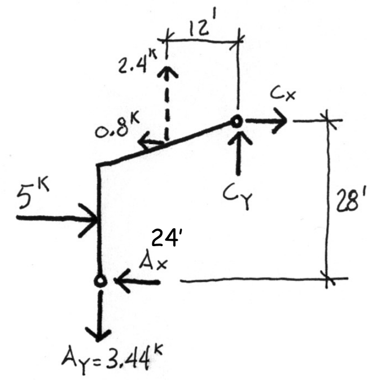
$$\Sigma M@A=0 = 5^k(10') - 0.8^k(24') - 2.4^k(12') - 2.4^k(36') + 0.8^k(24') + B_y(48')$$

$$B_y = 1.36^K$$

$$\Sigma M@B=0 = 5^k(10') + 2.4^k(36') + 2.4^k(12') - 0.8^k(24') + 0.8^k(24') - A_y(48')$$

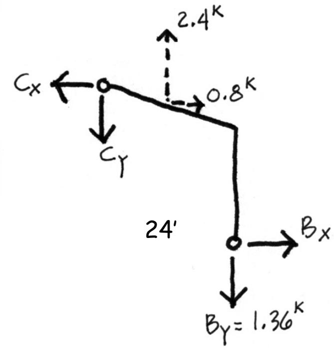
$$A_y = 3.44^K$$

3-Hinged Arch even supports
example 1



3. Draw an FBD of each side of the arch.
   - split at the hinge.
4. Find the horizontal reactions
   - sum moments at hinge.

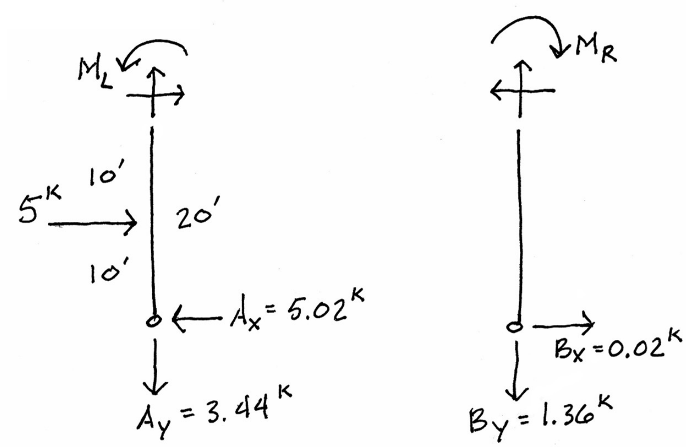
$$\Sigma M_c = 0 = 2.4^k(12') + 0.8^k(4') - 5^k(18') - 3.44^k(24') + A_x(28')$$
$$A_x = 5.02^k \leftarrow$$



$$\Sigma M_c = 0 = -2.4^k(12') - 0.8^k(4') + 1.36^k(24') - B_x(28')$$
$$B_x = 0.02k \rightarrow$$

3-Hinged Arch even supports
example 1

5. Find internal moments
   - cut additional FBDs (e.g., at the knees).



left wall right wall

$$\Sigma M_L = 0 = +5.02^k(20') - 5^k(10') - M_L$$
$$M_L = 50.4^{'-k}$$

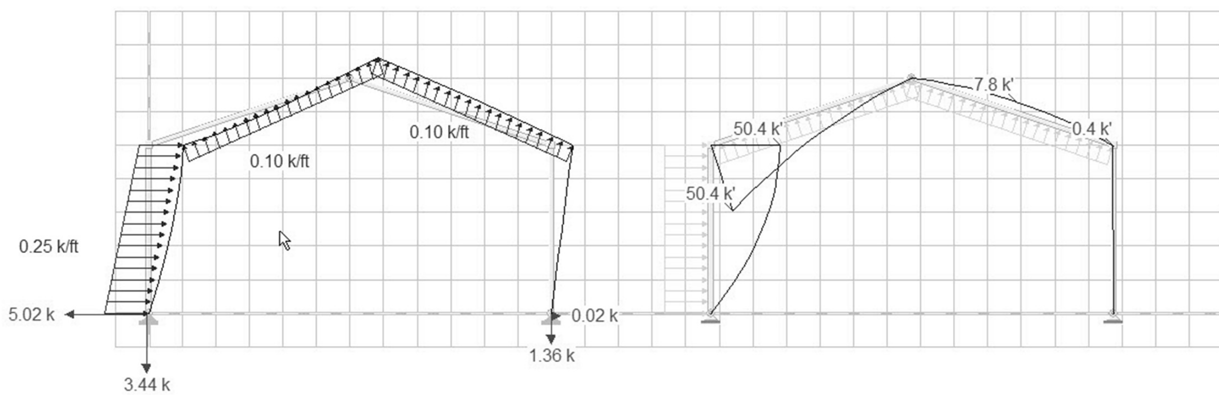
$$\Sigma M_R = 0 = -0.02^k(20') + M_R$$
$$M_R = 0.4^{'-k}$$

3-Hinged Arch even supports
example 1

Internal moments can be calculated taking appropriate sections and FBD's.

The moment diagram is traditionally **drawn on the tension side** (the opposite of the convention used for beams).

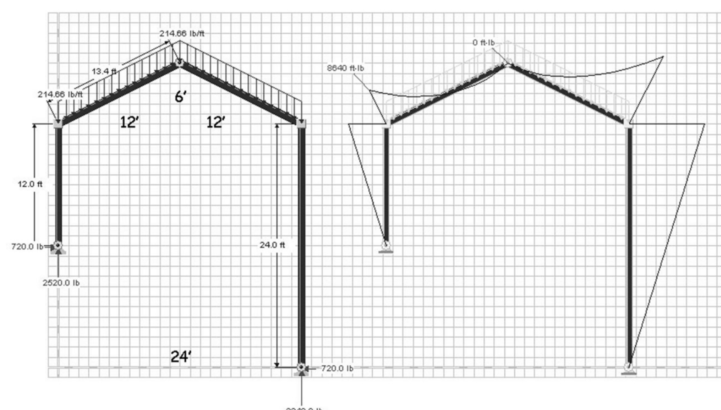
Tension on the inside is called a positive moment regardless of rotation direction.

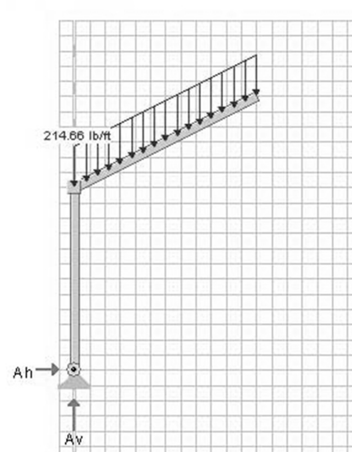


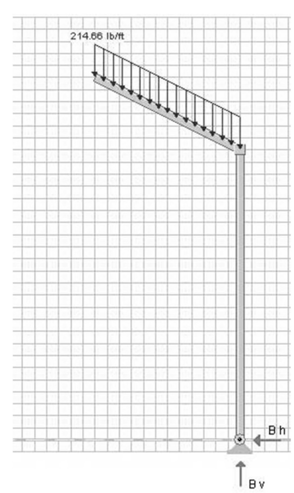
3-Hinged Arch **Uneven Supports**

procedure

1. Sum moments at B
   - get an equation with Ah and Av.
2. On left FBD sum moments at hinge
   - get a second equation for Ah and Av
3. Solve the two equations for Ah and Av
4. Repeat for right side or sum vertical and horizontal forces.



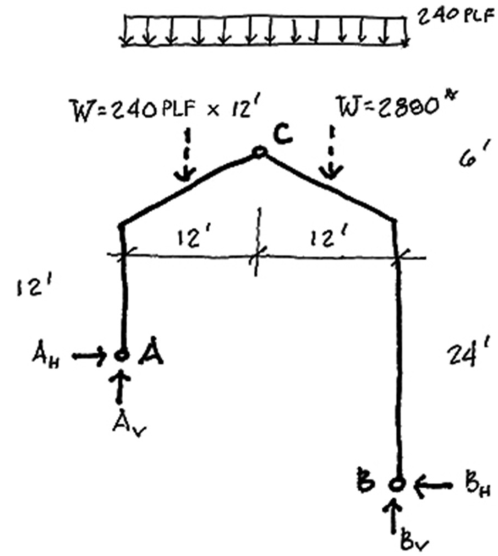




3-Hinged Arch **Uneven Supports**

example 2

1. Sum moments at B to get an equation with A_H and A_V.

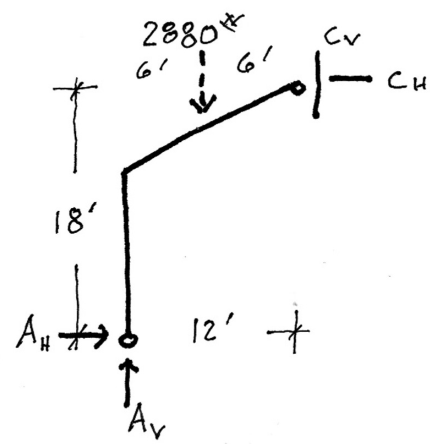


$$\Sigma M_{@B} = 0 = A_H(12') + A_V(24') - 2880^{\#}(18') - 2880^{\#}(6')$$
$$A_V = -A_H(0.5') + 2880$$

3-Hinged Arch **Uneven Supports**

example 2

2. On left FBD sum moments at hinge to get a second equation for Ah and Av



$$\Sigma M_{@C} = 0 = -A_H(18') + A_V(12') - 2880(6')$$
$$A_V = A_H(1.5') + 1440$$

combining the two equations gives:

$$-A_H(0.5') + 2880 = A_H(1.5') + 1440$$
$$A_H = 720^{\#}$$

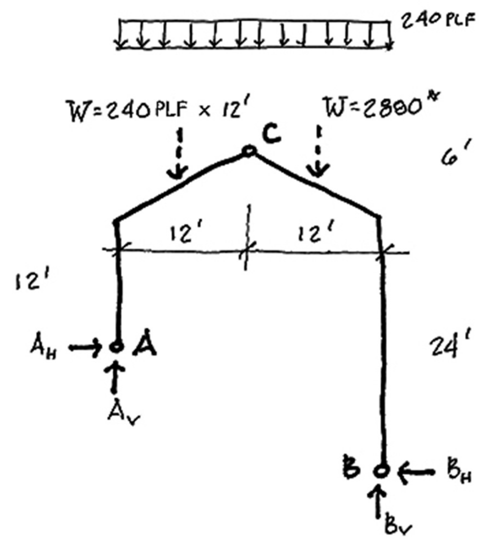
2. Solve the two equations for Ah and Av

$$A_V = A_H(1.5) + 1440$$
$$A_V = 720(1.5) + 1440$$
$$A_V = 2520^{\#}$$

3-Hinged Arch **Uneven Supports**

example 2

4. Repeat for right side or sum vertical and horizontal forces.



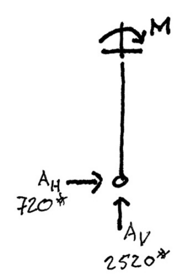
$$\Sigma F_V = 0 = 2520^{\#} - 2880^{\#} - 2880^{\#} + B_V$$
$$B_V = 3240^{\#}$$

$$\Sigma F_H = 0 = 720^{\#} - B_H$$
$$B_H = 720^{\#}$$

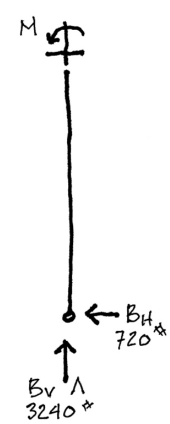
3-Hinged Arch **Uneven Supports**

example 2

5. Cut FBDs at knees to find internal moments



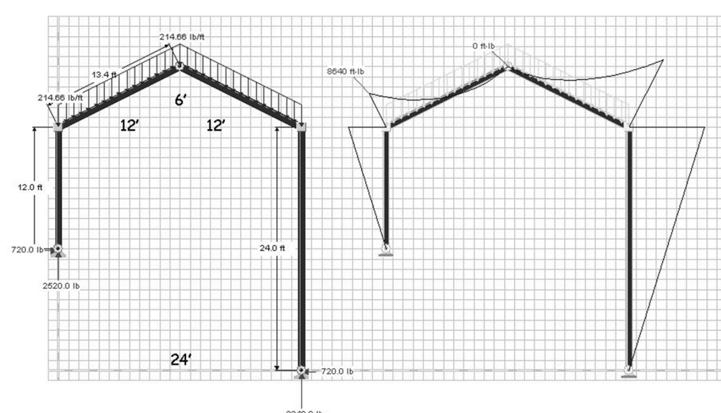
left knee



right knee

$$\Sigma M @ \text{LEFT KNEE} = 0$$
$$M_L - 720^{\#}(12') = 0$$
$$M_L = 8640\ '\text{-}\#$$

$$\Sigma M @ \text{RIGHT KNEE} = 0$$
$$-M_R + 720^{\#}(24') = 0$$
$$M_R = 17280\ '\text{-}\#$$



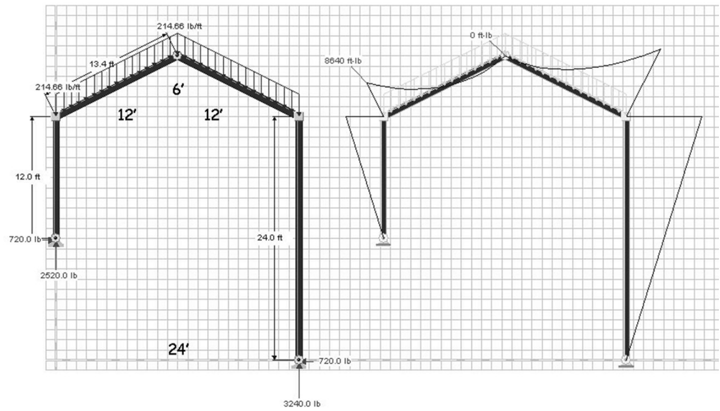
Sign Convention for Frames

Draw the moment on the tension side of the member.

The traditional convention is:

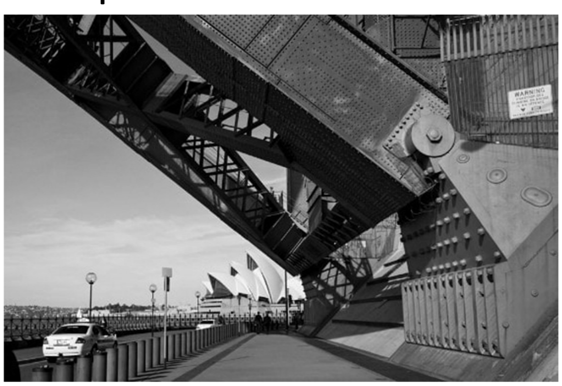
tension outside -

tension inside +

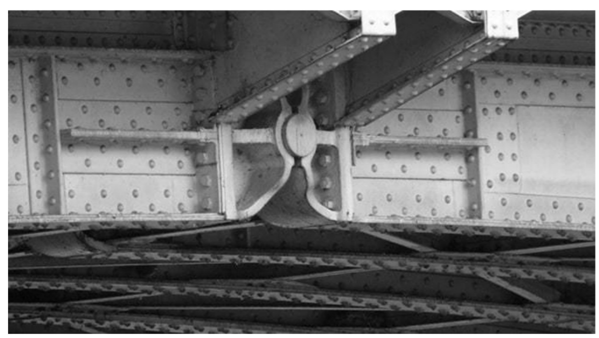


Here both knees have a negative moment

Examples and Details



Sydney Harbour Bridge



Center Hinge

The Iron Bridge
Telford England

Hinged Glulam Timbers

Keystone Wye Bridge

Glulam arches, spanning 160 ft.

built 1967-68 in South Dakota