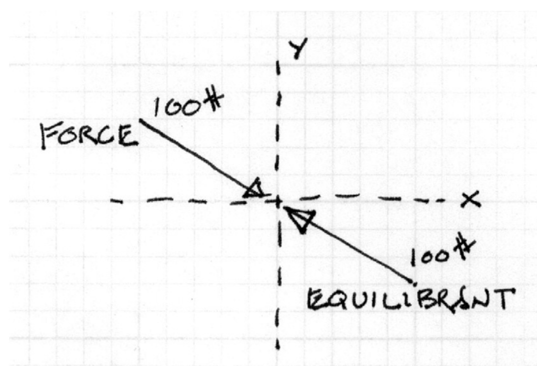
Equilibrium of Rigid Bodies

- Equilibrium
- Parallel Force Resultant
- Load Distribution
- External Reactions

Newton's First Law

An object at rest will remain at rest unless acted upon by an outside, external net force.

$$\sum \mathbf{F}_x = 0 \quad \sum \mathbf{F}_y = 0 \quad \sum \mathbf{M} = 0$$

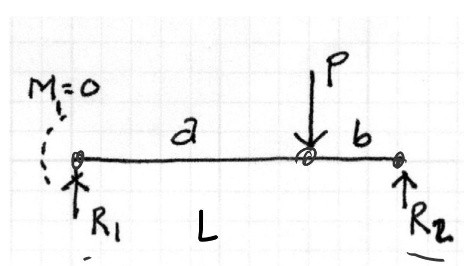


Horizontal Equilibrium

$$\sum \mathbf{F}_x = 0$$

Vertical Equilibrium

$$\sum \mathbf{F}_y = 0 = \mathbf{R}_1 + \mathbf{R}_2 - \mathbf{P} \qquad \mathbf{R}_1 + \mathbf{R}_2 = \mathbf{P}$$



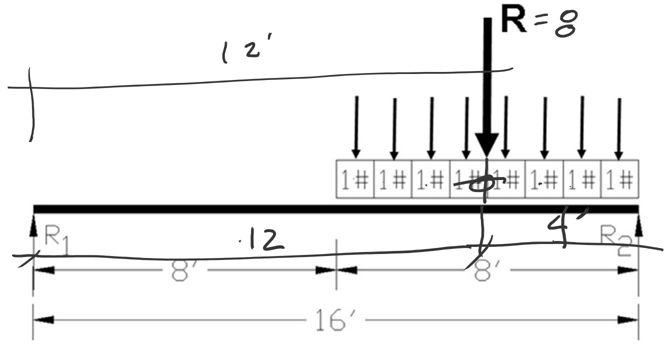
Rotational Equilibrium

$$\sum \mathbf{M}_1 = 0 = \mathbf{Pa} - \mathbf{R}_2\mathbf{L} \qquad \mathbf{R}_2 = \frac{\mathbf{Pa}}{\mathbf{L}}$$

Parallel Force Resultant

The resultant is a single force that has the same effect as a group of forces.

The resultant is located at the center or *centroid* of the group of forces.



$$\sum(\mathbf{F}\times d)=\mathbf{R}\times\bar{d}$$

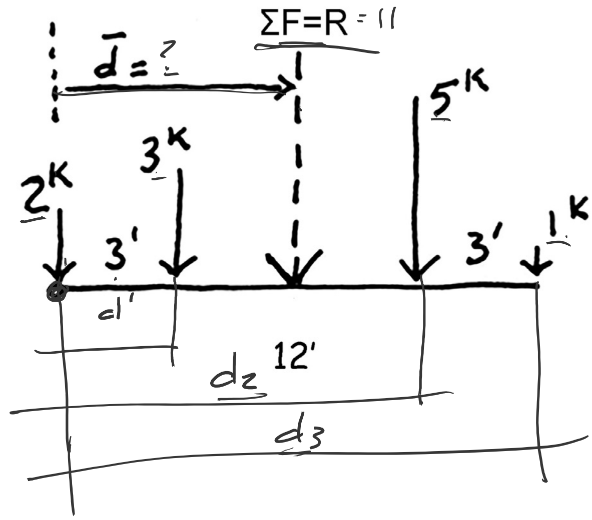
$$\mathbf{R}=\sum\mathbf{F}$$

$$\bar{d}=\frac{\sum(\mathbf{F}\times d)}{\sum\mathbf{F}}$$

Parallel Force Resultant

The resultant is a single force that has the same effect as a group of forces.

Since the resultant is equivalent to the group of forces, it can be used in place of the group.



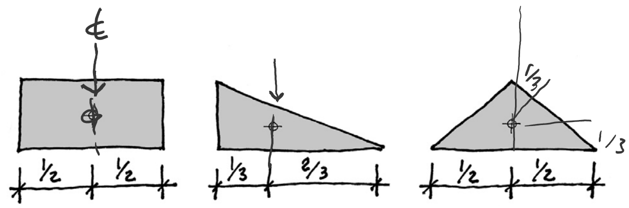
$$\sum(\mathbf{F}\times d)=\mathbf{R}\times\bar{d}$$

$$\mathbf{R}=\sum\mathbf{F}$$

$$\bar{d}=\frac{\sum(\mathbf{F}\times d)}{\sum\mathbf{F}}$$

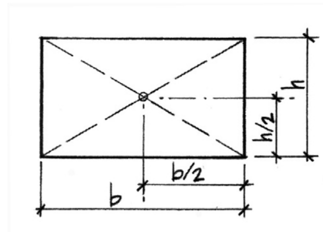
Center of Area (centroid)

In determining external reactions, the total load can be represented as a single (resultant) load at the center of gravity. In 2 dimensions this is the center of area or the centroid.



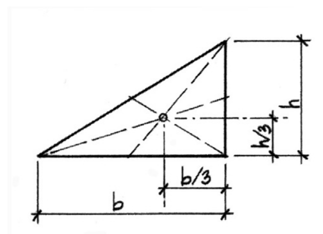
Centroids:

rectangles = midpoint



triangles = 1/3 point

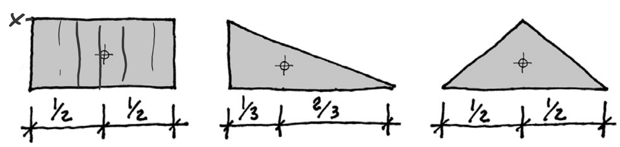
symetric = center



Load Distribution through the Centroid

Self Load

Through center of gravity



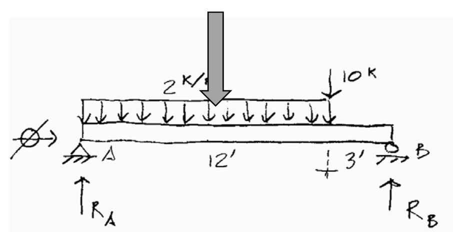
Uniform Load

Constant over length

examples:

beam selfweight

rectangular floor system



Uniformly Varying Load

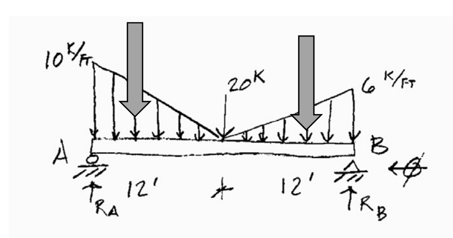
Linear change over length

examples:

snow drifts

fluid pressure

triangular floor areas



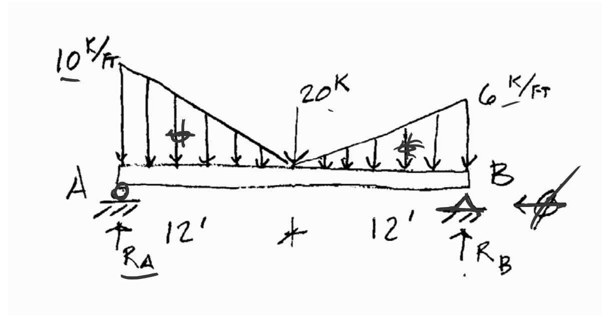
Equilibrium of Forces

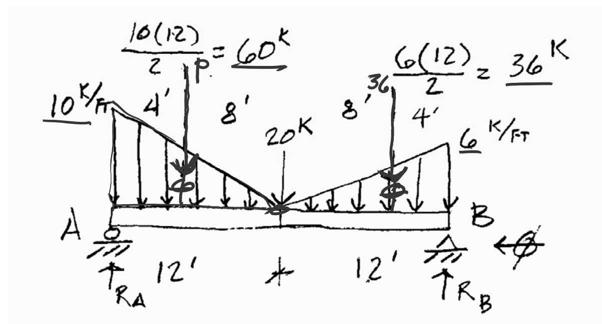
Example: Beam End Reactions

1. Label components of reactions. Depending on the support condition, include vertical, horizontal and rotational.

2. Convert area loads to point loads through the centroid of the area.

3. Since there is only one horizontal force, it must equal zero.





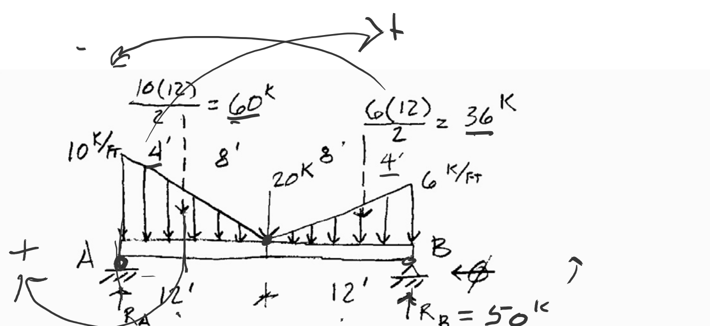
Equilibrium of Forces

Example: Beam End Reactions

4. Use the summation of moments about A to find R_B.

5. Use the summation of moments about B to find R_A.

6. Check calculation by summing vertical forces.



$\Sigma M@A = 0$

$= +60^K(4') + 20^K(12') + 36^K(20') - R_B(24')$

$R_B(24) = 1200$

$R_B = 50 \uparrow$

$\Sigma M@B = 0$

$= +R_A(24) - 60(20) - 20(12) - 36^K(4')$

$R_A(24) = 1584$

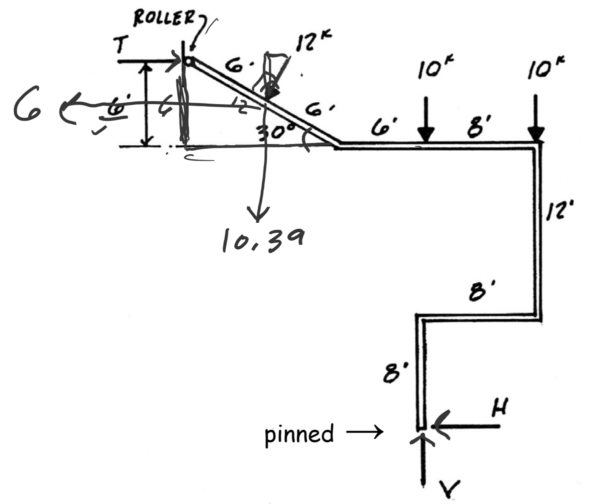
$R_A = 66 \uparrow$

CHECK

$\Sigma F_V = 0 = +66 - 60 - 20 - 36 + 50 = 0$ ✓

Ridged Body Supports

Example 1



1. Label components of reactions. Depending on the support condition, include vertical, horizontal and rotational.

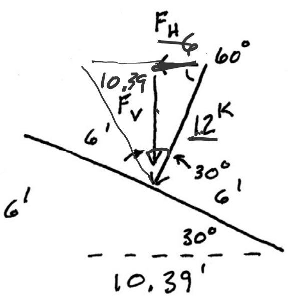
2. Convert all point loads into x and y components.

$F_H = \sin 30(12) = 6^K$

OR

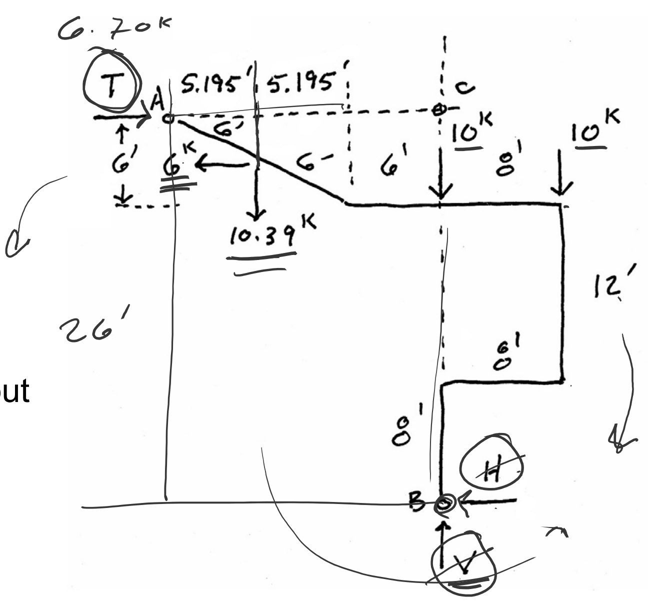
$\frac{6'}{12'} : \frac{F_H}{12^K}$ $F_H = 6^K$

$F_V = \sqrt{12^2 - 6^2} = 10.39^K$



Ridged Body Supports

Example 1



3. Since there is only one unknown vertical force (V), find that first.

4. Use the summation of moments about B to find T.

$\Sigma F_V = 0 = -10.39^K - 10^K - 10^K + V$

$V = 30.39^K \uparrow$

$\Sigma M_{@B} = 0 = \vec{T}(26') - 6^K(23') - 10.39^K(11.195') + 10^K(8')$

$T(26') = 138^{K-'} + 116.3^{K-'} - 80^{K-'} = 174.3^{K-'}$

$T = 6.70^K \rightarrow$

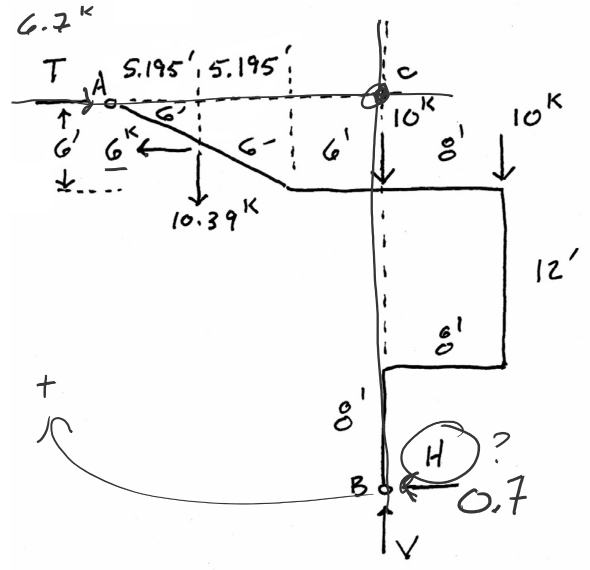
Ridged Body Supports

Example 1

5. Use the summation of moments about C to find H.

6. Note that each solution was independent of other calculated values.

7. Finally check calculations by summing horizontal forces. They should balance out to zero.



$$\Sigma M_C = 0 = 6^K(3') - 10.39^K(11.195') + 10^K(8') + H(26')$$

$$H(26') = -18^{K\text{-}'} + 116.3^{K\text{-}'} - 80^{K\text{-}'} = 18.3^{K\text{-}'}$$

$$H = 0.70^K \leftarrow$$

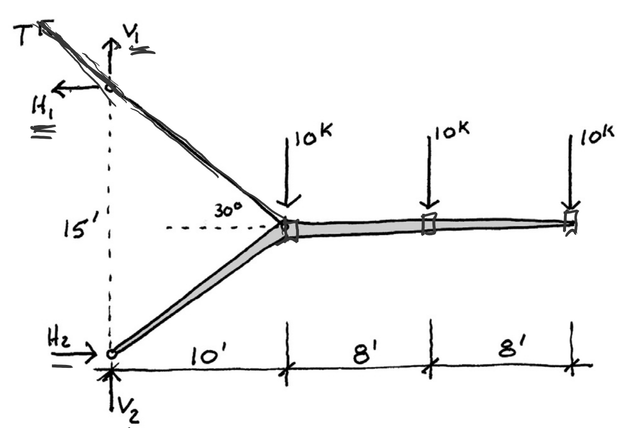
CHECK

$$\Sigma F_H = +6.7^K - 6^K - 0.7^K = 0 \checkmark$$

Cantilever Frame

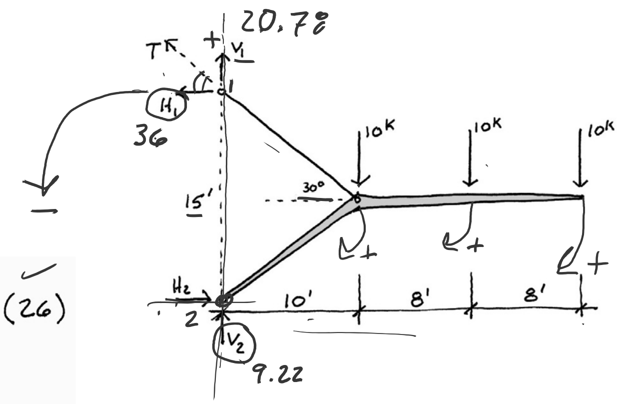
Find the reactions of the cable supported frame.

Hint: $V_1 / H_1 = \tan 30°$



Cantilever Frame

Find the reactions of the cable supported frame.



$$\Sigma M_{R_2} = 0 = -H_1(15) + 10(10) + 10(18) + 10(26)$$

$$H_1 = 540/15 = 36^K \leftarrow$$

$$\tan 30° = 0.57735 = V_1/H_1 = V_1/36$$

$$V_1 = 20.78^K \uparrow$$

$$\Sigma F_V = 0 = V_2 + 20.78 - 10 - 10 - 10$$

$$V_2 = +9.22^K \uparrow$$

$$\Sigma F_H = 0 = H_2 - 36$$

$$H_2 = 36^K \rightarrow$$

Other Examples