Architecture 314

Structures I

Stress and Strain

- Stress
- Strain
- Analysis – ASD vs. LRFD
- Modes of Failure

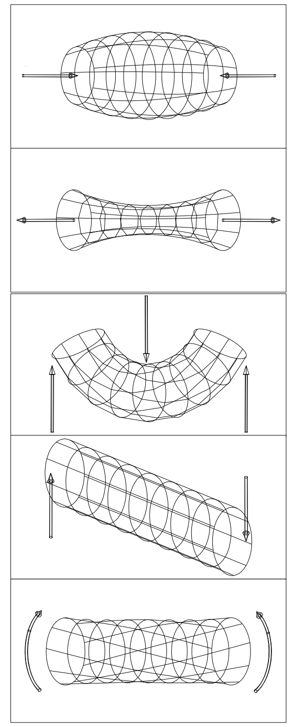
$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{M\ c}{I}$$

$$\tau = \frac{P}{A} or \frac{VQ}{Ib}$$

$$\tau = \frac{T\ r}{J}$$



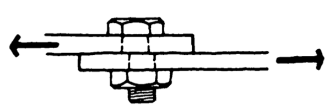
Stress

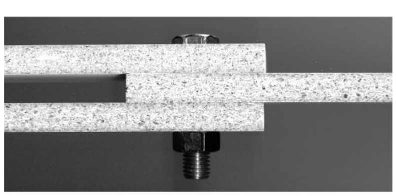
Stress is the result of a force being applied to the area of a material.

$$\sigma = \frac{P}{A}$$

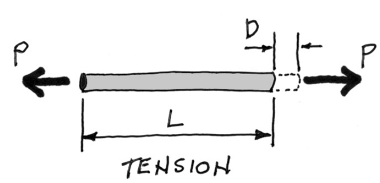
FORCE

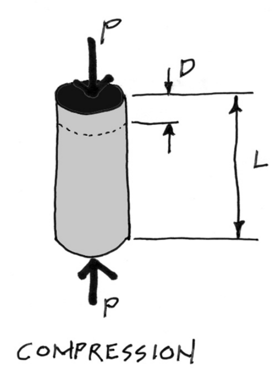
AREA





Shear Stress

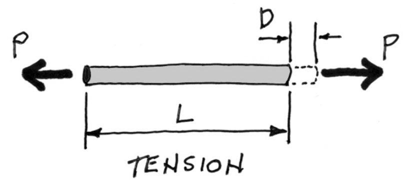




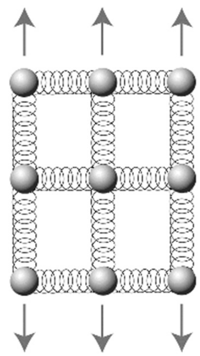
Strain

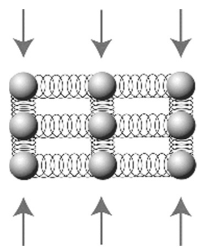
Strain is the amount of deformation in the material, per unit length.

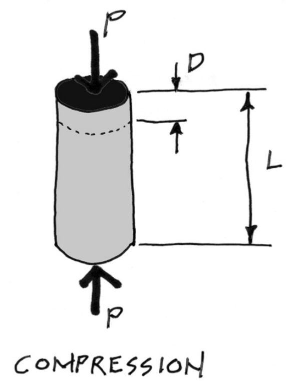
$$\varepsilon = \frac{D}{L}$$



Deformation occurs either in stretching (tension) or in compressing (compression) but not always at the same rate.

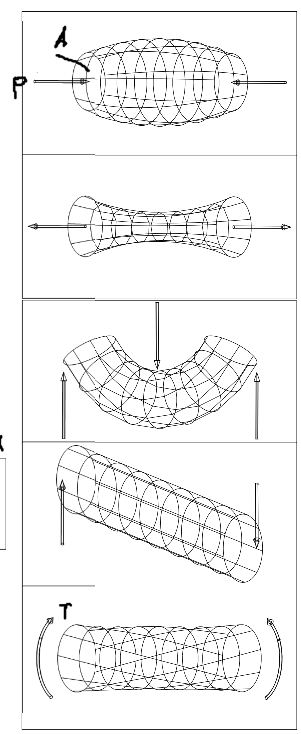






Types of Stress

- Compression $\sigma = \frac{P}{A}$
- Tension $\sigma = \frac{P}{A}$
- Flexure $\sigma = \frac{M\,c}{I}$
- Shear $v = \frac{P}{A} \; or \; \frac{VQ}{Ib}$ (BEAM)
- Torsion $\tau = \frac{T\,r}{J}$



Stress Analysis

Allowable Stress Design (ASD) OLDER

- use applied loads (no F.S. on loads)
- reduce stress by a Factor of Safety F.S.

$$f_{actual} = \frac{P}{A}$$

$$f_{actual} \leq F_{allowable}$$

$$F_{allowable} = F.S. \cdot f_{yield}$$

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use factor on nominal strength ϕ

$$\uparrow \quad P_{load} = \gamma \cdot P_{applied}$$

$$\downarrow \quad P_{resisting} = \phi \cdot P_{material}$$

$$P_{load} \leq P_{resisting}$$

Stress Calculations - example

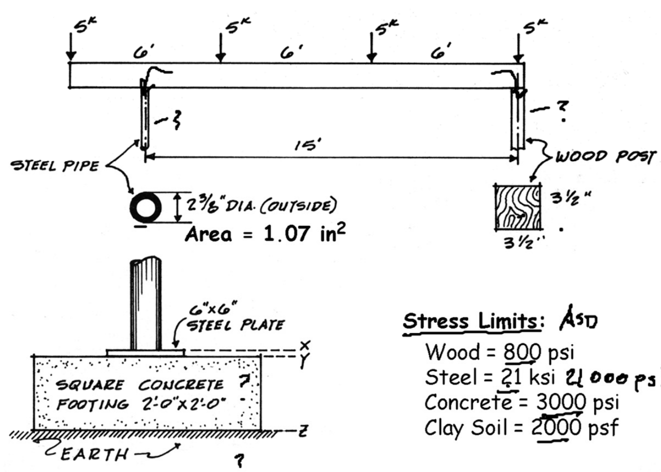
Find the stress in each material:

- wood
- steel
- concrete
- soil

Axial Compression

The stress equals the force spread over an area.

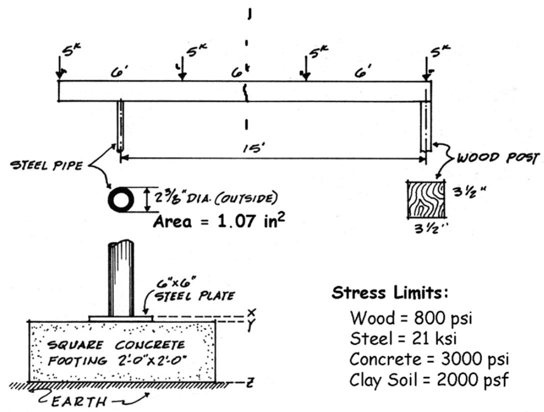
$$\sigma = \frac{P}{A}$$

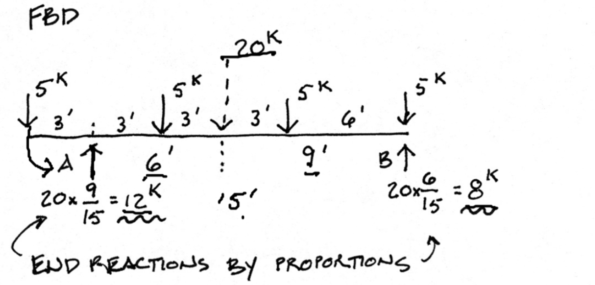


Stress Calculations

Find the force on the members

FBD to find the end reactions





OR

ΣM AT EACH REACTION

$\Sigma M @ A = -5^K(3') + 5^K(3') + 5^K(9') + 5^K(15') - B(15')$

$B = 8^K$

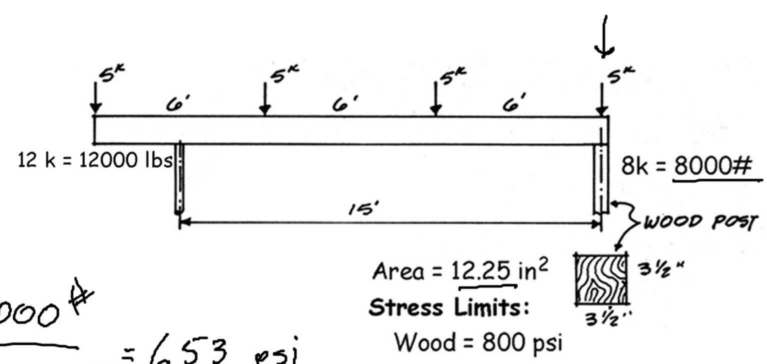
$\Sigma F_V = 0 = -20 + A + 8 = 0$

$A = 12^K$ ✓

Stress Calculations

for the right side (wood)

The stress equals the force on the member, spread over the sectional area of the member.



$$\sigma = \frac{P}{A} = \frac{8000\#}{12.25\ in^2} = 653\ psi$$

653 < 800 ALLOW ✓ GOOD

Stress in Wood:

f = P/A

f = 8000lbs/12.25 in²

f = 653 psi

F = 800 psi ALLOW

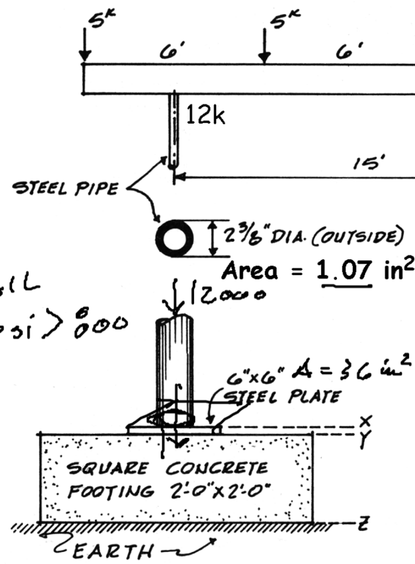
f < F ok ✓

Stress Calculations

for the left side (steel pipe)

The stress equals the force spread over the area.

$$\sigma = \frac{P}{A}$$



WOOD

$\frac{12000}{12.25\ in^2} = 979\ psi > 800$ FAIL

STEEL

$\frac{12000\#}{1.07\ in^2} = 11200\ psi$

11,200 ksi

CONC.

$\frac{12000}{1.07\ A_{PIPE}} = 11200 > 3000$

Stress Limits:
Steel = 21 ksi
Concrete = 3000 psi

Stress in Steel:
f = P/A = 12k / 1.07 in²
f = 11.2 ksi < F = 21 ksi ok

Stress in Concrete:
f = 12000 lbs / 1.07 in²
f = 11200 psi > 3000 psi FAILS!

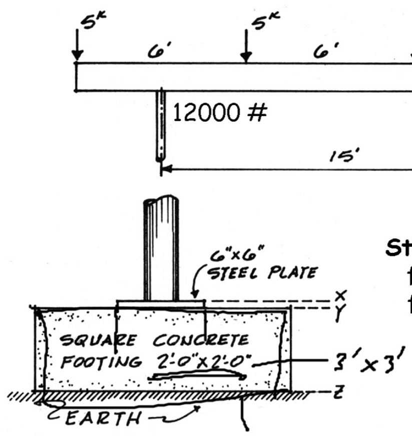
f = 12000 / 36 in² = 333 psi
333 psi < 3000 psi ok

Stress Calculations

for the left side (foundation)

The stress equals the force spread over an area.

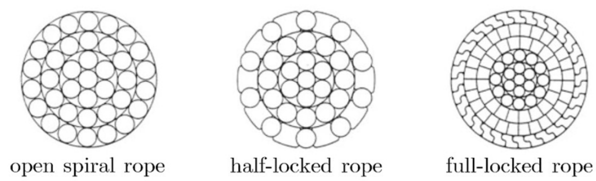
$$\sigma = \frac{P}{A}$$



Stress in Soil:
f = 12000 # / 4 sf = 3000 psf
f = 3000 psf > 2000 psf FAILS!

Stress Limits:
Clay Soil = 2000 psf

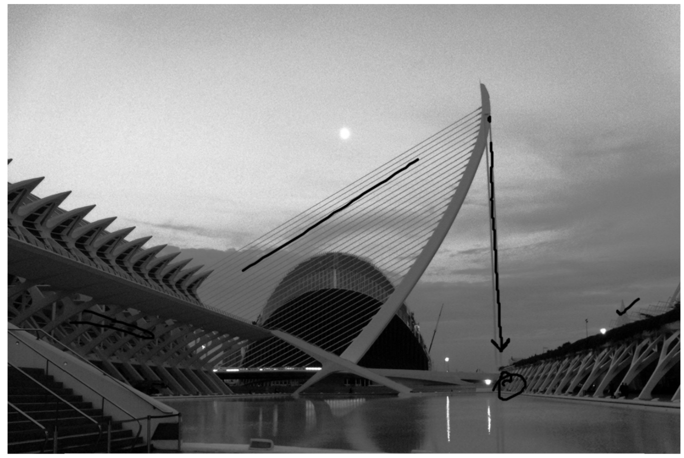
Stress Calculations



Axial Tension

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$



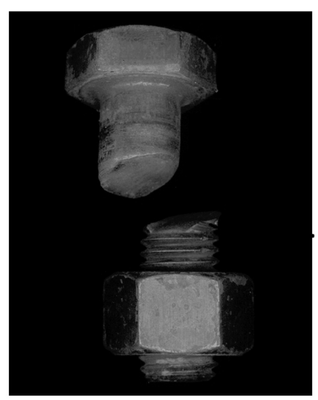
Santiago Calatrava - Serreria Bridge - Valencia 2008

Stress Calculations

Shear

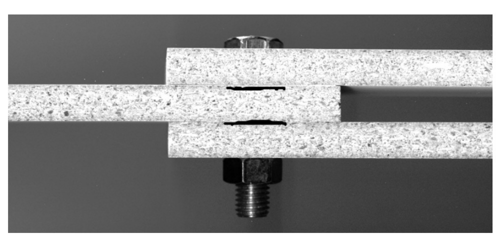
The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$

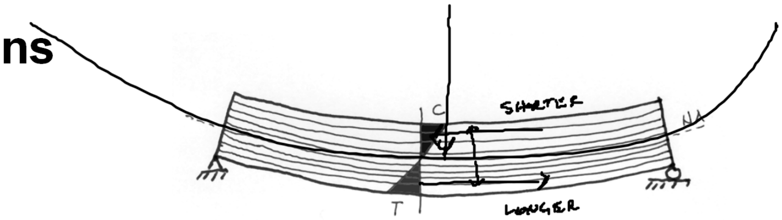


$P/A = \sigma$

$P = \sigma A$



Stress Calculations



Bending

Flexure Stress

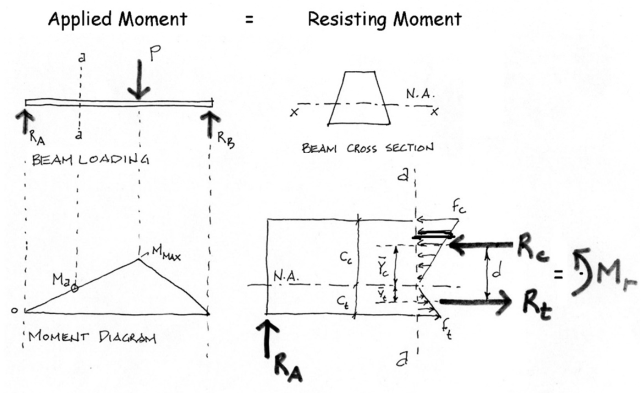
The stress is on the “fibers” or longitudinal layers

$$\sigma = \frac{M\ c}{I}$$

Shear Stress

The stress is between the longitudinal layers.

$$\tau = \frac{VQ}{Ib}$$



Modes of Failure

Strength

- Tension rupture
- Compression crushing

Stability

- Column buckling
- Beam lateral torsional buckling

Serviceability

- Beam deflection
- Building story drift
- cracking