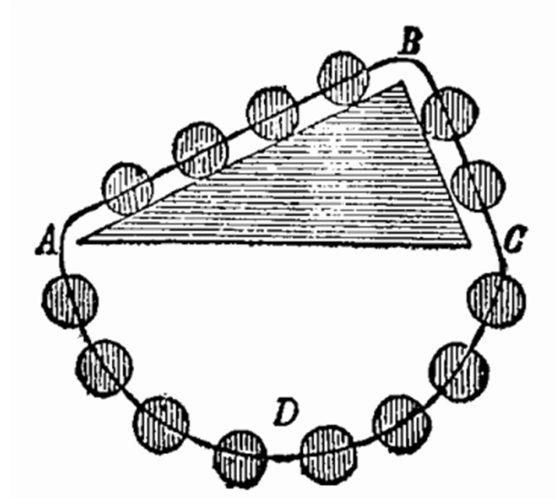


Statics and Force Vectors

- Components
- Resultants & Equilibrants
- Graphic method
- Analytic method

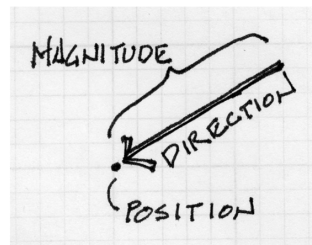


Simon Steven from *Weeghconst* (1586)

Force Definitions

Single vector

- Magnitude
- Direction
- Point of Application

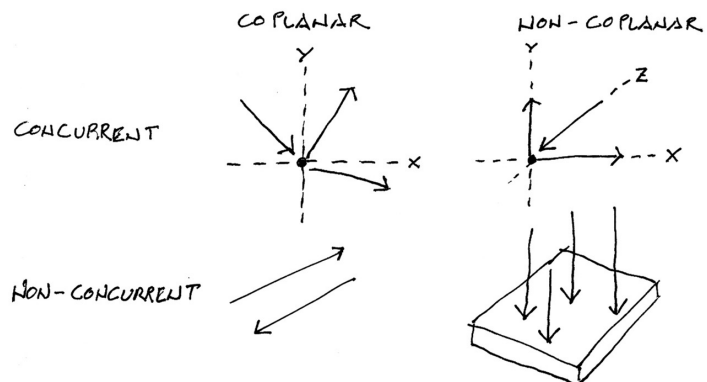


Force Transmissibility

- A force can be resolved at any point along its line of action
- The external affect on a body is unchanged

Force Systems

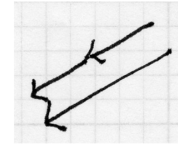
- Concurrent – Coplanar
- Non-concurrent - Coplanar
- Concurrent – Non-coplanar
- Non-concurrent – Non-coplanar



Force Addition

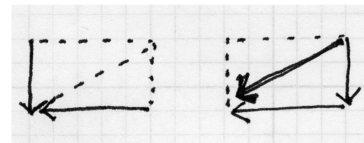
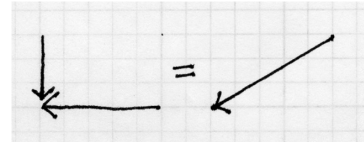
Inline forces

- By linear addition



Orthogonal forces

- Pythagorean Theorem

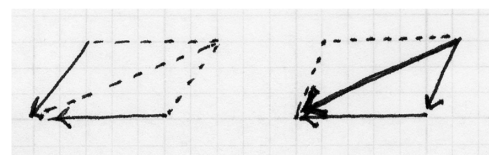


Graphic Method

Addition of Two Forces

Force Parallelogram

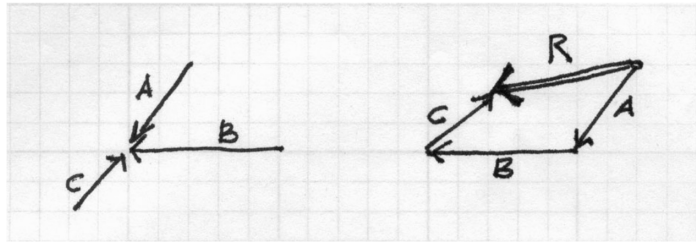
The diagonal is the vector addition
of the two sides



Resultant

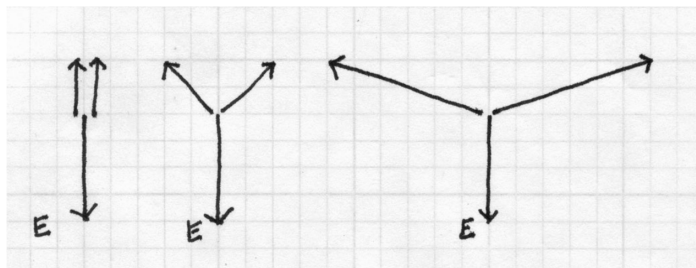
Addition of two or more forces

- Force parallelogram
- Force polygon



Equilibrant

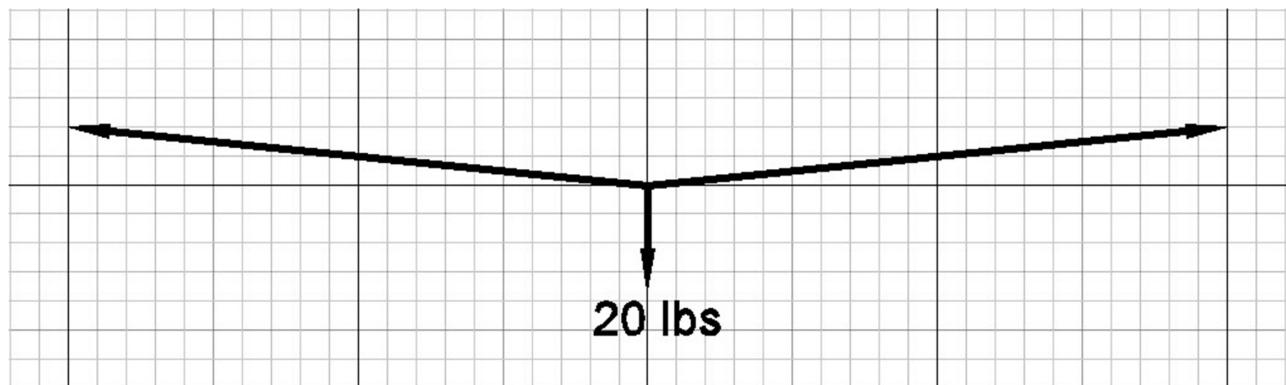
Opposite and equal to the resultant



Lecture Quiz - Find the Balancing Forces

Use the graphic approach to determine the force components in the rope with a suspended load of 20 pounds. The slope of the rope is 1:10.

What is the total force in the rope?

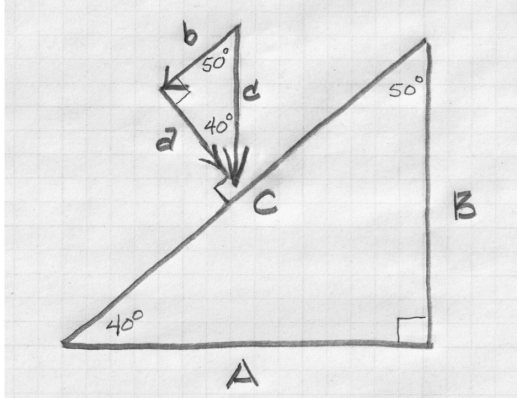


Force Components

Orthogonal

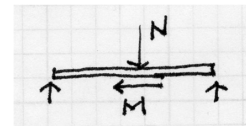
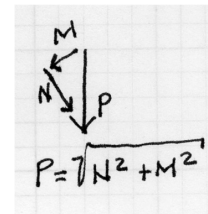
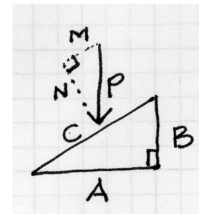
- Horizontal
- Vertical

Force Decomposition



$$\frac{C}{c} = \frac{A}{a} = \frac{B}{b}$$

University of Michigan, TCAUP



Structures I

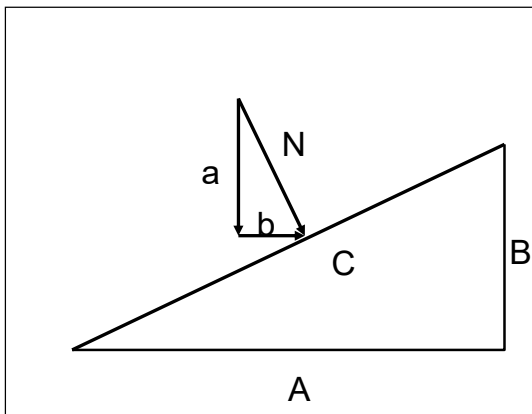
Slide 7 of 13

Force Components

Orthogonal

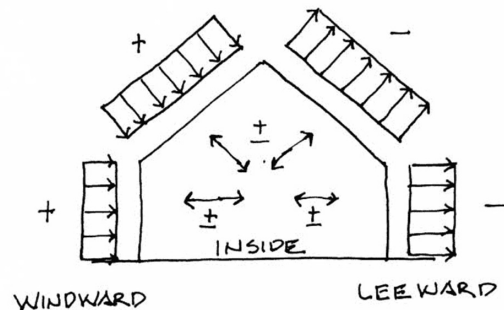
- Horizontal
- Vertical

Decomposition of a Normal Force



$$\frac{C}{N} = \frac{A}{a} = \frac{B}{b}$$

University of Michigan, TCAUP



Structures I

Slide 8 of 13

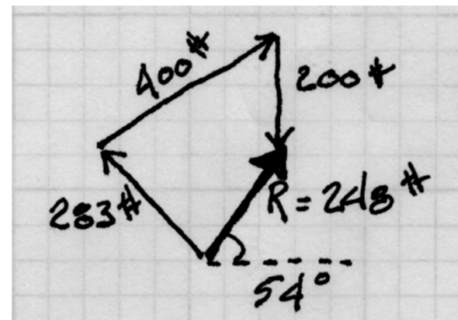
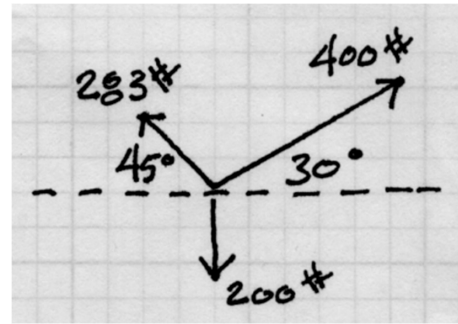
Graphic Method

Addition of Multiple Forces

Force Polygon

Forces add "Head to Tail"

The resultant closes the figure "Tail to Head"



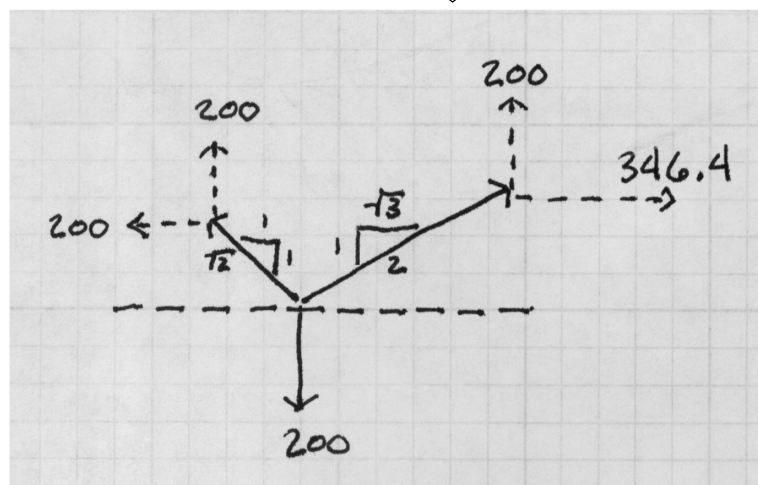
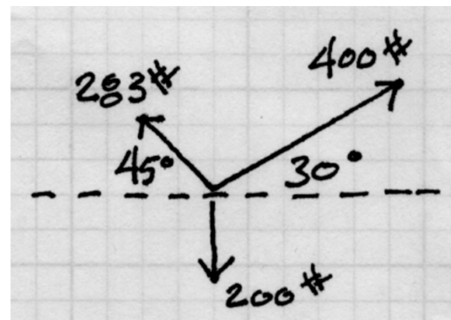
Analytic Method

Addition of Multiple Forces

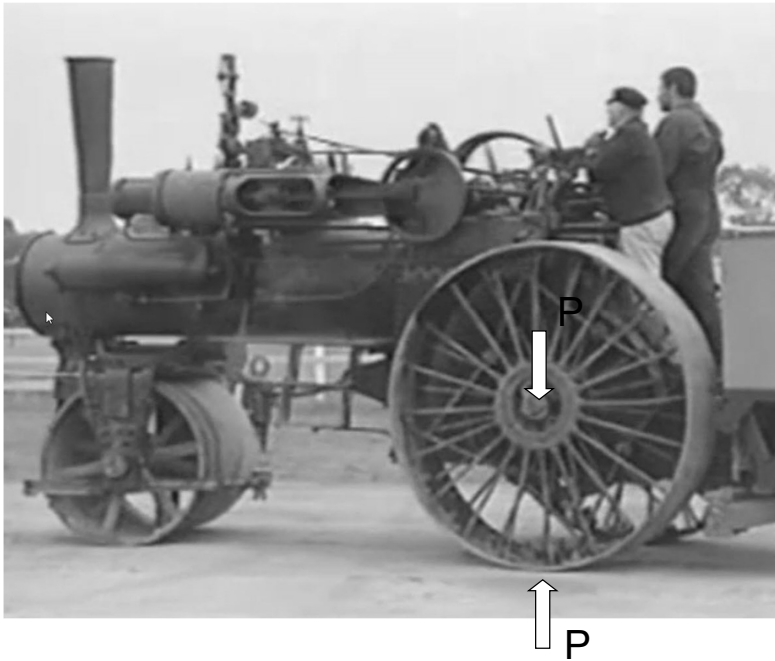
Break each force into orthogonal components

Sum all vertical and sum all horizontal

Find the resultant of the orthogonal resultants



Graphic Method



Trig Formulas

Addition of Two Forces

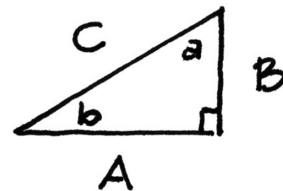
or

Decomposition of One Force

Orthogonal

Orthogonal

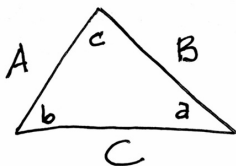
- Pythagorean Theorem



$$C = \sqrt{A^2 + B^2}$$

Non-orthogonal

- Law of Sines
- Law of Cosines



$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

$$C^2 = A^2 + B^2 - 2AB \cos c$$

$$a = \arctan A/B$$

$$a = \arccos B/C$$

$$a = \arcsin A/C$$

$$b = \arctan B/A$$

$$b = \arccos A/C$$

$$b = \arcsin B/C$$

$$C = B \frac{1}{\sin b}$$

$$C = A \frac{1}{\sin a}$$

$$A = C \cos b$$

$$A = C \sin a$$

$$B = C \sin b$$

$$B = C \cos a$$

Simon Stevin

Originator of Vector Analysis

The vector analysis of a “perpetual motion machine”, from *Weeghconst* (1586)

1. Take G_1 and G_2 to be the gravitational force on the balls (weight).
2. Break these two unequal forces into orthogonal components, normal to and along the side (N and S)
3. Because G is normal to the base, the orthogonal component triangles will be similar.
4. S_1 and S_2 can be seen to be equal and proportional to the height of the original triangle. If G forces are scaled 1:1 with lengths L , then $S_1 = S_2 = h$, therefore the forces down each slope are balanced.

