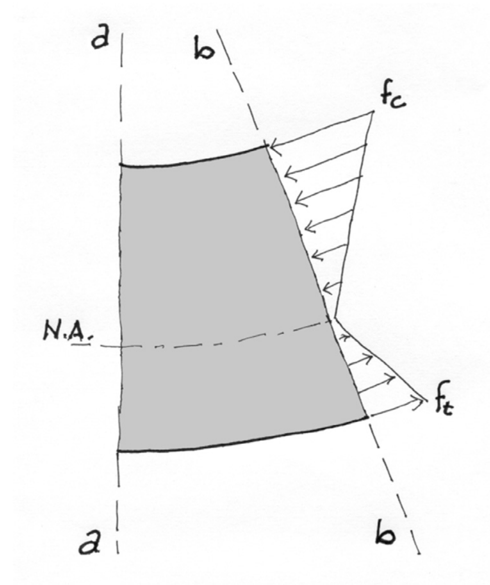
Architecture 314

Structures I

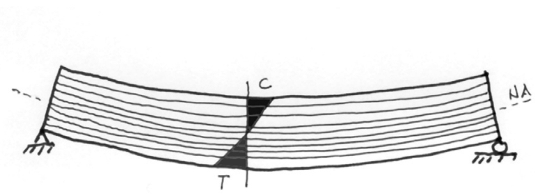
Bending Stresses in Beams



- Elastic Bending
- Stress Equation
- Section Modulus
- Flexure Capacity Analysis
- Flexure Beam Design

Elastic Bending

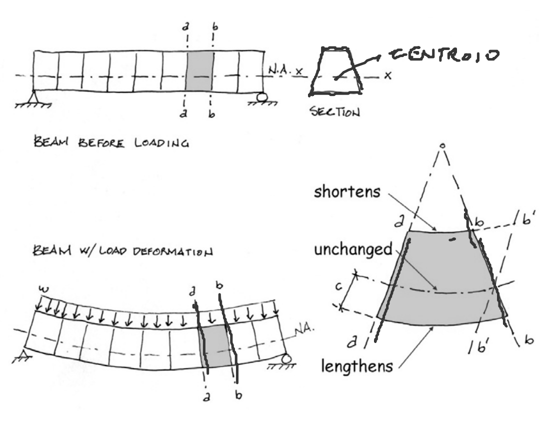
Flexure results in internal tension and compression forces, the resultants of which form a couple which resists the applied moment.



In the initial unloaded state, all transverse sections are parallel.

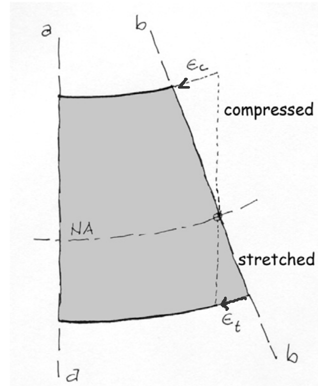
The application of load causes the member to bend in a curve. This means the initial parallel plane sections, while remaining plane, now follow the radii of the curves.

Notice that by the geometry of the curved member the top edge is shortened and the bottom edge is lengthened. Only the neutral axis remains its original length.

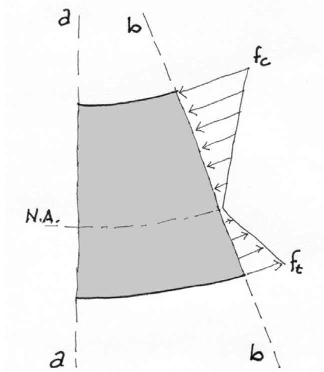


Elastic Bending

The change in lengths, top and bottom, results in the material straining. For a simple span with downward loading, the top is compressed and the bottom stretched. The change in length is linear and proportional to the distance from the Neutral Axis.



The material strains result in corresponding stresses. By **Hooke's Law**, these stresses are proportional to the strains which are proportional to the change in length of the radial arcs of the beam "fibers". This assumes that the Modulus of Elasticity is constant across the section.



Elastic Bending

The applied moment at any point on the beam is equal to the resisting moment which is formed by the internal force couple, R_c and R_t.

$$M_{applied} = M_{resisting}$$

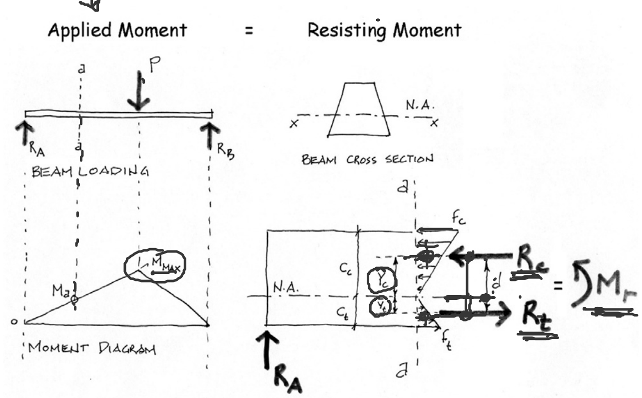
Balance of the external and internal moments

$$R_{comp.} = R_{tens.}$$

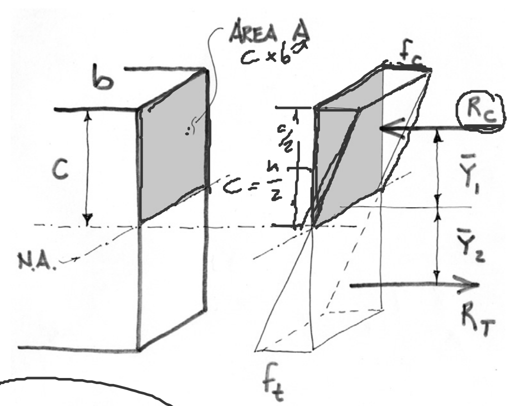
Balance of the internal force couple

$$M_r = R_c \cdot y_c + R_t \cdot y_t$$
$$M_r = R_c \cdot d$$
$$M_r = R_t \cdot d$$

Expressions of the internal resisting moment



Elastic Bending



The internal moment, M_r, can be expressed as the result of the couple R_c and R_t.

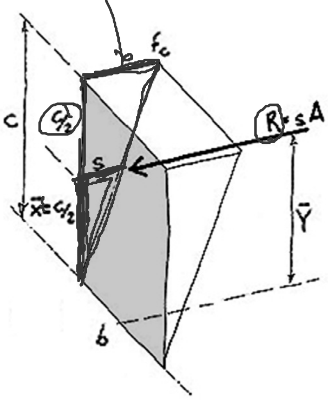
$$M_r = R_c \cdot \bar{y}_1 + R_t \cdot \bar{y}_2$$

In turn, the forces R_c and R_t, can be written as the resultants of the “stress volumes” acting through the centroids of those volumes. The stress volumes equal c times b times the average stress (s). The average unit stress, s = fc/2. The resultant R is the area times s:

$$R = A \cdot s$$

Using similar triangles, s can be expressed as:

$$\frac{s}{f_c} = \frac{\bar{x}}{c} \quad \text{and} \quad s = \frac{f_c \cdot \bar{x}}{c}$$



Substituting these values back into the moment equation gives:

$$M_r = \frac{f_c A_c \bar{x}_1 \bar{y}_1}{c_c} + \frac{f_t A_t \bar{x}_2 \bar{y}_2}{c_t}$$

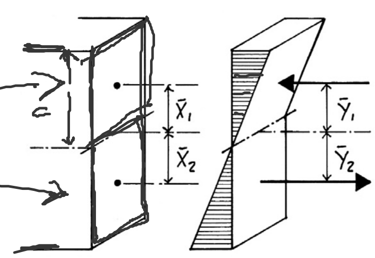
Elastic Bending

By definition:

$$\mathrm{I}_x = \mathrm{A}\overline{xy}$$

And for homogeneous materials with $E_c = E_t$

$$M_r = \frac{f\,I_1}{c} + \frac{f\,I_2}{c} = \frac{f}{c}\left(I_1 + I_2\right)$$



Or using the I for the whole section:

$$M_r = \frac{f\,I}{c}$$

And so,

$$f = \frac{M\,c}{I}$$

The Section Modulus is:

$$S = \frac{I}{c}$$

With c = h/2 at extreme fibers of a symmetric section.

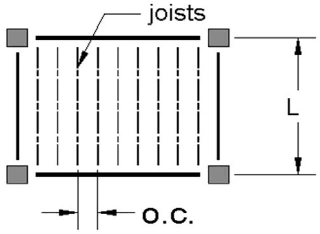
So, at extreme fibers:

$$M = f\,S$$

And:

$$f = \frac{M}{S}$$

Beam Analysis



Allowable Capacity (ASD):

$$M = F_b S$$

for steel: $F_b = (0.66 \text{ to } 0.6)\, F_y$ ksi

for wood: $F_b = 1000 \text{ to } 600$ psi

Applied Load:

$$M = \frac{wl^2}{8}$$

(uniform load)

Pass

$$M = F_b S \;>\; M = \frac{wl^2}{8}$$

Fail

$$M = F_b S \;<\; M = \frac{wl^2}{8}$$

Capacity

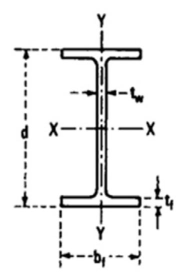
$$M = F_b S \;=\; M = \frac{wl^2}{8}$$ solve for w

Design

$$M = \frac{wl^2}{8} \;=\; M = F_b S$$ solve for S

Beam Capacity Analysis - procedure

1. Determine section properties. (from table)
2. Choose safe allowable stress. (depends on bracing)
3. Calculate allowable moment capacity. $M = F_b S$
4. Set equal to applied moment and find load. $M = \frac{wl^2}{8}$



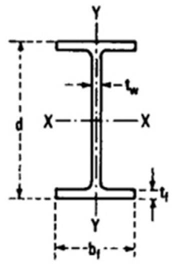
WIDE FLANGE SHAPES

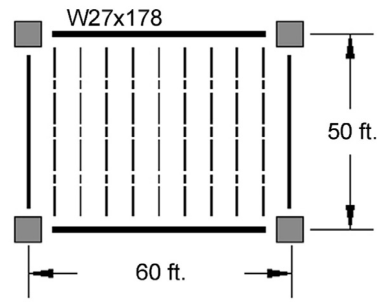
Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Axis X-X I_x	Axis X-X S_x	Axis X-X r_x	Axis Y-Y I_y	Axis Y-Y S_y	Axis Y-Y r_y	r_T
	lb	in.²	in.	in.	in.	in.	in.⁴	in.³	in.	in.⁴	in.³	in.	in.
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Beam Capacity Analysis - example

Given:

Beam = W27x178
S_x = 502 in^3
Fy = 50 ksi
Fb = .66Fy = 33 ksi (braced by joists)





Find:

Floor capacity

WIDE FLANGE SHAPES

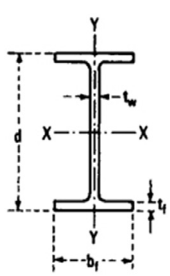
Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Axis X-X I_x	Axis X-X S_x	Axis X-X r_x	Axis Y-Y I_y	Axis Y-Y S_y	Axis Y-Y r_y	r_T
	lb	in.^{2}	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.	in.
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
	161	47.4	27.59	14.020	1.080	0.660	6280	455	11.5	497	70.9	3.24	3.70
	146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	3.68
W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
	102	30.0	27.09	10.015	0.830	0.515	3620	267	11.0	139	27.8	2.15	2.56
	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

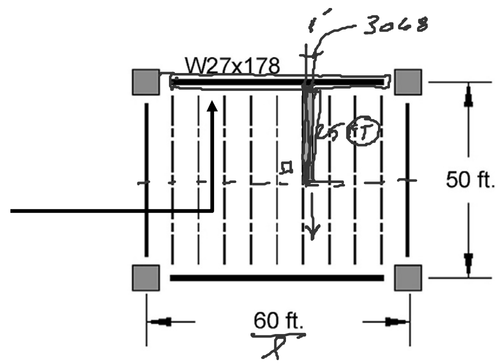
WF
DEEP

Beam Capacity Analysis

Given:

Beam = W27x178
Sx = 502 in3
Fy = 50 ksi
Fb = .66Fy = 33 ksi (fully braced)





Find:

Floor capacity

$M = F_b S_x$

$F_b = 50(.66)$

$M = 33^{KSI} \; 502 \, in^3 = 16566^{K-''} = 1380.5^{K-'}$

$M = 1380.5^{K-'}$

$M = \frac{w \ell^2}{8}$

$w = \frac{M\,8}{\ell^2} = \frac{1380.5\,(8)}{60^2} = 3.068^{K/'} = 3068^{\#/'}$

$PSF = \frac{w}{\ell_1/2} = \frac{3068}{50/2} = 123 \; PSF$

Quiz

Given:

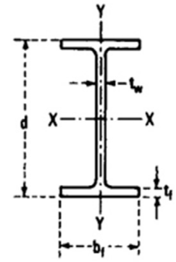
Beam = W27x114 $\qquad$ Sx = ______ in^3

Fy = 36 ksi

Fb = .6Fy = __________ ksi

Find:

$F_b S_x$ = Allowable Moment = __________ ft-lbs



WIDE FLANGE SHAPES

Section Number	Weight per Foot	Area of Section A	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Axis X-X I_x	Axis X-X S_x	Axis X-X r_x	Axis Y-Y I_y	Axis Y-Y S_y	Axis Y-Y r_y	r_T
	lb	in.²	in.	in.	in.	in.	in.⁴	in.³	in.	in.⁴	in.³	in.	in.
W27 x	178	52.3	27.81	14.085	1.190	0.725	6990	502	11.6	555	78.8	3.26	3.72
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W27 x	114	33.5	27.29	10.070	0.930	0.570	4090	299	11.0	159	31.5	2.18	2.58
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	94	27.7	26.92	9.990	0.745	0.490	3270	243	10.9	124	24.8	2.12	2.53
	84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	2.49

Section Properties

Section Modulus Table

Sorted by Sx for design selection

with:

S = I/c

f_b is actual stress

F_b is allowable stress

- for bracing < L_c, $F_b = 0.66F_y$
- for bracing < L_u, $F_b = 0.6F_y$

F_y is the yield stress

$M_r = .66Fy\ S_x$

So the design equations is:

$S_x = M_{applied}/F_b$ → 600

ALLOWABLE STRESS DESIGN SELECTION TABLE
For shapes used as beams S_x

F_y = 50 ksi L_c	F_y = 50 ksi L_u	F_y = 50 ksi M_R	S_x	Shape	Depth d	F_y'	F_y = 36 ksi L_c	F_y = 36 ksi L_u	F_y = 36 ksi M_R
Ft	Ft	Kip-ft	In.³		In.	Ksi	Ft	Ft	Kip-ft
10.6	**11.2**	**2130**	**776**	**W 44×198**	**42⅞**	**—**	**12.5**	**15.5**	**1540**
14.1	15.2	2110	769	W 40×199	38⅝	—	16.6	20.0	1520
11.8	45.7	2110	769	W 21×333	25	—	13.9	63.4	1520
14.2	19.8	2080	757	W 33×221	33⅞	—	16.7	27.6	1500
13.5	24.0	2050	746	W 30×235	31¼	—	15.9	33.3	1480
12.8	29.0	2040	742	W 27×258	29	—	15.1	40.3	1470
10.9	15.1	1980	719	W 36×210	36¾	—	12.9	20.9	1420
11.9	34.7	1970	718	W 24×279	26¾	—	14.0	48.2	1420
12.8	**16.7**	**1880**	**708**	**W 40×192**	**38¼**	**37.1**	**17.8**	**19.7**	**1400**
11.6	42.7	1900	692	W 21×300	24½	—	13.7	59.4	1370
14.1	17.9	1880	684	W 33×201	33⅝	—	16.6	24.9	1350
10.6	**12.3**	**1880**	**682**	**W 40×183**	**39**	**—**	**12.5**	**17.1**	**1350**
12.7	26.7	1850	674	W 27×235	28⅝	—	15.0	37.0	1330
10.9	13.9	1830	664	W 36×194	36½	—	12.8	19.4	1310
13.5	21.4	1820	663	W 30×211	31	—	15.9	29.7	1310
11.8	31.4	1770	644	W 24×250	26⅜	—	13.9	43.7	1280
11.5	39.2	1740	632	W 21×275	24⅛	—	13.6	54.5	1250
12.6	24.9	1720	624	W 27×217	28⅜	—	14.9	34.5	1240
10.8	49.0	1720	624	W 18×311	22⅜	—	12.7	68.1	1240
10.8	**13.1**	**1710**	**623**	**W 36×182**	**36⅜**	**—**	**12.7**	**18.2**	**1230**
10.4	**11.0**	**1650**	**599**	**W 40×167**	**38⅝**	**—**	**12.5**	**14.5**	**1190**
13.5	19.4	1640	598	W 30×191	30⅝	—	15.9	26.9	1180
11.7	29.0	1620	588	W 24×229	26	—	13.8	40.3	1160
10.8	12.2	1600	580	W 36×170	36⅛	—	12.7	17.0	1150
11.4	35.5	1560	569	W 21×248	23¾	—	13.5	49.3	1130
10.6	45.0	1550	564	W 18×283	21⅞	—	12.6	62.6	1120
12.6	22.4	1530	556	W 27×194	28⅛	—	14.8	31.1	1100
10.3	13.8	1510	549	W 33×169	33⅞	—	12.1	19.2	1090
10.7	**11.4**	**1490**	**542**	**W 36×160**	**36**	**—**	**12.7**	**15.7**	**1070**
13.4	17.5	1480	539	W 30×173	30½	—	15.8	24.2	1070
11.7	26.5	1460	531	W 24×207	25¾	—	13.7	36.7	1050
10.5	42.2	1410	514	W 18×258	21½	—	12.4	58.6	1020
8.5	**10.7**	**1410**	**512**	**W 40×149**	**38¼**	**—**	**11.9**	**12.6**	**1010**
11.4	32.7	1400	510	W 21×223	23⅜	—	13.4	45.4	1010
10.5	11.3	1390	504	W 36×150	35⅞	—	12.6	14.6	998
12.6	20.1	1380	502	W 27×178	27¾	—	14.9	27.9	994
11.6	24.7	1350	491	W 24×192	25½	—	13.7	34.3	972
10.4	12.2	1340	487	W 33×152	33½	—	12.2	16.9	964
10.4	38.8	1280	466	W 18×234	21	—	12.3	53.8	923
11.3	29.8	1270	461	W 21×201	23	—	13.3	41.3	913
12.6	18.3	1250	455	W 27×161	27⅝	—	14.8	25.4	901
11.5	22.8	1240	450	W 24×176	25¼	—	13.6	31.7	891

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Beam Design - procedure

1. Choose a steel grade and allowable stress.
2. Determine the applied moment (e.g. moment diagram)
3. Calculate the section modulus, S_x
4. Choose a safe section. (from S_x table)

$$S_x = \frac{M}{F_b}$$

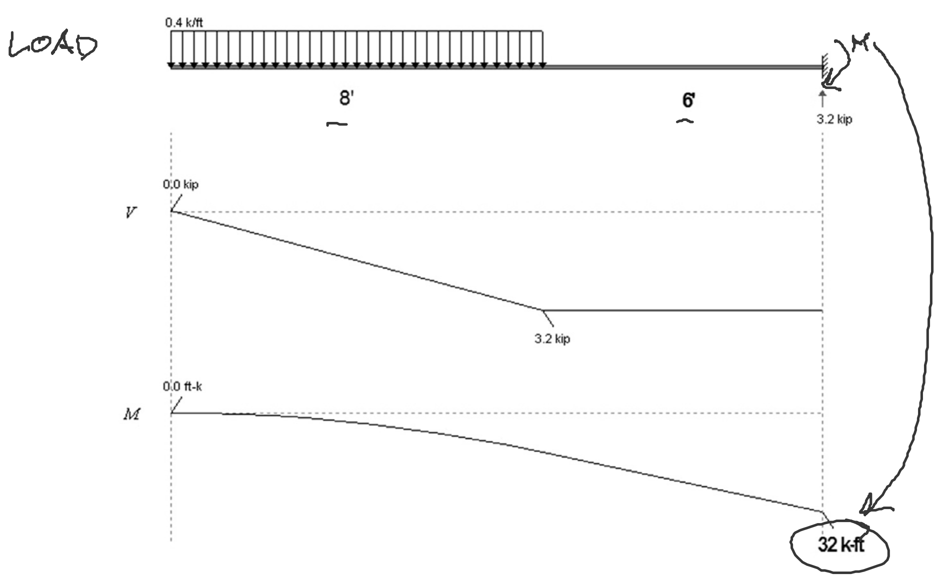
ALLOWABLE STRESS DESIGN SELECTION TABLE
For shapes used as beams S_x

F_y = 50 ksi			S_x	Shape	Depth d	F'_y	F_y = 36 ksi		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	In³		In	Ksi	Ft	Ft	Kip-ft
2.9	**3.6**	**47**	**17.1**	**W 12×16**	**12**	**—**	**4.1**	**4.3**	**34**
5.4	14.4	46	16.7	W 6×25	6⅜	—	6.4	20.0	33
3.6	4.4	45	16.2	W 10×17	10⅛	—	4.2	6.1	32
4.7	7.1	42	15.2	W 8×18	8⅛	—	5.5	9.9	30
2.5	**3.6**	**41**	**14.9**	**W 12×14**	**11⅞**	**54.3**	**3.5**	**4.2**	**30**
3.6	3.7	38	13.8	W 10×15	10	—	4.2	5.0	27
5.4	11.8	37	13.4	W 6×20	6¼	62.1	6.4	16.4	27
5.3	12.5	36	13.0	M 6×20	6	—	6.3	17.4	26
1.9	**2.6**	**33**	**12.0**	**M 12×11.8**	**12**	**—**	**2.7**	**3.0**	**24**
3.6	5.2	32	11.8	W 8×15	8⅛	—	4.2	7.2	23
2.8	3.6	30	10.9	W 10×12	9⅞	47.5	3.9	4.3	22

Beam Design - steel

Using **Steel W section**:

1. Choose a steel grade: Using F_y = 50 ksi $F_b = 0.6\ F_y$
2. Determine the applied moment



Beam Design – steel

$$S_x = \frac{M}{F_b}$$

Using Steel W section:

2. Calculate section modulus, S_x

$$S_x = M/F_b = \frac{32 (k\text{-}ft)(12)}{0.6(50\ ksi)}$$

$$S_x = 12.8\ in^3$$

Beam Design – steel

Using Steel W section:

3. Choose a safe section. (from S_x table)

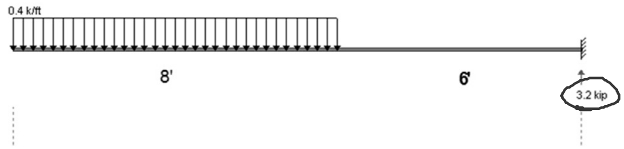
$S_x \geq 12.8\ in^3$

W12x14 is the lightest section with Sx > 12.8

ALLOWABLE STRESS DESIGN SELECTION TABLE
For shapes used as beams S_x

F_y = 50 ksi			S_x	Shape	Depth d	F'_y	F_y = 36 ksi		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	In^3		In	Ksi	Ft	Ft	Kip-ft
2.9	**3.6**	**47**	**17.1**	**W 12×16**	**12**	—	**4.1**	**4.3**	**34**
5.4	14.4	46	16.7	W 6×25	6⅜	—	6.4	20.0	33
3.6	4.4	45	16.2	W 10×17	10⅛	—	4.2	6.1	32
4.7	7.1	42	15.2	W 8×18	8⅛	—	5.5	9.9	30
2.5	**3.6**	**41**	**14.9**	**W 12×14**	**11⅞**	**54.3**	**3.5**	**4.2**	**30**
3.6	3.7	38	13.8	W 10×15	10	—	4.2	5.0	27
5.4	11.8	37	13.4	W 6×20	6¼	62.1	6.4	16.4	27
5.3	12.5	36	13.0	M 6×20	6	—	6.3	17.4	26
1.9	**2.6**	**33**	**12.0**	**M 12×11.8**	**12**	—	**2.7**	**3.0**	**24**
3.6	5.2	32	11.8	W 8×15	8⅛	—	4.2	7.2	23
2.8	3.6	30	10.9	W 10×12	9⅞	47.5	3.9	4.3	22

Beam Design – Glulam



Using **Glulam Timber**:

F_b = 1250 psi (DF grade L3)

$$S_x = \frac{M}{F_b}$$

$$S_x = \frac{M_{APPLIED}}{F_b} = \frac{32000\ \#\text{-}'(12)}{1250\ psi} = 307.2\ in^3$$

Table 5B Reference Design Values for Structural Glued Laminated Softwood Timber

(Members stressed primarily in axial tension or compression) (Tabulated design values are for normal load duration and dry service conditions. See NDS 5.3 for a comprehensive description of design value adjustment factors.)

Use with Table 5B Adjustment Factors

Combination Symbol	Species	Grade	All Loading: Modulus of Elasticity, For Deflection Calculations E (10^6 psi)	All Loading: Modulus of Elasticity, For Stability Calculations E_{min} (10^6 psi)	All Loading: Compression Perpendicular to Grain $F_{c\perp}$ (psi)	Axially Loaded: Tension Parallel to Grain, 2 or More Laminations F_t (psi)	Axially Loaded: Compression Parallel to Grain, 4 or More Laminations F_c (psi)	Axially Loaded: Compression Parallel to Grain, 2 or 3 Laminations F_c (psi)	Bending about Y-Y Axis (Loaded Parallel to Wide Faces of Laminations): Bending, 4 or More Laminations F_{by} (psi)	Bending, 3 Laminations F_{by} (psi)	Bending, 2 Laminations F_{by} (psi)	Shear Parallel to Grain(1)(2)(3) F_{vy} (psi)	Bending About X-X Axis (Loaded Perpendicular to Wide Faces of Laminations): Bending, 2 Laminations to 15 in. Deep(4) F_{bx} (psi)	Shear Parallel to Grain(3) F_{vx} (psi)	Fasteners: Specific Gravity for Fastener Design G
Visually Graded Western Species															
1	DF	L3	1.5	0.79	560	950	1550	1250	1450	1250	1000	230	1250	265	0.50
2	DF	L2	1.6	0.85	560	1250	1950	1600	1800	1600	1300	230	[illegible]	265	0.50
3	DF	L2D	1.9	1.00	650	1450	2300	1900	2100	1850	1550	230	2000	265	0.50
4	DF	L1CL	1.9	1.00	590	1400	2100	1950	2200	2000	1650	230	2100	265	0.50
5	DF	L1	2.0	1.06	650	1650	2400	2100	2400	2100	1800	230	2200	265	0.50

Section Properties

Using Glulam Timber:

Glulam Timbers – 8 ¾" wide

S_x required = 307.2 in^3

Use 8 ¾" x 15" Sx = 328.1 > 307.2 in^3

$$\frac{131 \times 1}{144} \times 24 = 21.82\ \#$$

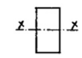
Table 1C Section Properties of *Western Species* Structural Glued Laminated Timber (Cont.)

Depth d (in.)	Area A ($in.^2$)	X-X Axis I_x ($in.^4$)	X-X Axis S_x ($in.^3$)	X-X Axis r_x (in.)	Y-Y Axis I_y ($in.^4$)	Y-Y Axis S_y ($in.^3$)
8-3/4 in. Width					**(r_y = 2.526 in.)**	
9	78.75	531.6	118.1	2.598	502.4	114.8
10-1/2	91.88	844.1	160.8	3.031	586.2	134.0
12	105.0	1260	210.0	3.464	669.9	153.1
13-1/2	118.1	1794	265.8	3.897	753.7	172.3
15	131.3	2461	328.1	4.330	837.4	191.4
16-1/2	144.4	3276	397.0	4.763	921.1	210.5
18	157.5	4253	472.5	5.196	1005	229.7
19-1/2	170.6	5407	554.5	5.629	1089	248.8
21	183.8	6753	643.1	6.062	1172	268.0

Section Properties

Sawn Lumber

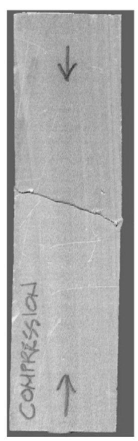
PROPERTIES OF SAWN LUMBER SECTIONS



Nominal Size b × d	Actual Size b × d	Area in.^{2}	I_x in.^{4}	S_x in.^{3}
1 × 4	3/4 × 3½	2.63	2.68	1.53
1 × 6	" × 5½	4.13	10.40	3.78
1 × 8	" × 7¼	5.44	23.82	6.57
1 × 10	" × 9¼	6.94	49.47	10.70
1 × 12	" × 11¼	8.44	88.99	15.83
2 × 4	1½ × 3½	5.25	5.36	3.06
2 × 6	" × 5½	8.25	20.80	7.56
2 × 8	" × 7¼	10.88	47.64	13.14
2 × 10	" × 9¼	13.88	98.93	21.39
2 × 12	" × 11¼	16.88	177.98	31.64
3 × 4	2½ × 3½	8.75	8.93	5.10
3 × 6	" × 5½	13.75	34.66	12.60
3 × 8	" × 7¼	18.13	79.39	21.90
3 × 10	" × 9¼	23.13	164.89	35.65
3 × 12	" × 11¼	28.13	296.63	52.73
4 × 4	3½ × 3½	12.25	12.50	7.15
4 × 6	" × 5½	19.25	48.53	17.65
4 × 8	" × 7¼	25.38	111.15	30.66
4 × 10	" × 9¼	32.38	230.84	49.91
4 × 12	" × 11¼	39.38	415.28	73.83

Modes of Failure

Strength
- Tension rupture
- Compression crushing
- Flexure
- Shear



Stability
- Column buckling
- Beam lateral torsional buckling

Serviceability
- Beam deflection
- Building story drift
- cracking