

## Equilibrium of a Particle

### Cable Systems

- Catenary Cable Systems
- Solving Cable Forces
- Cable Net Systems

ILEK , Stuttgart

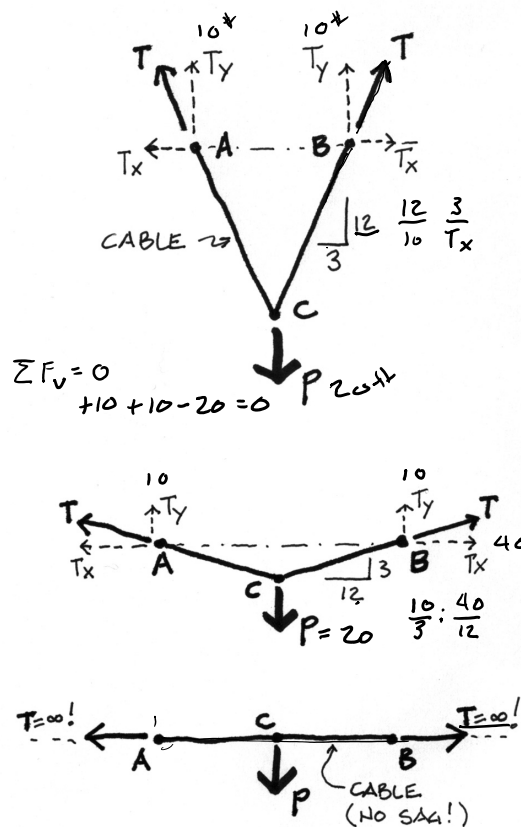


## Cables

Both the sag and the load determine the force in the cable.

The less the sag, the higher the force in the cable.

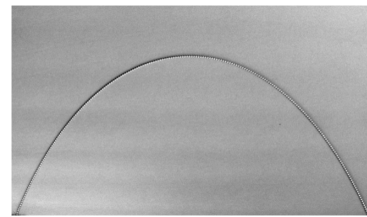
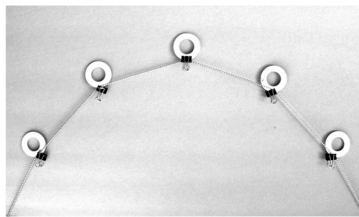
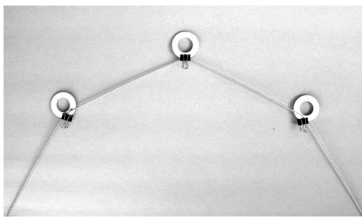
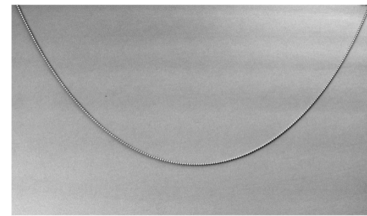
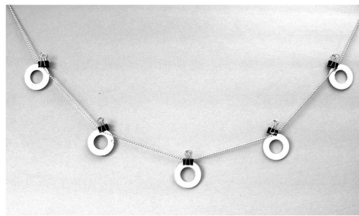
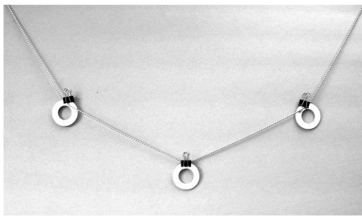
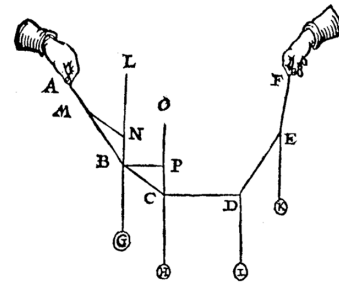
The vertical component of the cable force in the case shown remains constant .  $T_y = P/2$   
But since the resultant follows the direction of the cable,  $T_x$  becomes greater as sag decreases. With no sag,  $T_x$  is infinite !



# Catenary Shapes

The shape of the catenary depends on the loading. Simon Stevin showed this experimentally in 1585 with a weighted cord.

Because the cord has no resistance to moment, it assumes the shape (reversed) of the moment diagram for a beam with the same loading.



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Structures I

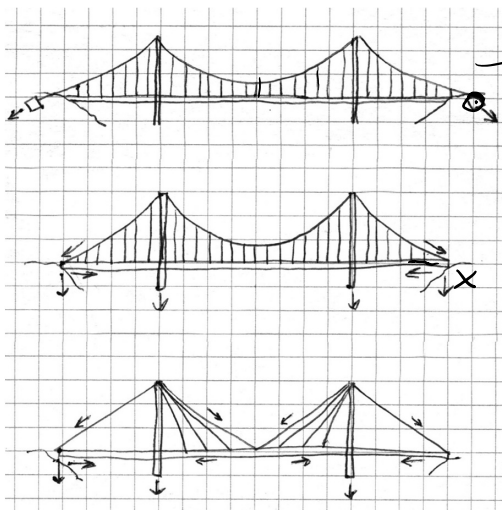
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## Cable Bridges

Long span bridges are often cable supported.

There are 3 common types:

- earth anchored cable
- deck anchored cable
- cable stayed



Gordie Howe International Bridge, Detroit.  
longest span: 0.53 mi total span: 1.55 mi



Golden Gate Bridge - center span: 1.28 mi total span: 1.7 mi

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Structures I

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# Solving Cables Forces

even supports – symmetric loading

Procedure:

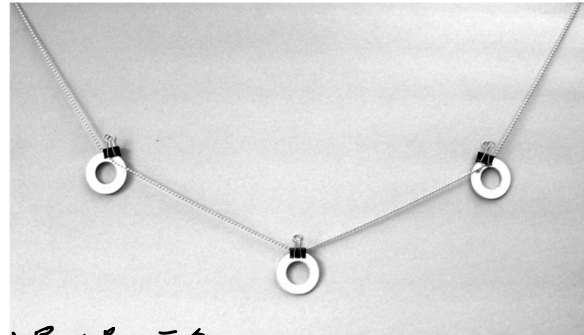
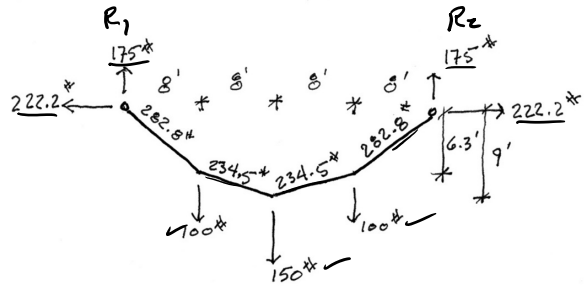
1. Solve all external forces (reactions)
  - If symmetric, then the reactions at each end are equal
  - Use 3 equilibrium equations to solve

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0$$

- The moment is also 0 at any point in the cable since the cable cannot support flexure,

2. Start at reactions and move inward
  - Draw FBD of system from reaction inward
  - Slope is proportional to forces
  - Use force equations to solve at cut
  - Find forces and slope of each section

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 = -100 - 150 - 100 + R_1 + R_2 = 0 \end{aligned}$$



## Cables Forces

### Example 1

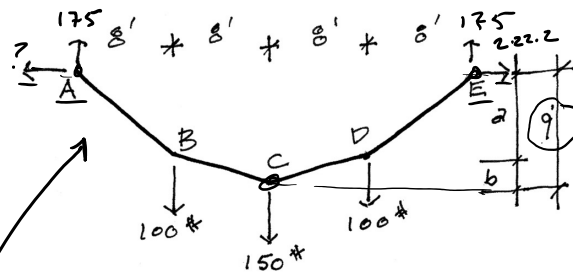
even supports – symmetric loading

Systems with supports on the same level can be solved without simultaneous equations since the horizontal reactions pass through the same point.

If the system is symmetric then the end reactions will be equal, and can be solved by summing vertical forces.

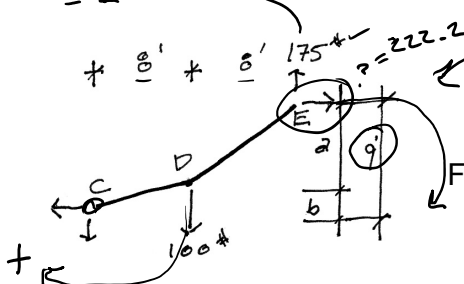
Using a FBD of a cut section and summing moments at a point with known dimensions (in this case C) the horizontal forces can be found.

Full Diagram



$$\begin{aligned} \sum F_y = 0 &= +A_y - 100 - 150 - 100 + E_y \\ A_y + E_y &= 350 \\ A_y &= E_y = 175 \uparrow \end{aligned}$$

FBD 1



FBD 1

$$\begin{aligned} \sum M_C = 0 &= +100(8') - 175(16') + E_x(9') \\ E_x(9') &= 2000 \text{ lb-ft} \\ E_x &= 222.2 \text{ lb} \\ \sum F_H = 0 &= -A_x + E_x = 0 \\ A_x &= E_x = 222.2 \text{ lb} \end{aligned}$$

# Cables Forces

## Example 1

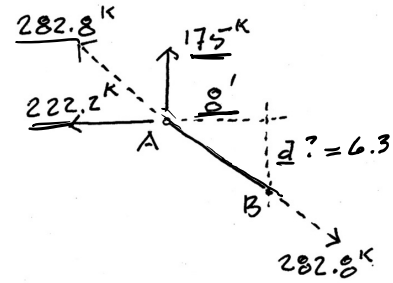
even supports – symmetric loading

Cable slopes and forces can now be found by ratios and Pythagorean formula.

CABLE A-B AND D-E

$$\frac{V}{H} = \frac{175}{222.2} = \frac{2}{8.1} \quad 2 = 6.3$$

$$\text{CABLE FORCE} = \sqrt{175^2 + 222.2^2} = 282.8^k$$



FBD 2

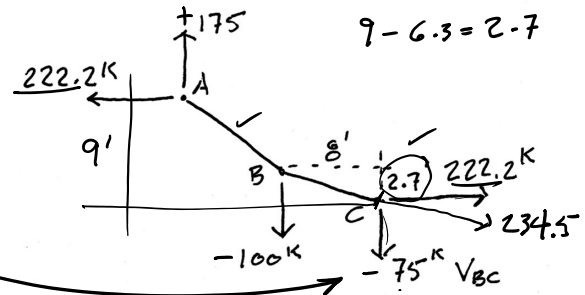
CABLES B-C AND C-D

$$9' - 6.3' = 2.7'$$

$$\sum F_V = 0 = 175 - 100 - V_{BC}$$

$$V_{BC} = 75^k$$

$$\text{CABLE FORCE} = \sqrt{75^2 + 222.2^2} = 234.5^k$$

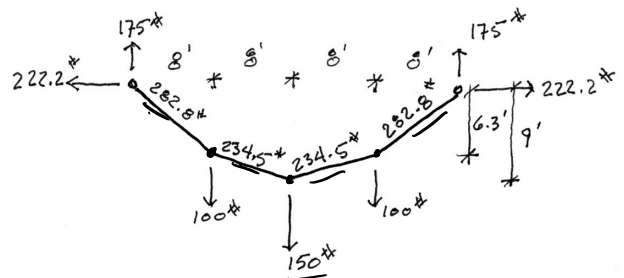


# Cables Forces

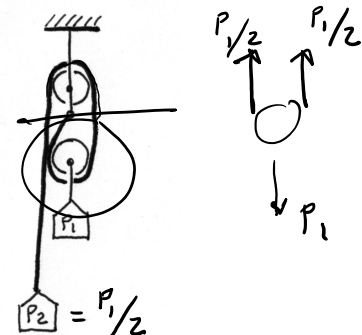
## Example 1

even supports – symmetric loading

Note the final tensile force distribution. The tension in the cables closer to the supports is higher and the slope is steeper (higher vertical component). The horizontal component of the force is constant across the cable (in this case 222.2k).



Note that the total force in each length of cable changes at each load connection. This type of system should not be confused with cable systems using pulleys, where the force remains the same throughout the length of the cable.

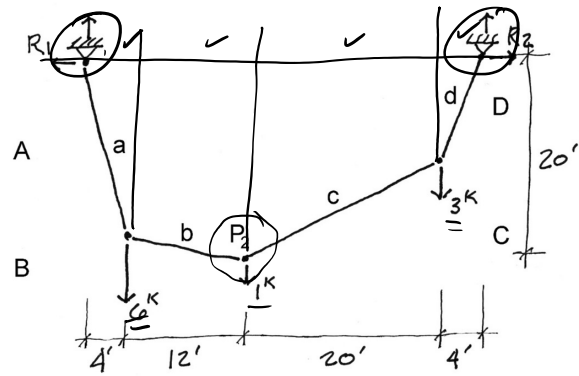


# Solving Cables Forces

even supports – asymmetric loading

Procedure:

1. The cable shape is defined by the supports, one other point, and horizontal position of loads.
2. Solve all external forces (reactions)
  - If asymmetric, then the vertical reactions are not equal.
  - With no applied horizontal loads, the horizontal reactions are equal.
  - The vertical reactions can be solved either by summing moments at each end or by proportions as in a beam.
  - The moment is also 0 at any point in the cable since the cable cannot support flexure,
3. Start at reactions and move inward
  - Draw FBD of system from reaction inward
  - Slope is proportional to forces
  - Use force equations to solve internal cable force at each cut point
  - Find forces and slope of each section



$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}$$

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## Cables Forces

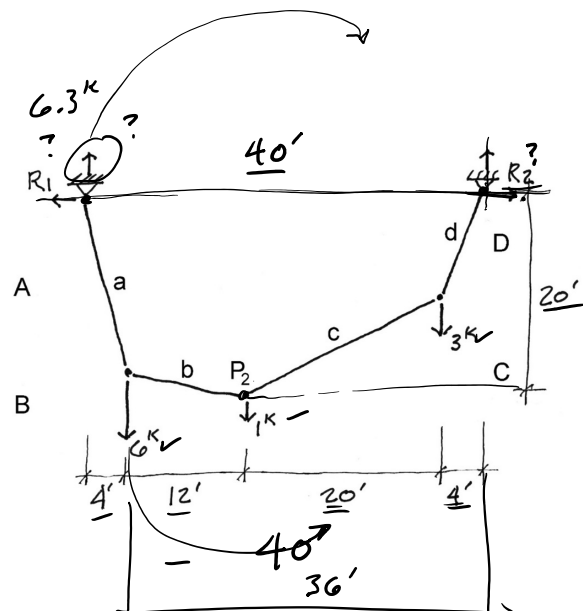
### Example 2

even supports – asymmetric loading

Systems with supports on the same level can be solved without simultaneous equations since the horizontal reactions pass through the same point.

If the system is asymmetric then summing moments about one reaction will give the vertical component of the other reaction.

To find the horizontal reaction component, cut a FBD at a point of known location (both x and y). So summing moments at  $P_2$  then finds the horizontal force.



$$\begin{aligned} \sum M_{R2} &= R1_v(40') - 6k(36') - 1k(24') - 3k(4') = 0 \\ (R1_v)(40') &= 216k \cdot 1 + 24k \cdot 1 + 12k \cdot 1 = 252k \cdot 1 \\ R1_v &= 6.3k \uparrow \end{aligned}$$

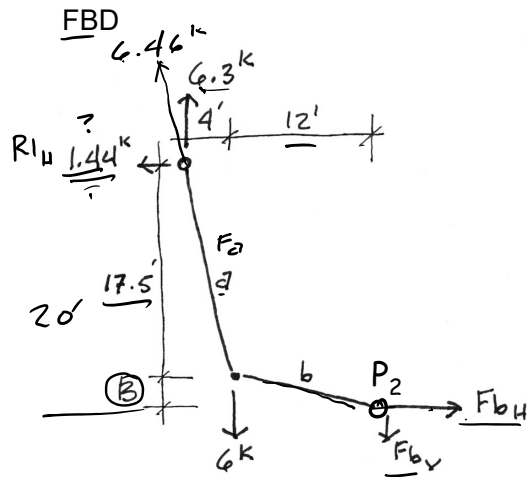
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# Cables Forces

Ex 2 - even supports – asymmetric loading



$$F_a = \sqrt{6.3^2 + 1.44^2} = 6.46 \text{ k T}$$

$$F_d = \sqrt{3.7^2 + 1.44^2} = 3.97 \text{ k T}$$

FIND A

$$\frac{R_{1V}}{R_{1H}} = \frac{6.3}{1.44} = \frac{A}{4}$$

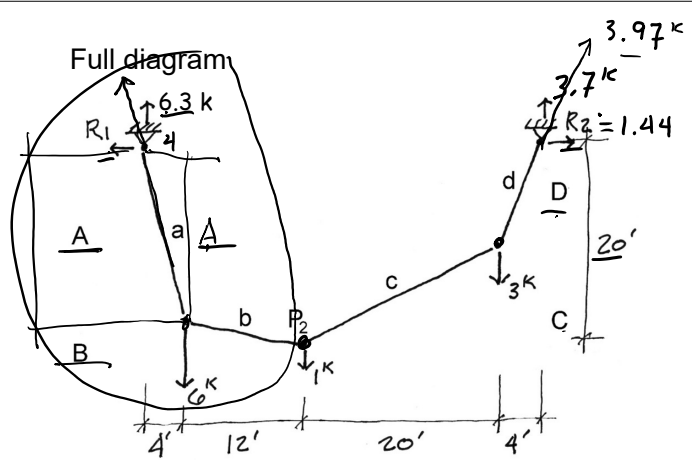
$$A = 17.5'$$

FIND D

$$\frac{3.7}{1.44} = \frac{D}{4}$$

$$D = 10.27'$$

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FBD

$$\sum M_{P_2} = 6.3(16') - R_{1H}(20') - 6(12') = 0$$

$$R_{1H}(20') = 100.8 - 72 = 28.8 \text{ k}\cdot\text{ft}$$

$$R_{1H} = 1.44 \text{ k} \leftarrow$$

Full diagram

$$\sum F_H = 0 = -1.44 \text{ k} + R_{2H}$$

$$R_{2H} = 1.44 \text{ k} \rightarrow$$

$$\sum F_V = 0 = 6.3 - 6 - 1 - 3 + R_{2V}$$

$$R_{2V} = 3.7 \text{ k} \uparrow$$

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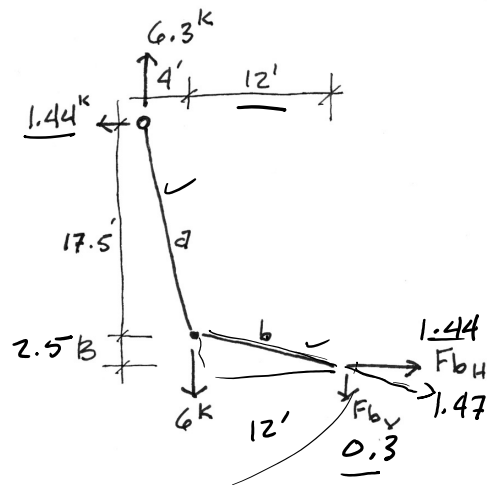
# Cables Forces

Example 2

even supports – asymmetric loading

Proceed working inward from the reactions by cutting a FBD just before the end of the next section.

Because the force is in the same axis as the cable and there is no flexure in the cable, the ratio of the force components is proportional to the slope of the cable.



$$\sum F_V = 0 = 6.3 - 6 - F_{bV}$$

$$F_{bV} = 0.3 \text{ k}$$

$$\sum F_H = 0 = -1.44 \text{ k} + F_{bH}$$

$$F_{bH} = 1.44 \text{ k}$$

$$F_b = \sqrt{0.3^2 + 1.44^2} = 1.47 \text{ k T}$$

FIND B

$$\frac{0.3}{1.44} = \frac{B}{12}$$

$$B = 2.5'$$

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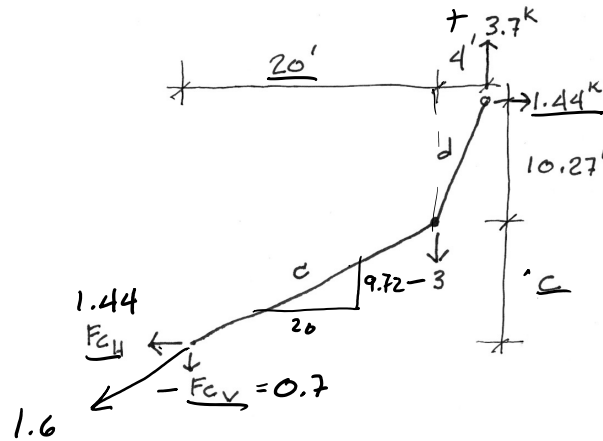
## Cables Forces

### Example 2

even supports – asymmetric loading

Proceed working inward from the reactions by cutting a FBD just before the end of the next section.

Because the force is in the same axis as the cable and there is no flexure in the cable, the ratio of the force components is proportional to the slope of the cable.



$$\sum F_V = 0 = 3.7^k - 3^k - F_{cV}$$

$$F_{cV} = 0.7^k$$

$$\sum F_H = 0 = 1.44^k - F_{cH}$$

$$F_{cH} = 1.44^k \rightarrow$$

$$F_c = \sqrt{0.7^2 + 1.44^2} = 1.60^k T$$

FIND C

$$\frac{0.7}{1.44} = \frac{C}{20}$$

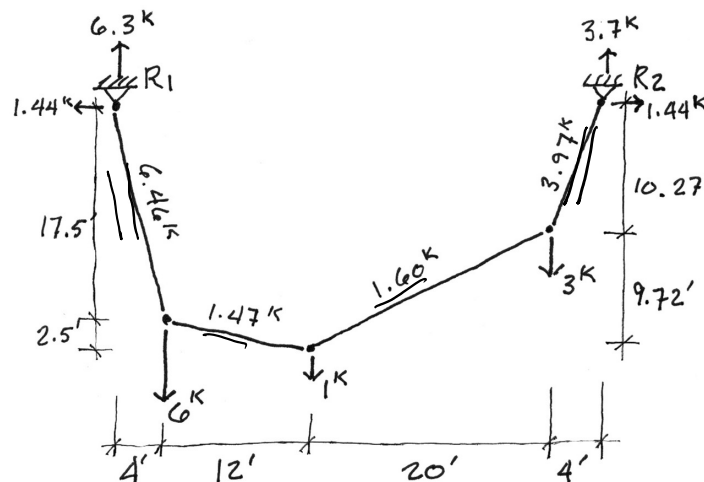
$$C = 9.72'$$

## Cables Forces

### Example 2

even supports – asymmetric loading

The final diagram compares the tension in each of the cable segments. Because of the asymmetry of the loading, the forces are also asymmetric. Note also that they generally increase toward the reactions.



# Cables Forces

## Example 3 uneven supports

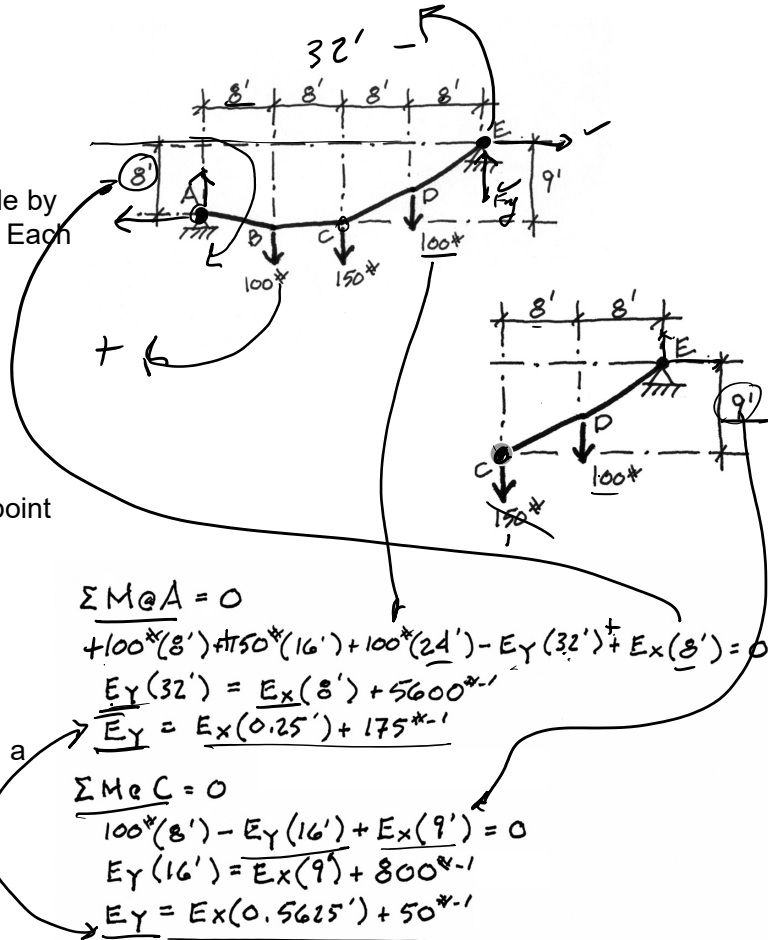
The analysis of a cable system is made by cutting a series of FBDs joint by joint. Each joint is analyzed as a concentric force system. All forces must sum to zero.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0$$

To define the shape, the end points, horizontal dimensions and one other point are needed.

The procedure usually begins with the summation of moments to determine the end reactions.

Since the location of point C is known, a FBD is cut at C of the right side. Summing moments at C will give a second equation for  $E_x$  and  $E_y$ .



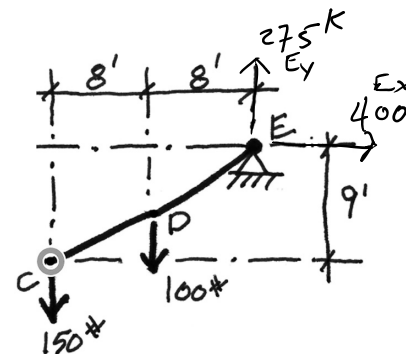
## Cables Forces - uneven supports

### Example 3

Setting the two equations equal to  $E_y$ , the value is found for  $E_x$ .

Then using  $E_x$ ,  $E_y$  can be solved.

Finally  $A_x$  and  $A_y$  are found by summing forces.

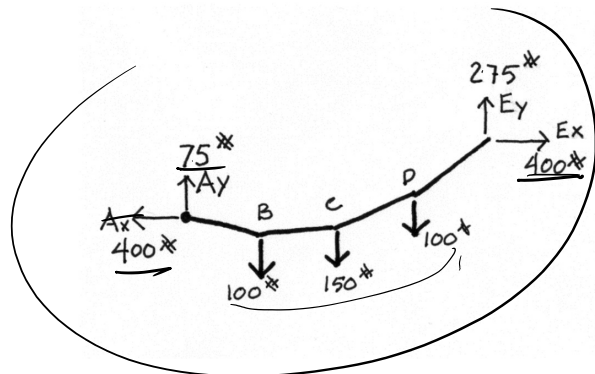


$$\begin{aligned} E_x(0.25) + 175 &= E_x(0.5625) + 50 \\ 125 &= E_x(0.3125) \\ E_x &= 400 \text{ k} \rightarrow \end{aligned}$$

$$\begin{aligned} E_y &= E_x(0.5625) + 50 \\ E_y &= 400(0.5625) + 50 \\ E_y &= 275 \text{ k} \uparrow \end{aligned}$$

$$\begin{aligned} \sum F_y = 0 &= A_y - 100 - 150 - 100 + 275 \\ A_y &= 75 \text{ k} \uparrow \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 &= -A_x + 400 \\ A_x &= 400 \text{ k} \leftarrow \end{aligned}$$



## Cables Forces: Ex. 3 - uneven supports

Working from the reactions inward, the slope of each segment is determined by the components of the force. The remaining forces can be found using the FBD of the joint..

FBD1

$$\frac{400}{75} : \frac{8}{y_1} \quad y_1 = 1.5'$$

FBD2

$$\frac{400}{275} : \frac{8}{y_2} \quad y_2 = 5.5'$$

FBD3

points B and C are both known. Slope =  $\frac{0.5}{8}$

$$\sum F_y = 0 = 75 - 100 + T_{bcy} \quad T_{bcy} = 25\#$$

$$\sum F_x = 0 = -400 + T_{bcx} \quad T_{bcx} = 400\#$$

FBD4

points C and D are both known. Slope =  $\frac{3.5}{8}$

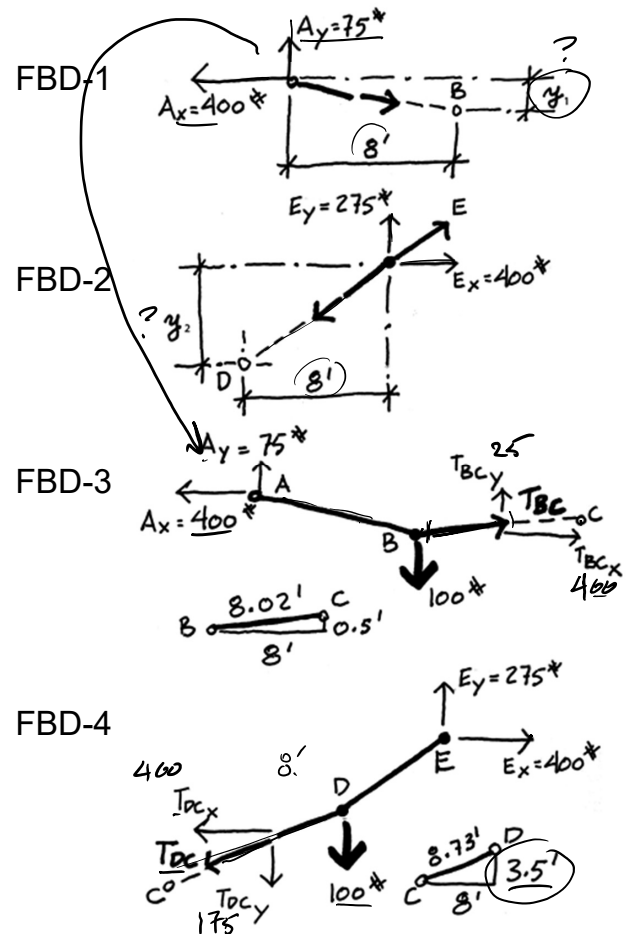
$$\sum F_y = 0 = 275 - 100 - T_{dcy} \quad T_{dcy} = 175\#$$

$$\sum F_x = 0 = 400 - T_{dex} \quad T_{dex} = 400\#$$

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## Cables Forces – uneven supports

### Example 3

Using FBDs of each joint in turn the force components and slopes can be determined.

Cable forces are found by adding the components with Pythagorean formula.

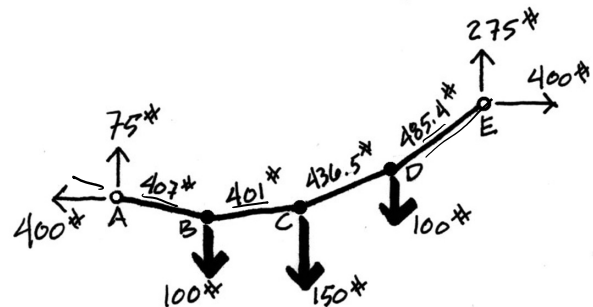
In summary the cable forces are:

$$T_{ab} = 407.0 \#$$

$$T_{bc} = 400.8 \#$$

$$T_{cd} = 436.5 \#$$

$$T_{de} = 485.4 \#$$

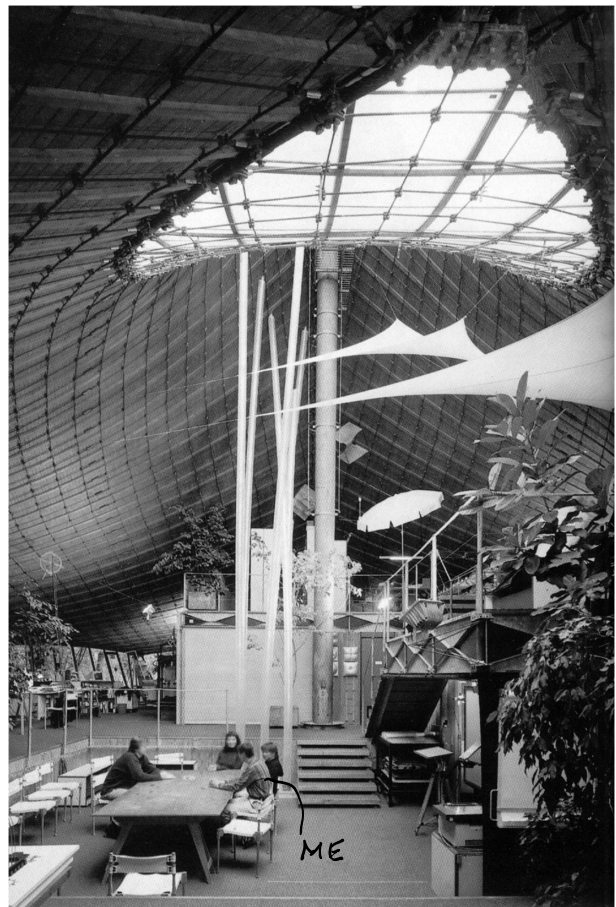


# Cable Structures



Munich Olympic Buildings  
Frei Otto with Günter Behnisch,  
1972

Institute for Lightweight Structures  
Frei Otto, 1965



# Cable Bracing



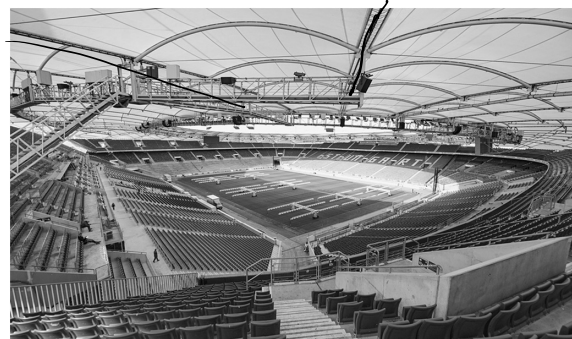
Grain silos in Michigan



Messe Tower in Leipzig  
Engineer: Schlaich Bergernam and Partners  
Architect: Gerkan, Marg and Partners  
Photo by Prolineserver

# Cable Structures

## Mercedes Benz Arena in Stuttgart



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