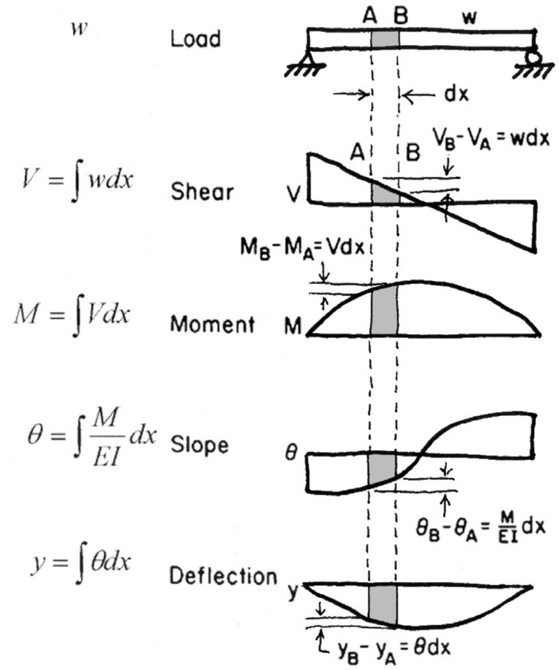
Bending and Shear in Simple Beams

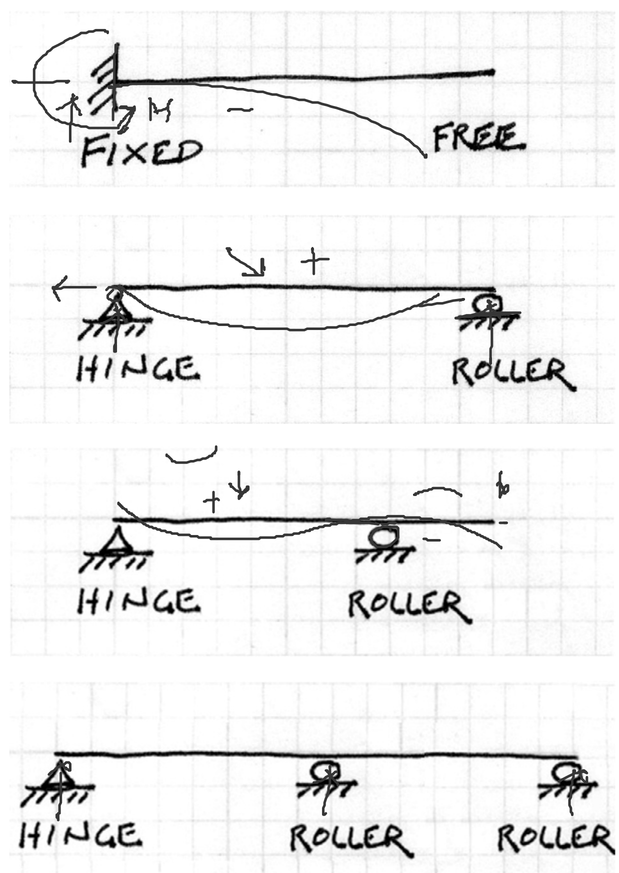
Part 1



- Free Body Diagrams of Shear and Moment in Beams
- Sign Conventions for Plotting V & M Diagrams
- Diagrams by Equilibrium (FBD)
- Diagrams by Integration

Beam Types

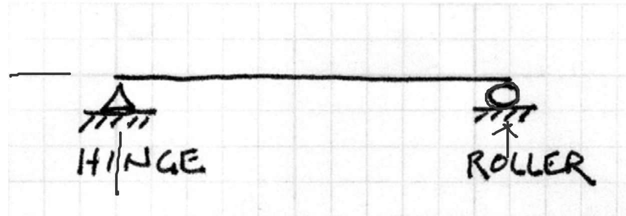
- Cantilever
- Simple
- Simple with Cantilever
- Continuous (multi-span)



Support Conditions

Roller

Fixed in Fy



Hinge (Pinned)

Fixed in Fx

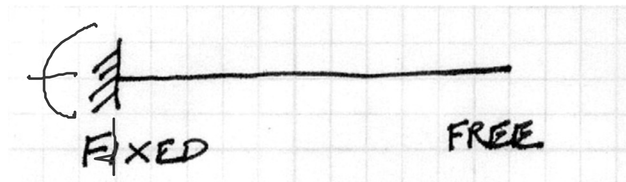
Fixed in Fy

Fixed

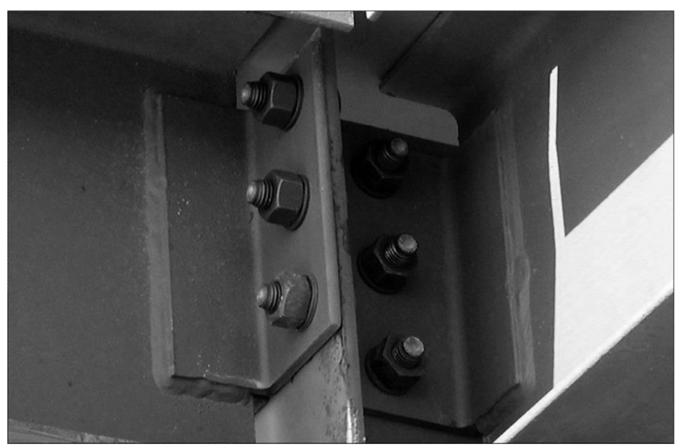
Fixed in Fx

Fixed in Fy

Fixed in Mz



Connection Types



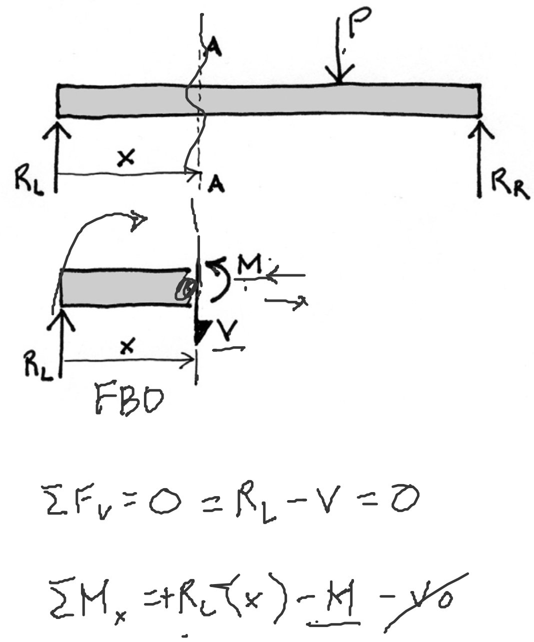
Bearing (or simple)

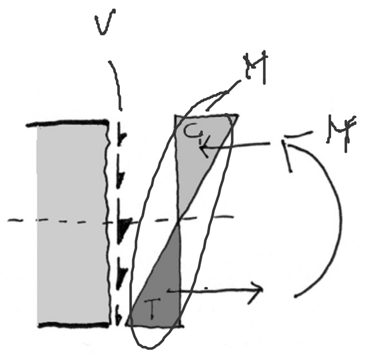
Slip Critical (or fixed)

Internal Shear and Moment

Cutting any section through a beam will reveal internal shear and moment forces necessary to maintain static equilibrium.

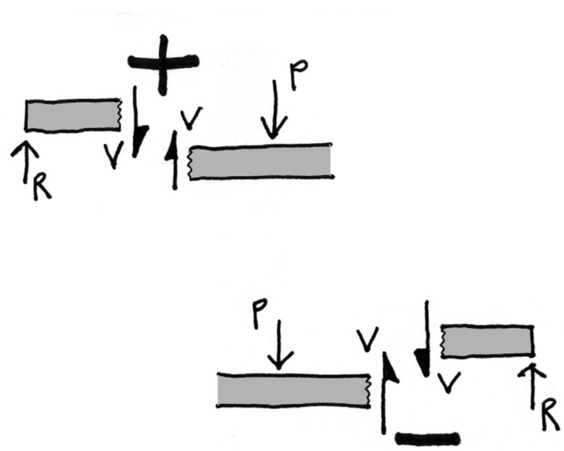
The shears can be determined by summing vertical forces and the moments by summing moments.





Sign Convention for Shear

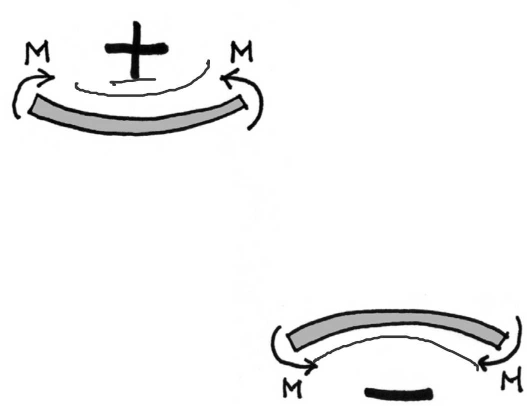
+ the sum of the vertical forces to the left of the cut is upwards
- the sum of the vertical forces to the left of the cut is downwards



Sign Convention for Moment

+ the top fibers are in compression
- the top fibers are in tension

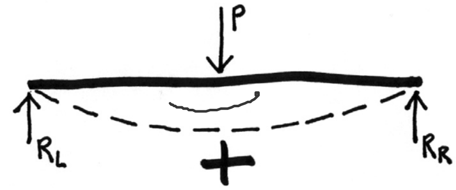
the European moment convention is the reverse

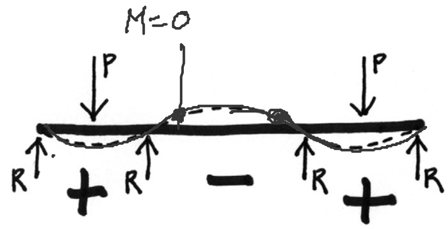


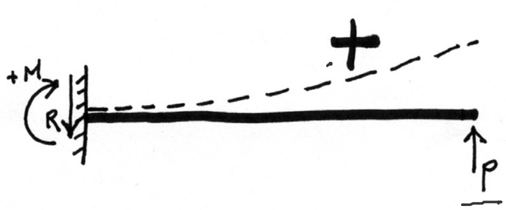
Sign Convention for Moment

- **+** positive curvature (holds water)
- **-** negative curvature (spills water)

the European moment convention is the reverse







Relationships of Forces and Deformations

There are a series of relationships among forces and deformations in a beam, which can be useful in analysis. Using either the deflection or load as a starting point, the following characteristics can be discovered by taking successive derivatives or integrals of the beam equations.

w Load — load $= \dfrac{d^4y}{dx^4} = \dfrac{dV}{dx}\dfrac{1}{EI} = \dfrac{w}{EI}$

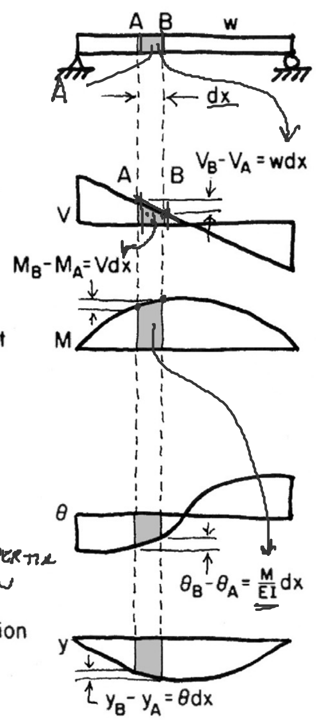
$V = \int w dx$ Shear — shear $= \dfrac{d^3y}{dx^3} = \dfrac{dM}{dx}\dfrac{1}{EI} = \dfrac{V}{EI}$

$M = \int V dx$ Moment — moment $= \dfrac{d^2y}{dx^2} = \dfrac{M}{EI}$

$\theta = \int \dfrac{M}{EI} dx$ Slope — slope $= \dfrac{dy}{dx}$

MATERIAL — MOMENT of INERTIA SECTION

$y = \int \theta dx$ Deflection — deflection $= y$



Methods to Determine Values of Shear and Moment

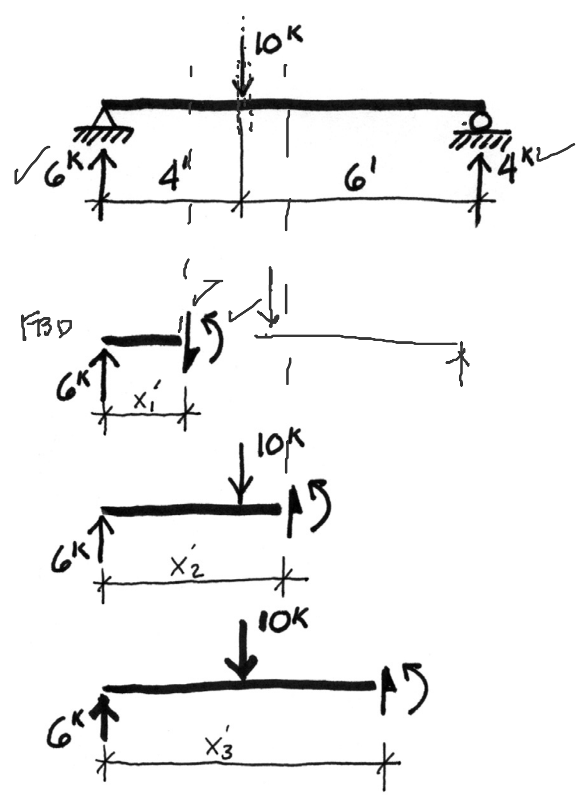
1. Equilibrium Method — FBD
   - Select a point along the beam
   - Cut a section and draw the FBD
   - Solve for the internal shear and moment forces at the section
2. Integration of Equations — PT 1
   - Write the equation of the load function
   - Integrate load equation to get shear equation
   - Solve integration constant (use end reaction)
   - Integrate shear equation to get moment equation
   - Solve integration constant (use point with zero moment, e.g. end point)
3. Semi-graphical Method
   - Draw load diagram and solve end reactions with equilibrium equations.
   - Start at left and construct the shear diagram using point loads and areas on load diagram
   - Calculate areas of shear diagram to find change in value on moment diagram
   - Find points of zero moment to begin moment diagram, e.g. end points
4. Superposition of Equations — PT 2
   - Break the loading into standard cases
   - Use given equations to solve shear and moment for each case
   - Add the cases to get combined values of original loading

1. Equilibrium Method - procedure

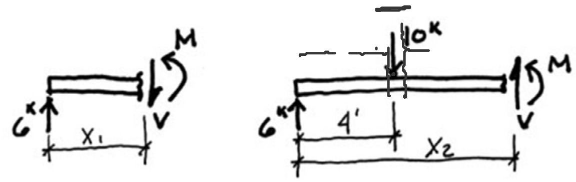
To plot the change of internal shear or moment forces, **a series of sections can be cut** along the beam. The exposed forces can be calculated.

A section should **not be cut "through" an applied force**, but either a bit to the left or to the right of the force.

Either the "left" or "right" free body diagram may be used to calculate the forces. The sign convention described earlier must be consistently applied.

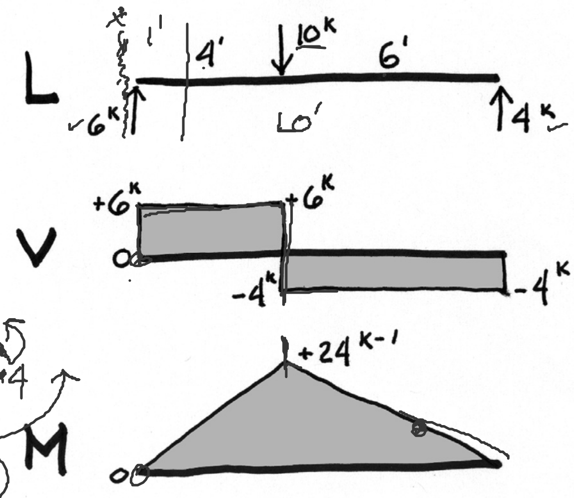


1. Equilibrium Method - example

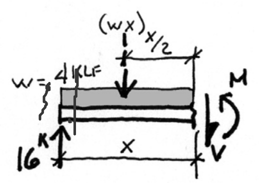


Tabulated Results of FBD Calculations

Cut Location From R_L (ft)	Shear V (k)	Moment M (k-ft)
0-	0	0
0+	6	0
1	6	6
2	6	12
3	6	18
4-	6	24
4+	-4	24
5	-4	20
6	-4	16
7	-4	12
8	-4	8
9	-4	4
10-	-4	0
10+	0	0

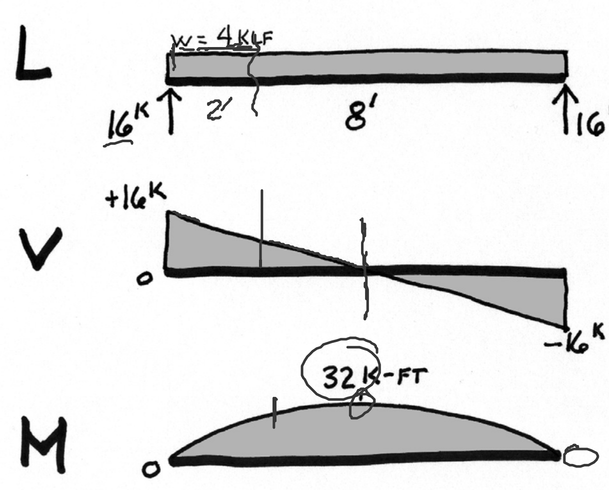


1. Equilibrium Method - example



Tabulated Results of FBD Calculations

Cut Location From R_L (ft)	Shear V (k)	Moment M (k-ft)
0-	0	0
0+	16	0
1	12	14
2	8	24
3	4	30
4	0	32
5	-4	30
6	-8	24
7	-12	14
8-	-16	0
8+	0	0



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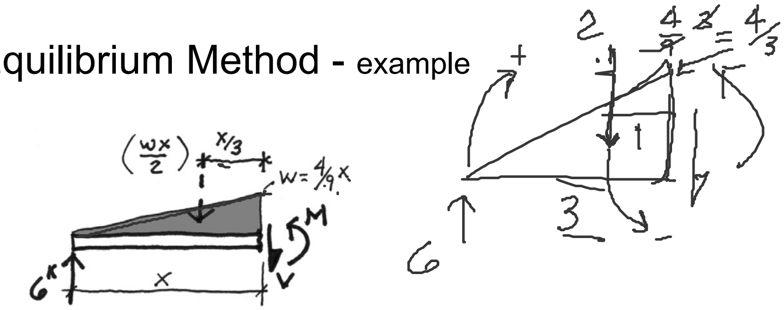


$\Sigma F_V = 6 - 2 - V = 0$

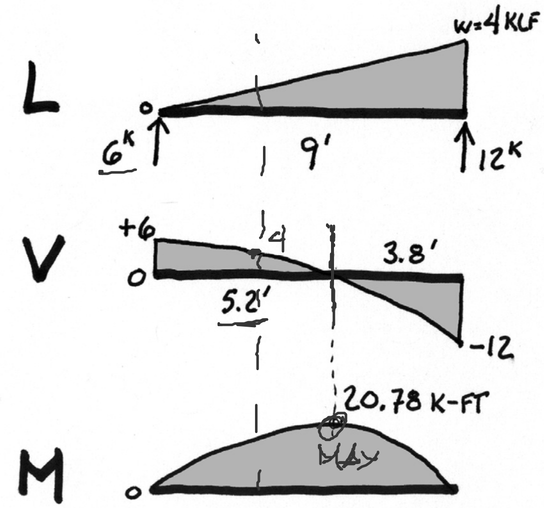
$V = 4^K$

$\Sigma M = 6(3) - 2(1) - M$

$M = 18 - 2 = 16$

Tabulated Results of FBD Calculations

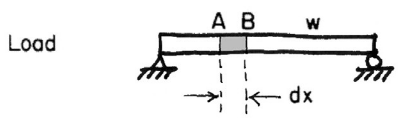
Cut Location From R_L (ft)	**Shear** **V** (k)	**Moment** **M** (k-ft)
0-	0	0
0+	6	0
1	5.78	6.9
2	5.11	11.4
3	4.00	16.0
4	2.44	19.3
5	0.44	20.74
5.2	0	20.78
6	-2.00	20.0
7	-4.90	16.6
8	-8.24	10.0
9-	-12.00	0
9+	0	0



Relationships of Forces and Deformations - procedure

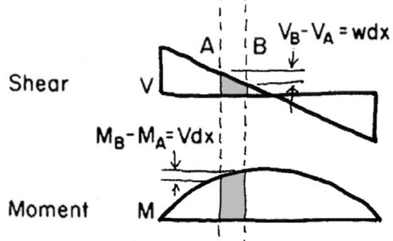
There are a series of relationships among forces and deformations in a beam, which can be useful in analysis. Using either the deflection or load as a starting point, the following characteristics can be discovered by taking successive derivatives or integrals of the beam equations.

w



load $= \dfrac{d^4y}{dx^4} = \dfrac{dV}{dx}\dfrac{1}{EI} = \dfrac{w}{EI}$

$V = \int w dx$

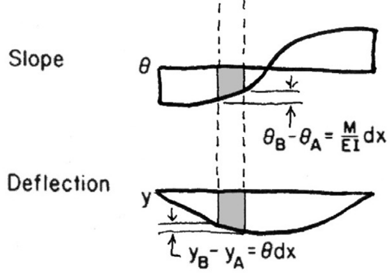


shear $= \dfrac{d^3y}{dx^3} = \dfrac{dM}{dx}\dfrac{1}{EI} = \dfrac{V}{EI}$

$M = \int V dx$

moment $= \dfrac{d^2y}{dx^2} = \dfrac{M}{EI}$

$\theta = \int \dfrac{M}{EI} dx$



slope $= \dfrac{dy}{dx}$

$y = \int \theta dx$

deflection $= y$

2. Shear and Moment by Integration - example

One method of solving shear and moment forces is to write the loading equation and solve the integration equations for the shear and moment. One problem using this method can be finding the constant of integration, particularly with discontinuous load functions.

$$-w$$

$$V = \int w dx$$

$$V = -wx + C$$

$$V = -wx + \frac{wl}{2}$$

$$M = \int V dx$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x + C$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x$$