Architecture 314

Structures I

Plane Trusses
Method of Joints

Definition and Assumptions

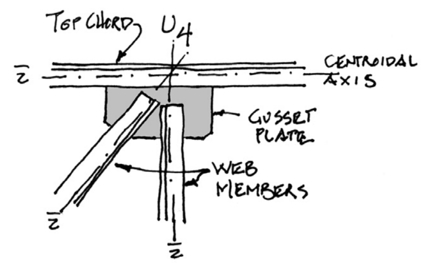
Nomenclature

Stability and Determinacy

Analysis by joints

Phaeodaria – Ernst Haeckel

Definitions and Assumtions of Truss Systems



2 Force Members

Pinned Joints

Concurrent Member Centroids at Joints

Joint Loaded

Straight Members

Small Deflections

Bullring Covering, Xàtiva, Spain
Kawaguchi and Engineers, 2007

Nomenclature

Panels

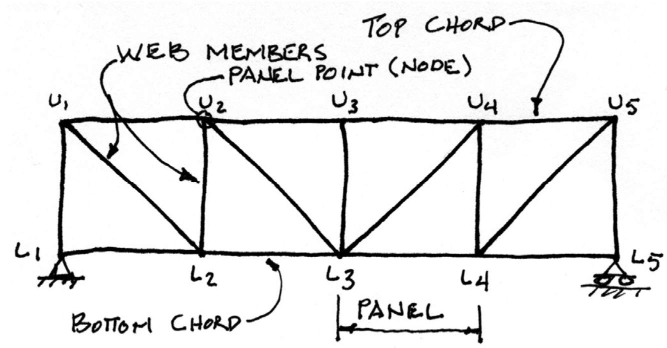
- Segments: left to right

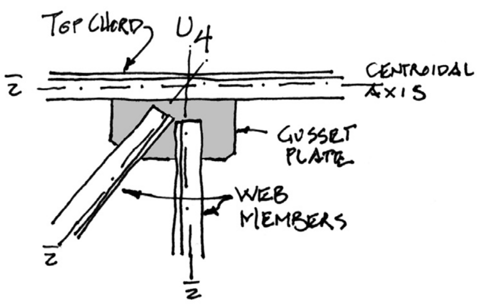
Joints

- Upper: U1, U2, U3...
- Lower: L1, L2, L3...

Members

- Chords
- Web





Trussed Force Systems

2D Trusses

- Concurrent Coplanar

3D Trusses

- Concurrent Non-Coplanar

Foster Bridge, 1889
Ann Arbor, Michigan

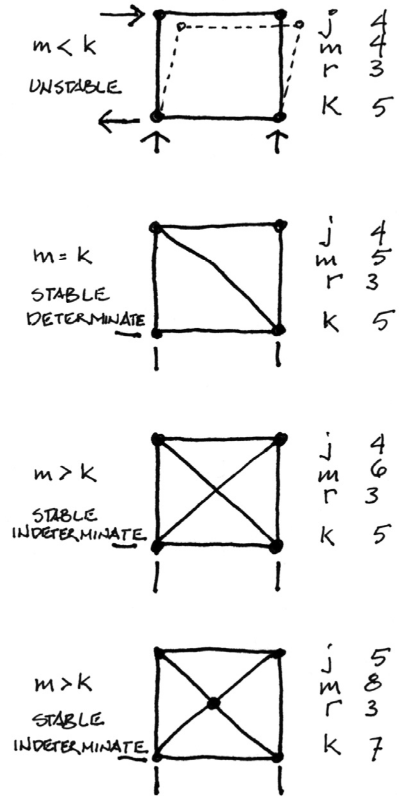
University of Michigan Architectural Research Lab
Unistrut System, Charles W. Attwood

Stability and Determinacy of 2D Trusses

For:

- j joints
- m members
- r reactions (restraints)

$$k = 2j - r$$



Three conditions

- m < k unstable
- m = k stable and determinate
- m > k stable and indeterminate

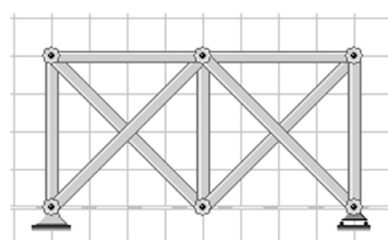
Quiz

For each of the following trusses, determine whether they are:

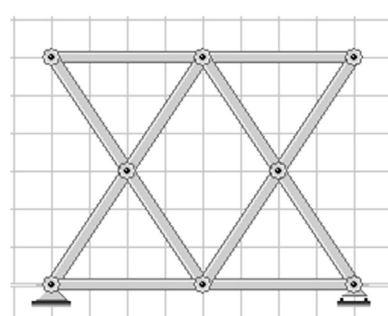
A) Stable
B) Unstable

$$k = 2j - r$$

- m < k unstable
- m = k stable and determinate
- m > k stable and indeterminate



Truss 1

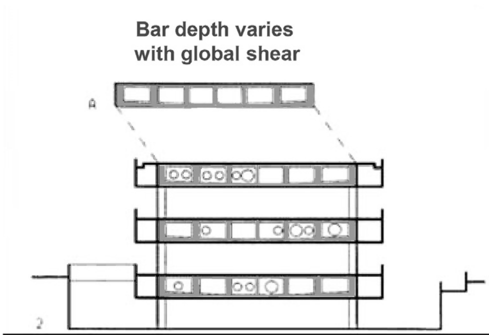


Truss 2

Vierendeel “Truss”

Not a true truss

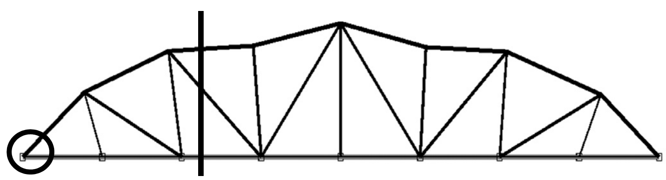
Moment frame structure
Rigid joints as moment connections
Flexure in members



Salk Institute, La Jolla.
Architect: Louis Kahn
Engineer: Komendant and Dubin

Vierendeel bridge
at Grammene, Belgium
Photo by Karel Roose

Truss Analysis



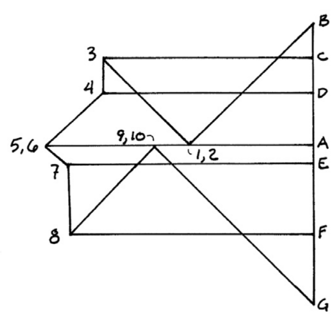
Method of Joints

Method of Sections

Graphic Methods

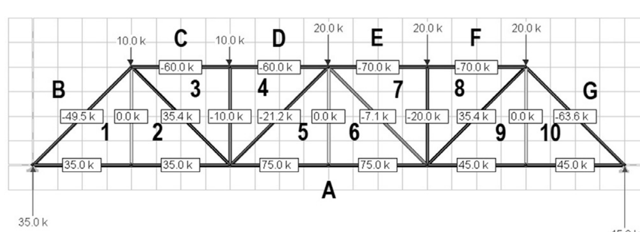
James Clerk Maxwell 1869
M. Williot 1877
Otto Mohr 1887
Heinrich Müller-Breslau 1904
William Baker, SOM

James Clerk Maxwell



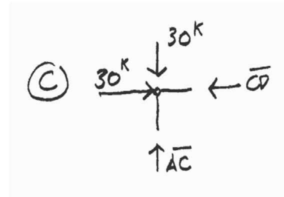
Computer Programs

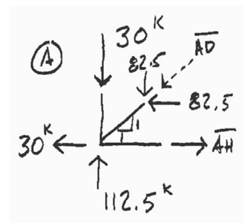
Dr. Frame (2D)
STAAD Pro (2D or 3D)
West Point Bridge Designer

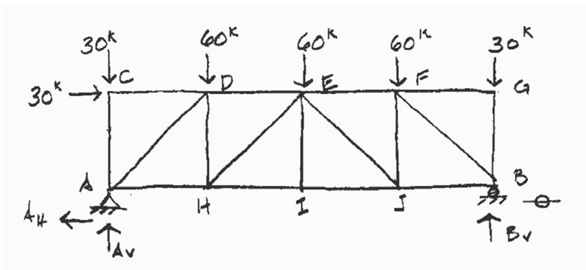


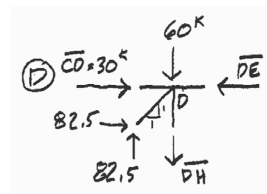
Method of Joints – procedure

1. Solve reactions (all external forces)
2. Inspect for zero force members (T's & L's)
3. Cut FBD of one joint
4. Show forces as orthogonal components
5. Solve with ΣF_H and ΣF_V (no ΣM)
6. Find resultant member forces
   (Pythagorean Formula)





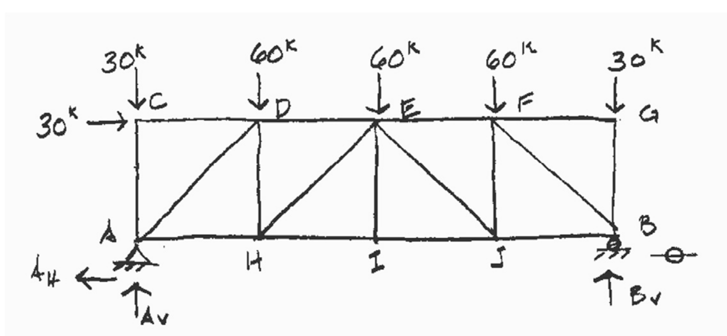


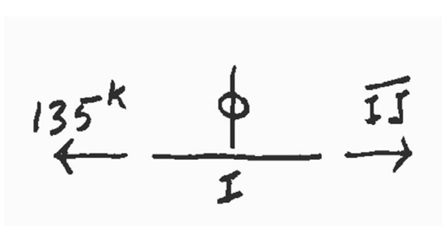


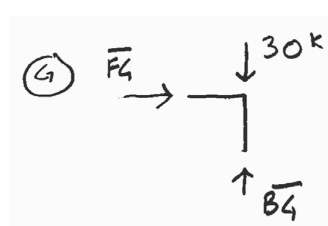
Inspection of Zero Force Members

T – joints

L – joints



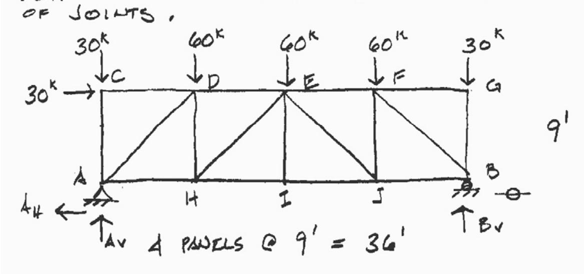




Method of Joints - example

1. Solve the external reactions for the whole truss.

DETERMINE ALL MEMBER FORCES USING THE METHOD OF JOINTS.



REACTIONS :

$\Sigma F_H = 0 = 30 - A_H \quad A_H = 30^k \leftarrow$

$\Sigma M @ A = 0 = 30(9) + 60(9) + 60(18) + 60(27) + 30(36) - B_V(36)$

$B_V(36) = 4590$

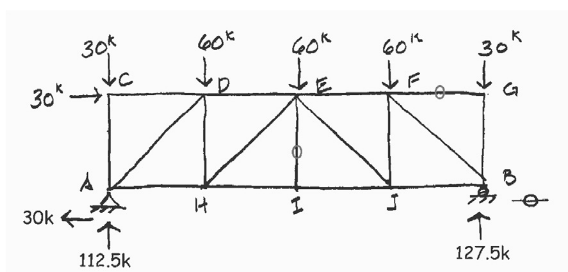
$B_V = 127.5^k \uparrow$

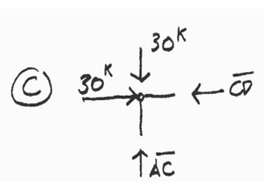
$\Sigma F_V = 0 = A_V + 127.5 - 2(30) - 3(60) = 0$

$A_V = 112.5^k \uparrow$

Method of Joints - example

2. T or L joints by inspection.
3. Cut FBD of joint
4. Show orthogonal components
5. Solve by ΣF horz. and vert.



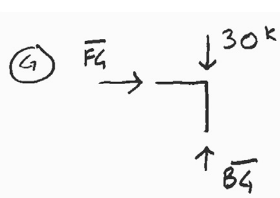


$\Sigma F_V = 0 = -30 + \overline{AC}$

$\overline{AC} = 30^k$ COMP

$\Sigma F_H = 0 = 30 - \overline{CD}$

$\overline{CD} = 30^k$ COMP



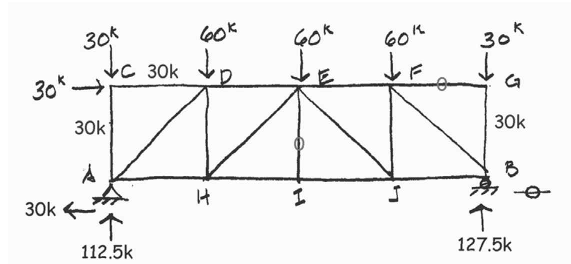
$\Sigma F_H = 0 = \overline{FG}$ (ZERO)

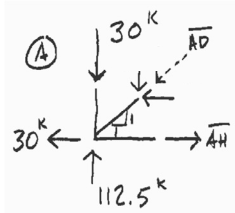
$\Sigma F_V = 0 = -30^k + \overline{BG}$

$\overline{BG} = 30^k$ COMP

Method of Joints - example

Continue with joints having only one unknown in either horizontal or vertical direction. Generally work starting at the reactions.





$\Sigma F_V = -30 + 112.5 - \overline{AD}_V = 0$

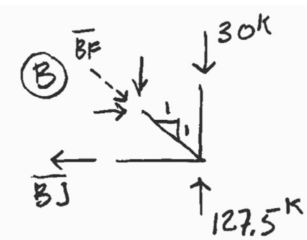
$\overline{AD}_V = 82.5^k$

$\overline{AD}_H = \overline{AD}_V = 82.5^k$

$\overline{AD} = \sqrt{82.5^2 + 82.5^2} = 116.67^k$ comp

$\Sigma F_H = 0 = -30 - 82.5 + \overline{AH}$

$\overline{AH} = 112.5^k$ TENSION



$\Sigma F_V = 0 = -30 + 127.5 - \overline{BF}_V$

$\overline{BF}_V = 97.5^k$

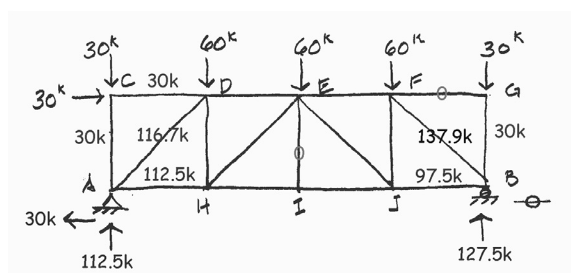
$BF_V = BF_H = 97.5^k$

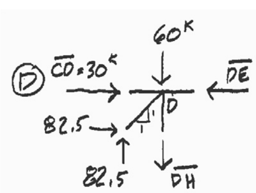
$\overline{BF} = \sqrt{97.5^2 + 97.5^2} = 137.9^k$ comp

$\Sigma F_H = 0 = 97.5 - \overline{BJ}$ $\quad \overline{BJ} = 97.5^k$ TENS

Method of Joints - example

Continue moving across the truss, joint by joint. Solve by ΣF_H and ΣF_V .



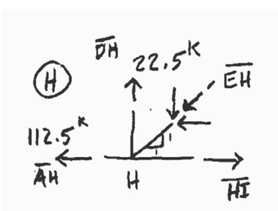


$\Sigma F_H = 30 + 82.5 - \overline{DE} = 0$

$\overline{DE} = 112.5^k$ comp

$\Sigma F_V = 82.5 - 60 - \overline{DH} = 0$

$\overline{DH} = 22.5^k$ TENSION



$\Sigma F_V = 0 = 22.5 - \overline{EH}_V$

$\overline{EH}_V = 22.5$

$\overline{EH}_V = \overline{EH}_H = 22.5$

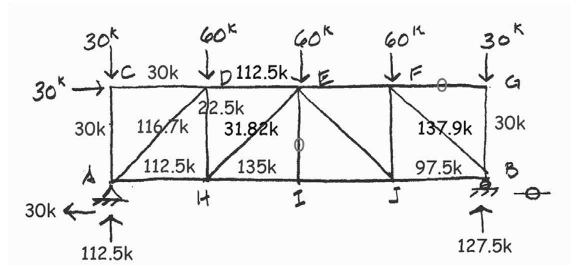
$\overline{EH} = \sqrt{22.5^2 + 22.5^2} = 31.82^k$ comp

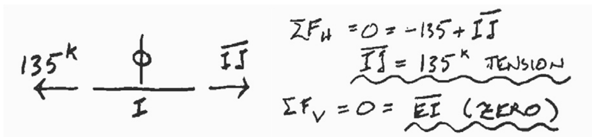
$\Sigma F_H = 0 = -112.5 - 22.5 + \overline{HI}$

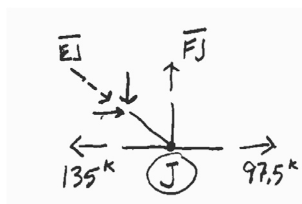
$\overline{HI} = 135^k$ TENSION

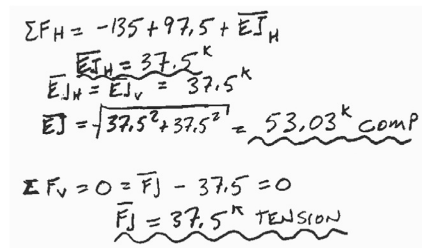
Method of Joints - example

Continue moving across the truss, joint by joint. Choose joints that have only one unknown in each direction, horizontal or vertical.





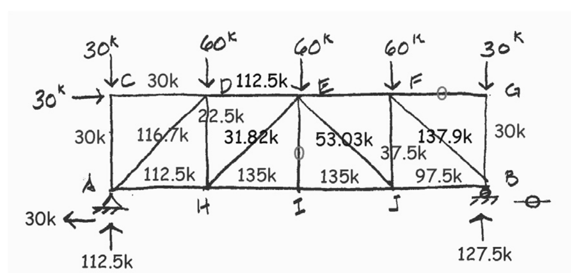


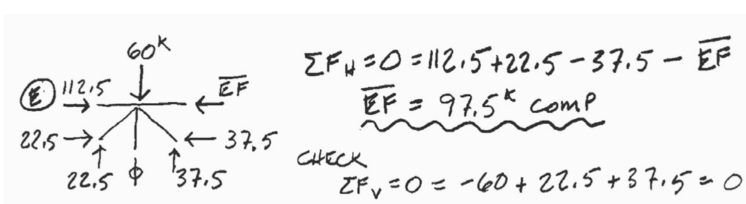


Method of Joints - example

Solve the joints with the most members last.

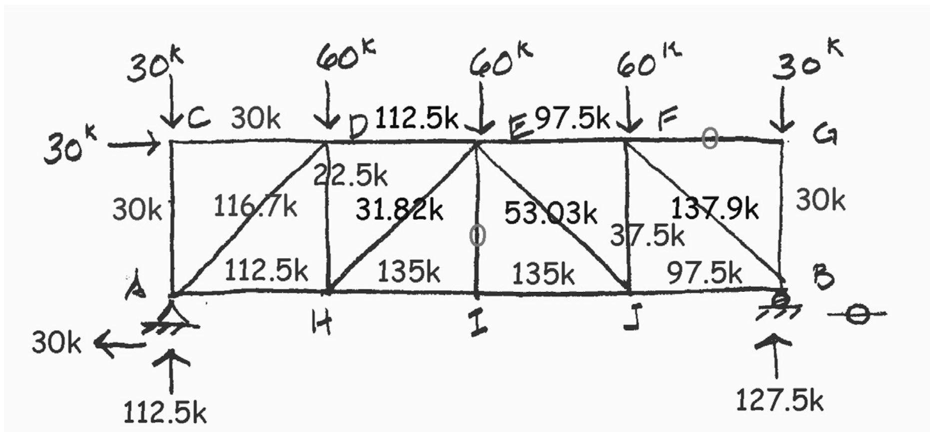
Check that all forces balance.





Method of Joints - example

Inspect the final solution to see that
it seems to make sense.



Qualitative T or C

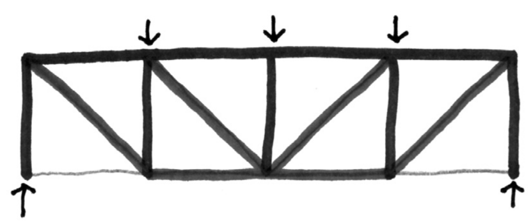
For typical gravity loading:
(tension=red compression=blue)

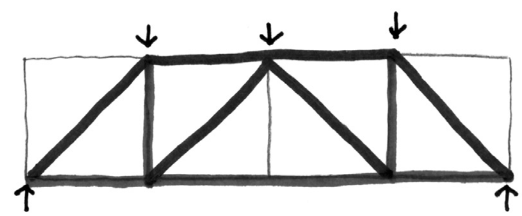
Top chords are in compression

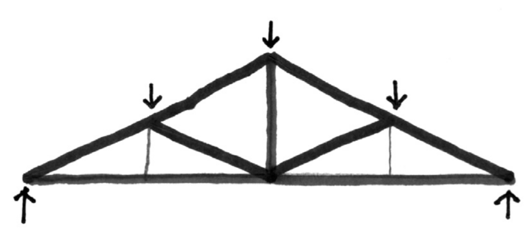
Bottom chords are in tension

Diagonals down toward center
are in tension (usually)

Diagonals up toward center are
in compression (usually)







Qualitative Force

For spanning trusses with uniform loading: (tension=blue compression=red)

Top and bottom chords greatest at center when flat (at maximum curvature or moment)

Diagonals greatest at ends (near reactions, i.e. greatest shear)