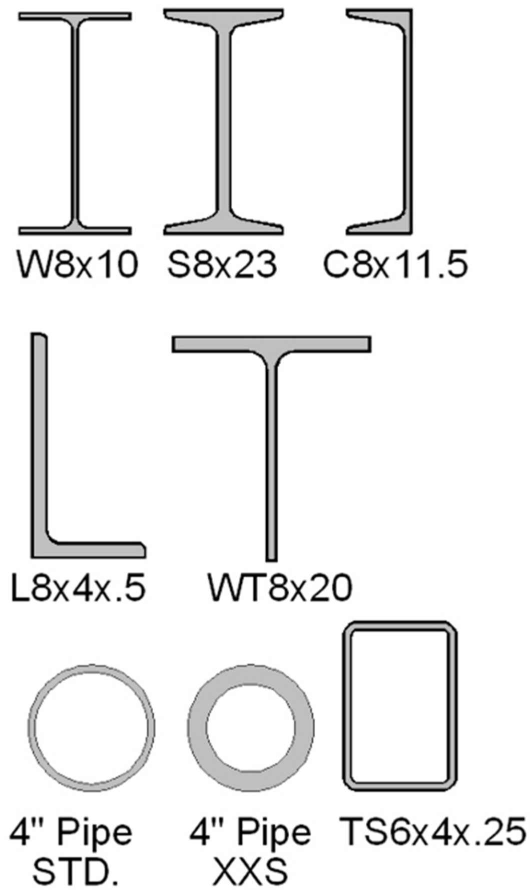
ARCHITECTURE 314

STRUCTURES I

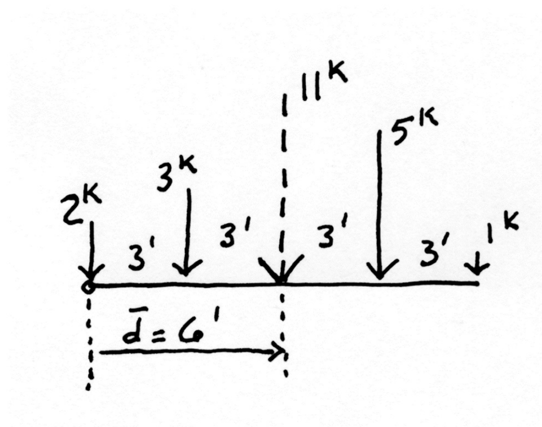
Cross-Sectional Properties of Structural Members



Resultant of Parallel Forces
Center of Gravity
Centroid of Area
First Moment of Area
Second Moment of Area
(Moment of Inertia)
Radius of Gyration

Parallel Force Resultant

The resultant is the single force that has the same effect as the group of forces.



$$\sum M = \sum (\mathbf{F} \times d) = \mathbf{R} \times \bar{d}$$

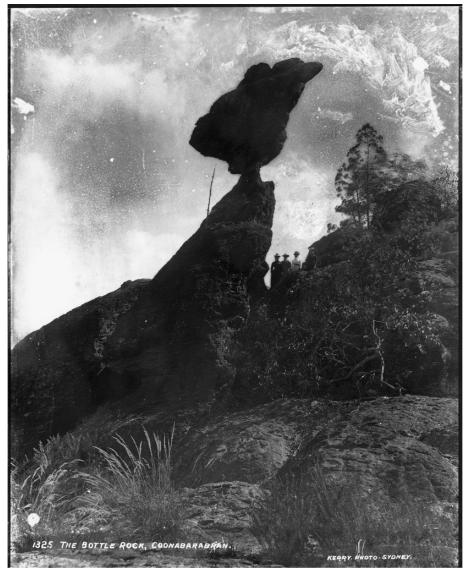
$$\sum \mathbf{F} = \mathbf{R}$$

$$\bar{d} = \frac{\sum (\mathbf{F} \times d)}{\sum \mathbf{F}}$$

Centers

The point about which a body may be balanced.

This is the point of application of the resultant weight.



Tyrrell Photographic Collection, Powerhouse Museum

Center of Gravity

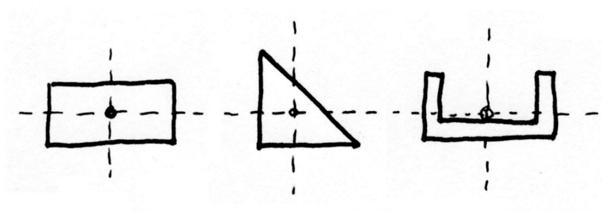
$$\bar{x} = \frac{\sum \mathbf{W} \times d_x}{\sum \mathbf{W}}$$

Center of Volume

$$\bar{x} = \frac{\sum \mathbf{V} \times d_x}{\sum \mathbf{V}}$$

Center of Area (centroid)

$$\bar{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}}$$

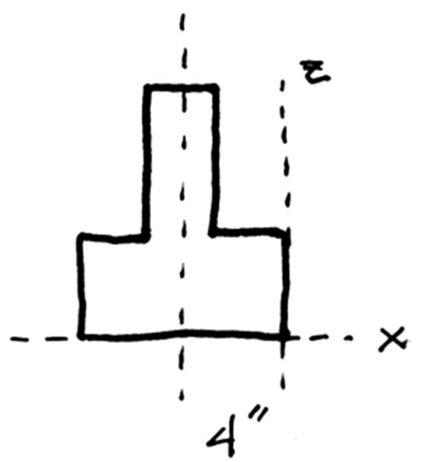


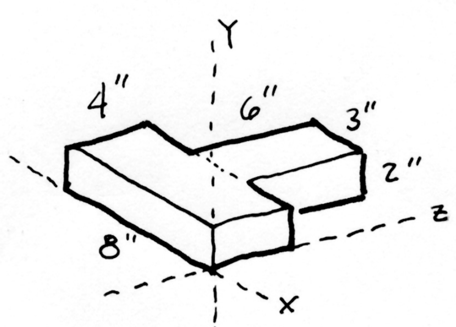
Center of Gravity (or Volume)

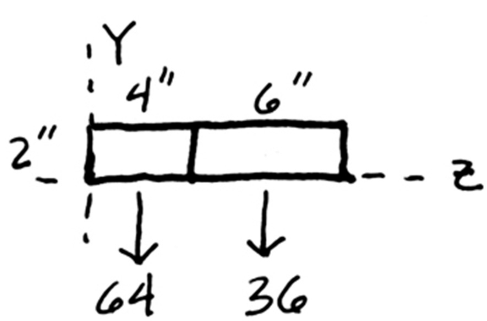
The Center of Gravity is located at the point defined by:

$$(\bar{x}, \bar{y}, \bar{z})$$

$$\bar{z} = \frac{\sum \mathbf{W} \times d_z}{\sum \mathbf{W}}$$







Center of Area - the Centroid

The “center of area” for a cross section.

$$\bar{x} = \frac{\sum(\mathbf{Area} \times d_x)}{\sum \mathbf{Area}} = \frac{\mathrm{A}\, x_A + \mathrm{B}\, x_B + \mathrm{C} x_c}{\mathrm{A} + \mathrm{B} + \mathrm{C}}$$

Area $_A$ = 2 x 7 = 14

Area $_B$ = 3 x 2 = 6

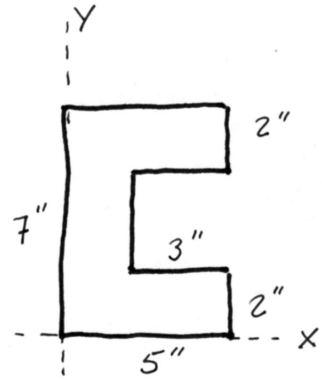
Area $_C$ = 3 x 2 = 6

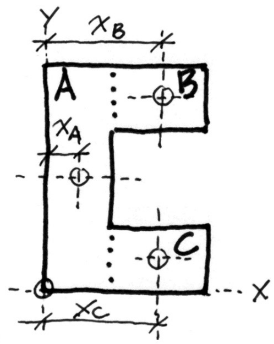
sum = 26

x_A = 1

x_B = 3.5

x_C = 3.5





Centroid Example 1 cont.

Area $_A$ = 2 x 7 = 14 x_A = 1

Area $_B$ = 3 x 2 = 6 x_B = 3.5

Area $_C$ = 3 x 2 = 6 x_C = 3.5

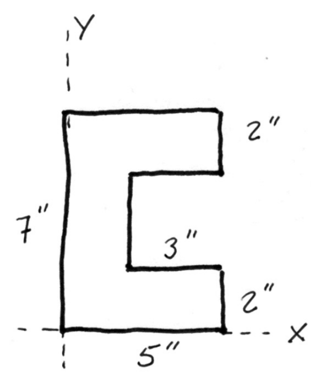
sum = 26

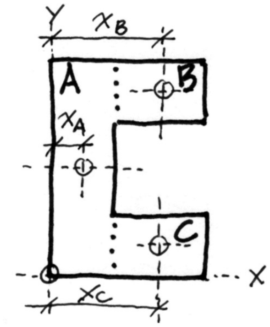
Calculation.

$$\bar{x} = \frac{\sum \mathbf{Area} \times d_x}{\sum \mathbf{Area}} = \frac{\mathrm{A}\, x_A + \mathrm{B}\, x_B + \mathrm{C} x_c}{\mathrm{A} + \mathrm{B} + \mathrm{C}}$$

$$\bar{x} = \frac{(14 \times 1) + (6 \times 3.5) + (6 \times 3.5)}{14 + 6 + 6}$$

$$\bar{x} = \frac{56}{26} = 2.15''$$





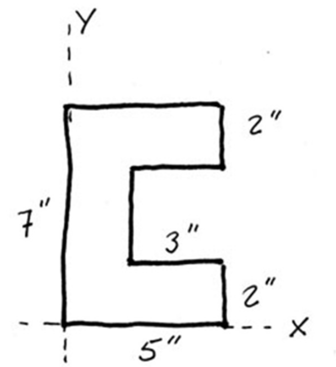
Centroid Example 1 cont.

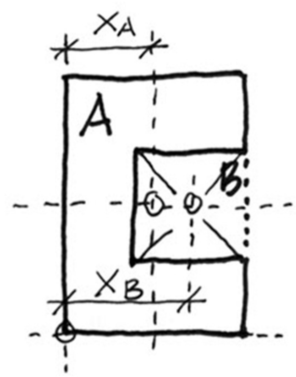
Calculation: by Solid - Void.

$$\bar{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}} = \frac{\mathrm{A}\, x_A - \mathrm{B}\, x_B}{\mathrm{A} - \mathrm{B}}$$

$$\bar{x} = \frac{\sum (35 \times 2.5) - (9 \times 3.5)}{\sum 35 - 9} = \frac{56}{26}$$

$$\bar{x} = 2.15''$$





Static Moment of Area

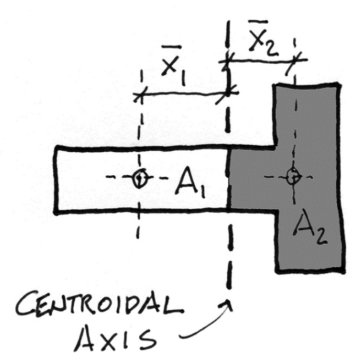
1st moment of area

The tendency of an area alone to rotate about an axis in the plane of that area.

$$Q = \mathrm{A}\bar{x}$$

At the Neutral Axis

$$A_1 \bar{x}_1 = A_2 \bar{x}_2$$

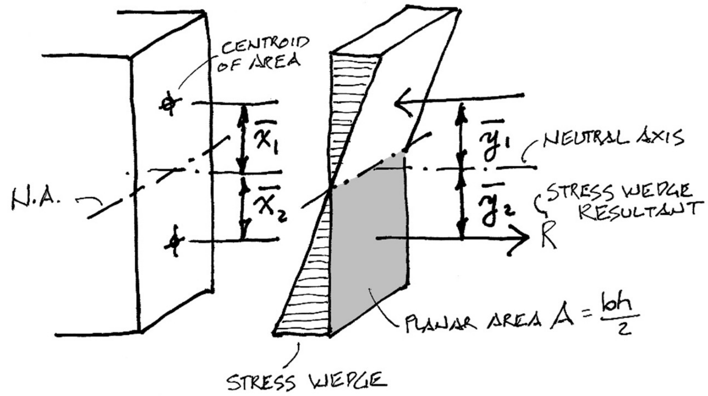


Moment of Inertia

2nd moment of area

By definition:

$$I_x = \mathrm{A}\,\bar{x}\,\bar{y}$$



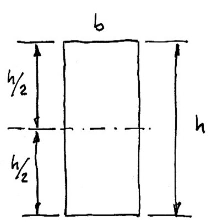
For a rectangle at the N.A.

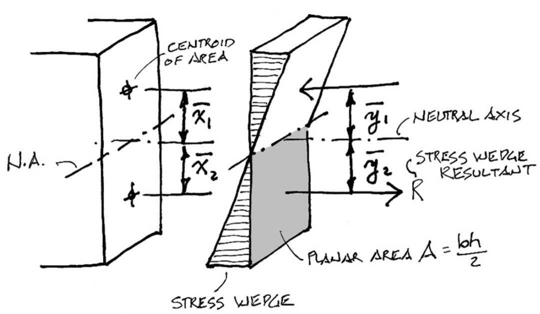
$$I_x = \frac{bh^3}{12}$$

Moment of Inertia

2nd moment of area

$$I_x = \mathrm{A}\,\bar{x}\,\bar{y}$$





FOR A RECTANGULAR SECTION:

$$A = b\,(h/2)$$
$$\bar{x} = h/2 \div 2 = h/4$$
$$\bar{y} = 2/3 \; h/2 = h/3$$

$$A\bar{x}\bar{y}\,(\text{TOP}) = \frac{bh}{2}\,\frac{h}{4}\,\frac{h}{3} = \frac{bh^3}{24}$$
$$A\bar{x}\bar{y}\,(\text{BOTTOM}) = \frac{bh}{2}\,\frac{h}{4}\,\frac{h}{3} = \frac{bh^3}{24}$$

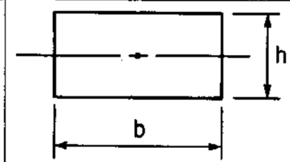
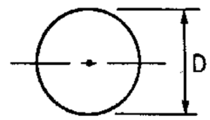
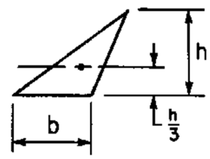
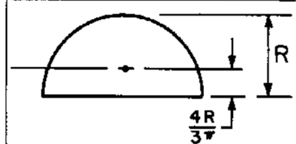
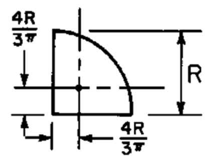
FOR TOTAL SECTION:

$$2 \times \frac{bh^3}{24} = \frac{bh^3}{12}$$

$$I_x = \frac{bh^3}{12}$$

Moment of Inertia

Solutions for basic shapes:

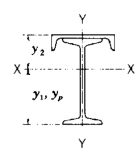
	Shape	Moment of inertia
Rectangle		$\bar{I} = \frac{1}{12}bh^3$
Circle		$\bar{I} = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$
Triangle		$\bar{I} = \frac{1}{36}bh^3$
Semicircle		$\bar{I} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4 = 0.11R^4$
Quarter circle		$\bar{I} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)\frac{R^4}{2} = 0.055R^4$

Moment of Inertia

Solutions for basic shapes:

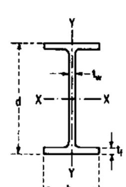
- Single Shapes
- Combination Shapes

COMBINATION SECTIONS
S shapes and channels
Properties of sections



Beam	Channel	Total Wt. per Ft	Total Area	AXIS X-X					AXIS Y-Y			
				I	$S_1 = I/y_1$	$S_2 = I/y_2$	r	y_1	I	S	r	r_T
		Lb.	In.^{2}	In.^{4}	In.^{3}	In.^{3}	In.	In.	In.^{4}	In.^{3}	In.	In.
S 10× 25.4	C 8×11.5	36.9	10.84	176	27.2	46.6	4.02	6.45	39.4	9.85	1.91	2.44
× 25.4	C 10×15.3	40.7	11.95	186	27.6	52.9	3.94	6.73	74.2	14.8	2.49	3.16
S 12× 31.8	C 8×11.5	43.3	12.73	299	39.8	63.2	4.84	7.50	42.0	10.5	1.82	2.38
× 31.8	C 10×15.3	47.1	13.84	316	40.4	71.4	4.78	7.82	76.8	15.4	2.36	3.06
× 40.8	C 10×15.3	56.1	16.49	377	50.1	80.0	4.78	7.53	81.0	16.2	2.22	2.94

WIDE FLANGE SHAPES

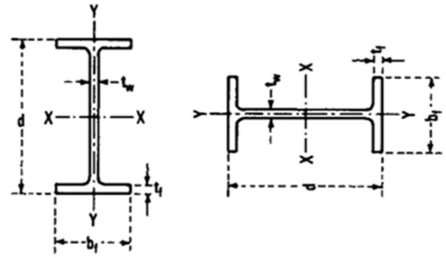


Theoretical Dimensions and Properties for **Designing**

Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Axis X-X I_x	Axis X-X S_x	Axis X-X r_x	Axis Y-Y I_y	Axis Y-Y S_y	Axis Y-Y r_y	r_T
	lb	in.^{2}	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.	in.
W27 x	**178**	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	**114**	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Section Properties

WIDE FLANGE SHAPES



Theoretical Dimensions and Properties for Designing

Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Axis X-X I_x	Axis X-X S_x	Axis X-X r_x	Axis Y-Y I_y	Axis Y-Y S_y	Axis Y-Y r_y	r_T
	lb	in.^{2}	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.	in.
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Section Properties

Rectangular :

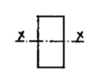
A = bd

$I = db^3/12$

$S = I/c$

c = d/2 (maximum)

PROPERTIES OF SAWN LUMBER SECTIONS

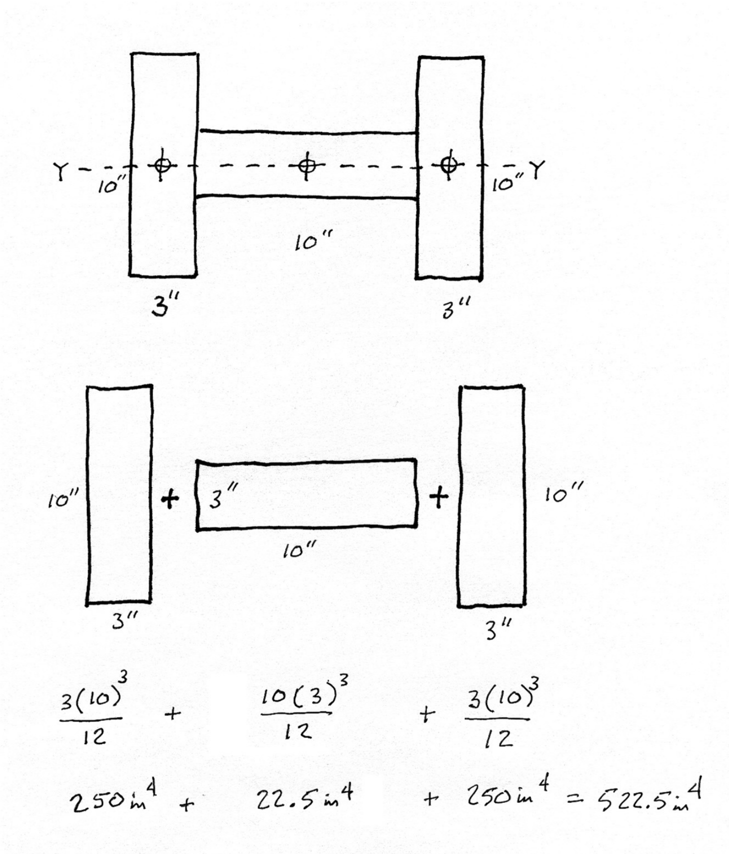


Nominal Size b × d	Actual Size b × d	Area in.^{2}	I_x in.^{4}	S_x in.^{3}
1 × 4	3/4 × 3½	2.63	2.68	1.53
1 × 6	″ × 5½	4.13	10.40	3.78
1 × 8	″ × 7¼	5.44	23.82	6.57
1 × 10	″ × 9¼	6.94	49.47	10.70
1 × 12	″ × 11¼	8.44	88.99	15.83
2 × 4	1½ × 3½	5.25	5.36	3.06
2 × 6	″ × 5½	8.25	20.80	7.56
2 × 8	″ × 7¼	10.88	47.64	13.14
2 × 10	″ × 9¼	13.88	98.93	21.39
2 × 12	″ × 11¼	16.88	177.98	31.64
3 × 4	2½ × 3½	8.75	8.93	5.10
3 × 6	″ × 5½	13.75	34.66	12.60
3 × 8	″ × 7¼	18.13	79.39	21.90
3 × 10	″ × 9¼	23.13	164.89	35.65
3 × 12	″ × 11¼	28.13	296.63	52.73
4 × 4	3½ × 3½	12.25	12.50	7.15
4 × 6	″ × 5½	19.25	48.53	17.65
4 × 8	″ × 7¼	25.38	111.15	30.66
4 × 10	″ × 9¼	32.38	230.84	49.91
4 × 12	″ × 11¼	39.38	415.28	73.83

Moment of Inertia

Shapes with common centroidal axes

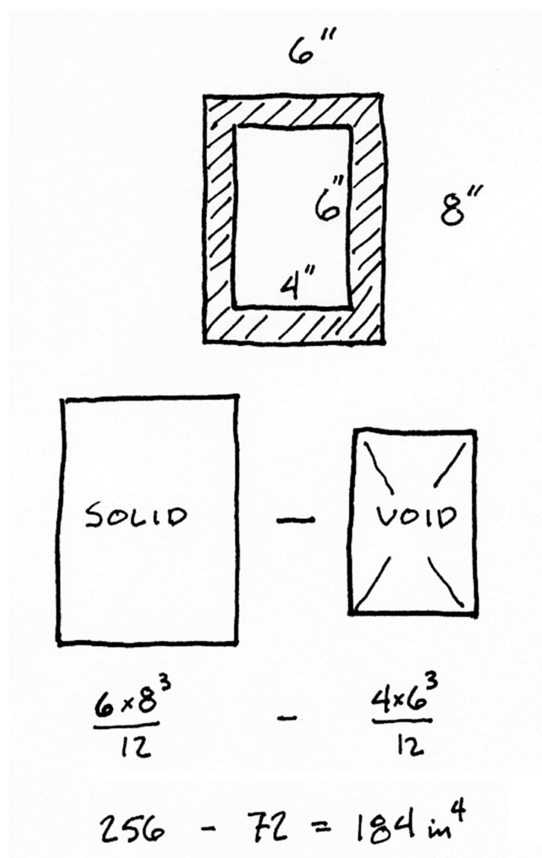
I solid + I solid = I x



Moment of Inertia

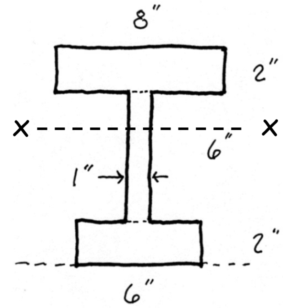
Shapes with common centroidal axes

I solid - I void = I x



Moment of Inertia

The **Transfer Equation** or **Parallel Axis Theorem**, taken about the x-x axis:

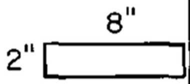
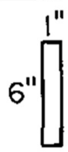
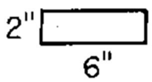


$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$I_x = \sum \bar{I}_x + \sum Ad^2$$

y-bar = 186/34 = 5.48”

Ix = 27.3+439.4 = 466.7 in^4

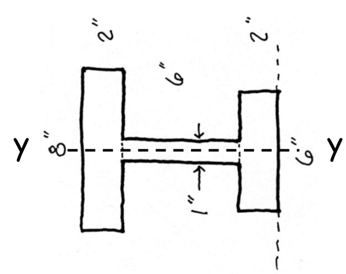
Shape	A	y	Ay	$\bar{I}_x$	d, in.	Ad^2
	(2)(8) = 16	9	144	$(\frac{1}{12})(8)(2)^3$ = 5.3	3.52	$(16)(3.52)^2$ = 198
	(1)(6) = 6	5	30	$(\frac{1}{12})(1)(6)^3$ = 18	0.48	$6(0.48)^2$ = 1.4
	(2)(6) = 12	1	12	$(\frac{1}{12})(6)(2)^3$ = 4	4.48	$12(4.48)^2$ = 240
	$\sum A$ = 34	$\sum Ay$ = 186		$\sum \bar{I}_x$ = 27.3		$\sum Ad^2$ = 439.4

y-bar = 186/34 = 5.48”

Ix = 27.3+439.4 = 466.7 in^4

Moment of Inertia

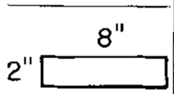
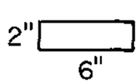
The Transfer Equation or Parallel Axis Theorem:



$$I_y = \sum \bar{I}_y + \sum Ad^2$$

Taken about the y-y axis:

Iy = 121.8+0 = 121.8

Shape	A	$\bar{I}_Y$	d	Ad^2
	16	$(\frac{1}{12})(2)(8)^3$ = 85.3	0	0
	6	$(\frac{1}{12})(6)(1)^3$ = 0.5	0	0
	12	$(\frac{1}{12})(2)(6)^3$ = 36.0	0	0
		$\sum \bar{I}_Y$ = 121.8		0

SUMMARY:

I_x = 466.7 in^4

I_y = 121.8 in^4

Radius of Gyration

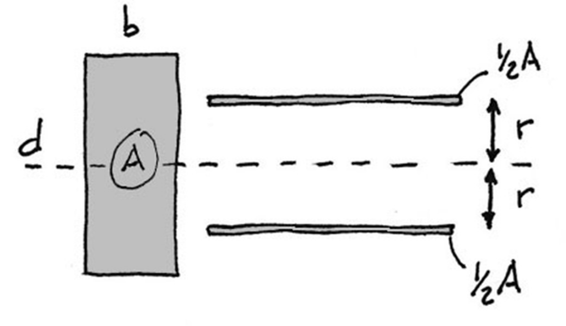
The distance from the centroid where all area could be collected to yield an equivalent Moment of Inertia.

$$I = A\,r^2$$

$$r = \sqrt{\frac{I}{A}}$$

r = 0.289 d

for a rectangle about the N.A



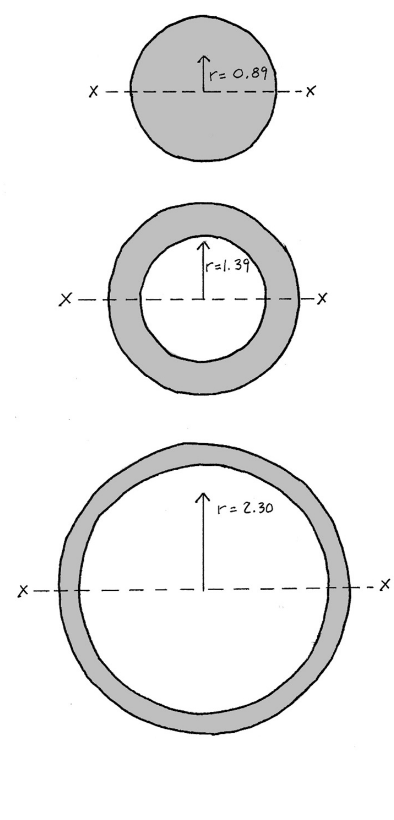
Radius of Gyration

The larger the radius of gyration, the more resistant the section is to buckling.

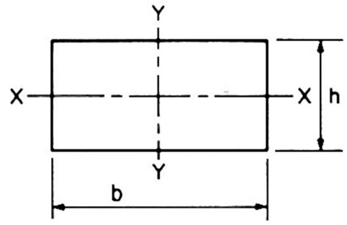
The areas in the table below are constant, while diameters increase.

OD	ID	t	A	r
3.57	0.00	1.78	10.00	0.89
3.71	1.00	1.35	10.00	0.96
4.09	2.00	1.05	10.00	1.14
4.66	3.00	0.83	10.00	1.39
5.36	4.00	0.68	10.00	1.67
6.14	5.00	0.57	10.00	1.98
6.98	6.00	0.49	10.00	2.30
7.86	7.00	0.43	10.00	2.63
8.76	8.00	0.38	10.00	2.97
9.68	9.00	0.34	10.00	3.30
10.62	10.00	0.31	10.00	3.65

$$Pcr = \frac{\pi^2 E}{\left(KL/r\right)^2}$$



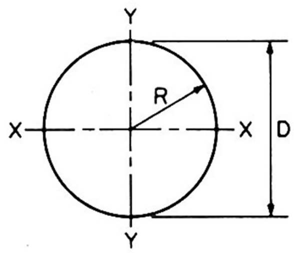
Section Formulas



Rectangle.

Rectangle

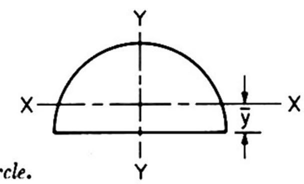
$$A = bh,$$
$$I_x = \tfrac{1}{12}bh^3,$$
$$r_x = \sqrt{I_x/A} = 0.288h.$$



Circle.

Circle

$$A = \tfrac{1}{4}\pi D^2 = \pi R^2,$$
$$I_x = \frac{\pi D^4}{64} = \frac{\pi R^4}{4},$$
$$r_x = \sqrt{I_x/A} = \frac{D}{4} = \frac{R}{2},$$
$$J = I_x + I_y = \frac{\pi D^4}{32} = \frac{\pi R^4}{2}.$$



Semicircle.

Semicircle

$$A = \tfrac{1}{8}\pi D^2 = \tfrac{1}{2}\pi R^2,$$
$$\bar{y} = \frac{4r}{3\pi},$$
$$I_x = 0.00682D^4 = 0.11R^4,$$
$$I_y = \frac{\pi D^4}{128} = \frac{\pi R^4}{8},$$
$$r_x = 0.264R.$$