



Recitation 004

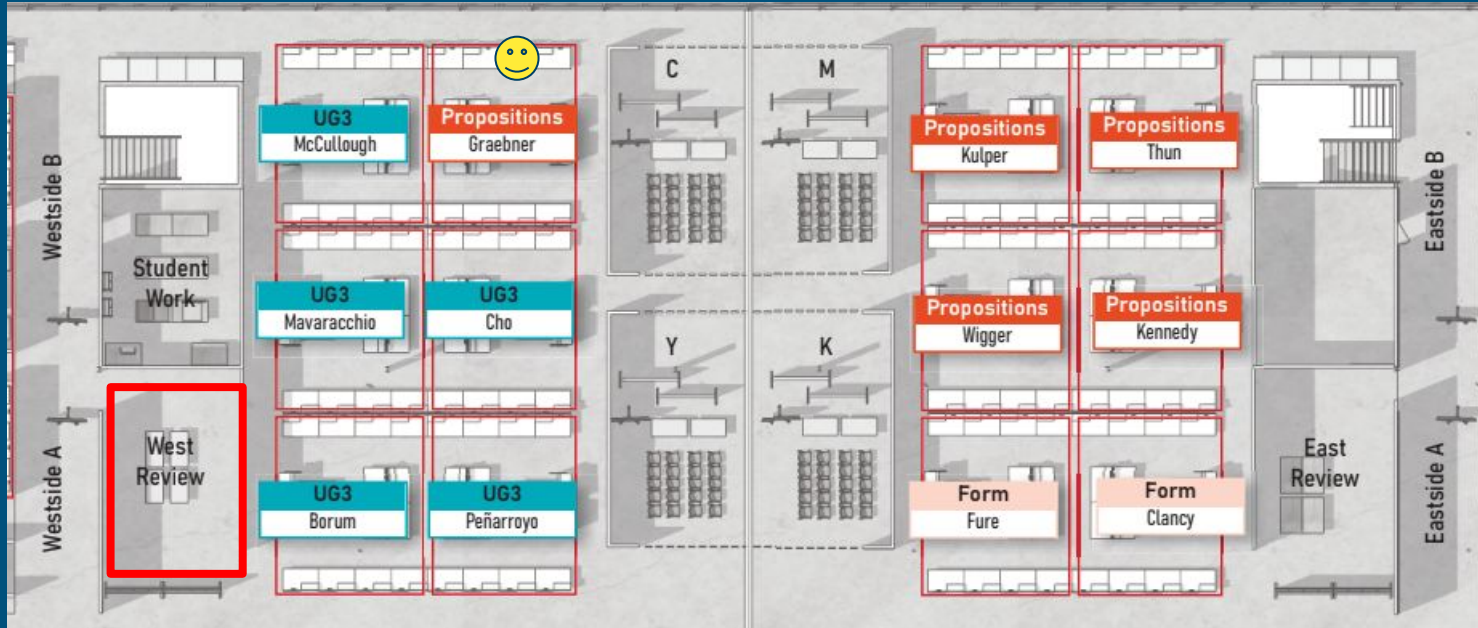
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GSI Info

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Questions

HW #8

8. Three Hinged Arches

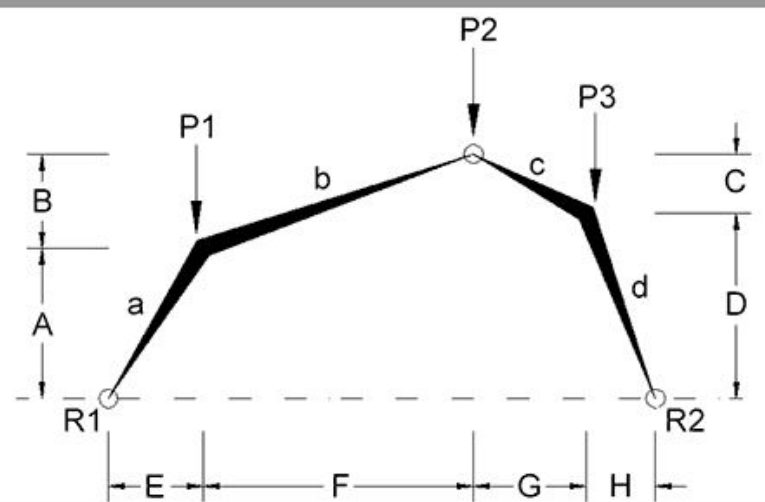
For the three-hinged arch shown, determine horizontal and vertical components of each reaction, and the moments at the knees.

DATASET: 1

-2-

-3-

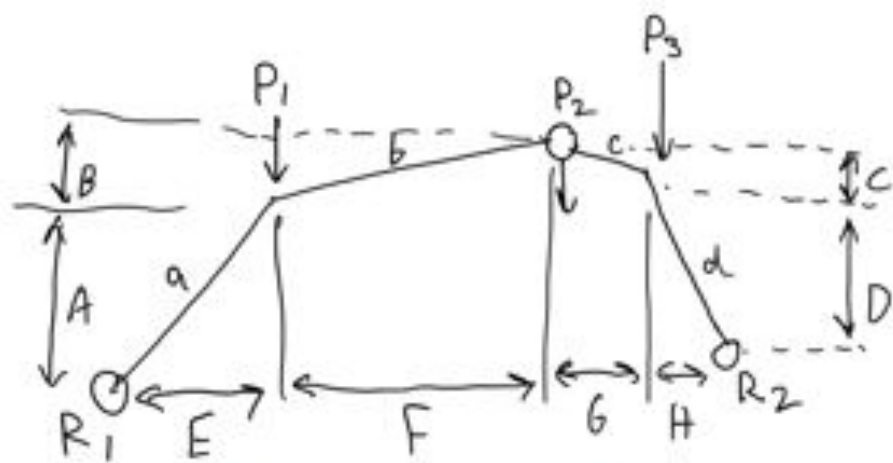
Height A	15 FT
Height B	3 FT
Height C	9 FT
Height D	9 FT
Length E	4 FT
Length F	14 FT
Length G	14 FT
Length H	4 FT
Force P1	6 KIPS
Force P2	19 KIPS
Force P3	10 KIPS



#

Question

- 1 HORIZONTAL component of R1 (+ = to the right)
- 2 VERTICAL component of R1 (+ = upward)
- 3 HORIZONTAL component of R2 (+ = to the right)
- 4 VERTICAL component of R2 (+ = upward)
- 5 Moment at M1 (+ = tension inside)
- 6 Moment at M2 (+ = tension inside)
- 7 Axial force in member "a" (+ is compression)
- 8 Axial force in member "b" (+ is compression)
- 9 Axial force in member "c" (+ is compression)
- 10 Axial force in member "d" (+ is compression)



$$A = 15'$$

$$B = 3'$$

$$C = 9'$$

$$D = 9'$$

$$E = 4'$$

$$F = 14'$$

$$G = 14'$$

$$H = 4'$$

$$P_1 = 6 \text{ kips}$$

$$P_2 = 19 \text{ kips}$$

$$P_3 = 10 \text{ kips}$$

1.) Find Vertical Components

$$\sum M @ R_2 = 0 = P_3(H) + P_2(G+H) + P_1(F+G+H) - R_{1y}(E+F+G+H)$$

$$R_{1y} = \frac{P_3(H) + P_2(G+H) + P_1(F+G+H)}{E+F+G+H}$$

$$R_{1y} = \frac{10k(4') + 19k(14'+4') + 6k(14'+14'+4')}{4' + 14' + 14' + 4'}$$

$$R_{1y} = 15.944 \text{ kips}$$

↑ #2, upwards so positive

$$\Sigma F_y = 0 = R_{1y} - P_1 - P_2 - P_3 + R_{2y}$$

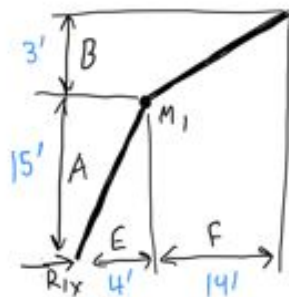
✓ From #2.

$$= 15.944_k - 6k - 19k - 10k + R_{2y}$$

$$R_{2y} = 19.0556$$

↑ #4, upwards so positive

2.) Solve for horizontal components



$$\sum M = 0 = R_{1y}(E+F) - R_{1x}(A+B) - P_1(F)$$

$$R_{1x} = \frac{P_1(F) - R_{1y}(E+F)}{(A+B)}$$

From #2

$$= \frac{6k(14') - 15.94(4' + 14')}{(15' + 3')}$$

$$R_{1x} = 11.2778 \text{ kips}$$

↑ #1, to the right
So positive

$$\sum F_x = 0 \equiv R_{1x} + R_{2x}$$

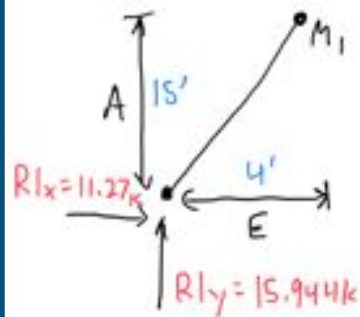
$$R_{2x} = -R_{1x} = -11.2778 \text{ kips}$$

↙ from #1.

$$R_{2x} = -11.2778 \text{ kips}$$

↑ #3, to the left
So negative

3.) Calculate Moments @ knees



$$\sum M_{@knee} = 0 = -R_{1y}(E) + R_{1x}(A) + M_1$$

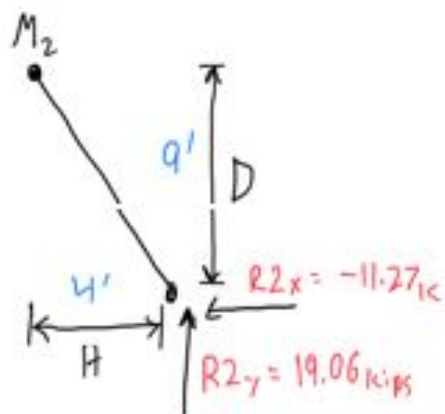
$$M_1 = R_{1y}(E) - R_{1x}(A)$$

$$= 15.944(4') - 11.277(15')$$

↑ From #2 ↓ From #2

$$M_1 = 105.3774 \text{ kips}$$

↑ #5. tension inside
So positive



$$\sum M @ \text{knee} = 0 = -R_{2y}(H) + R_{2x}(D) + M_2$$

$$M_2 = R_{2y}(H) - R_{2x}(D)$$

$$= 19.06(4') - 11.27(9')$$

From #4 \uparrow \quad \downarrow From #3

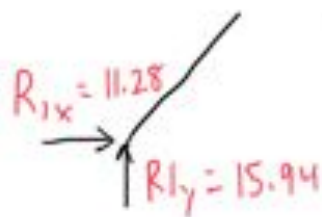
$$M_2 = -25.2778 \text{ kips}$$

\uparrow #6. No tension inside, so negative

Moment diagram is drawn on the tension side, opposite of the convention for beams.

4.) Determine Axial Forces

member a)

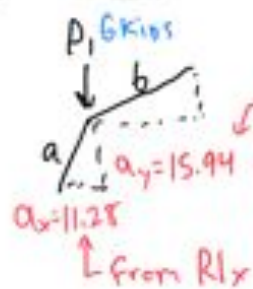


$$a = \sqrt{11.28^2 + 15.94^2} = 19.53 \text{ kips}$$

↑ from #1 ↑ from #2

7, forces are
pointed to the joint
so it is in compression

member b)



$$\sum F_x = 0 = a_x - b_x$$
$$b_x = 11.28$$

$$\sum F_y = 0 = a_y - P_1 - b_y$$

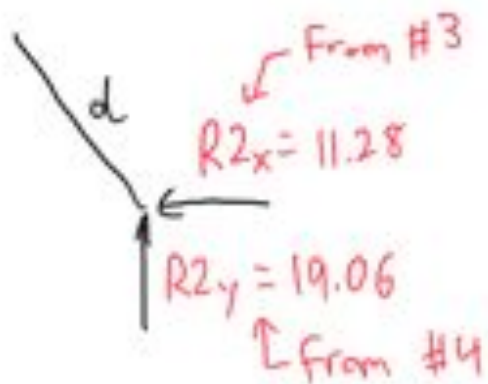
$$b_y = 15.94 - 6k$$

$$b_y = 9.94k$$

$$b = \sqrt{11.28^2 + 9.94^2} = 15.036 \text{ kips}$$

↑ # of forces are towards joint so it is in compression

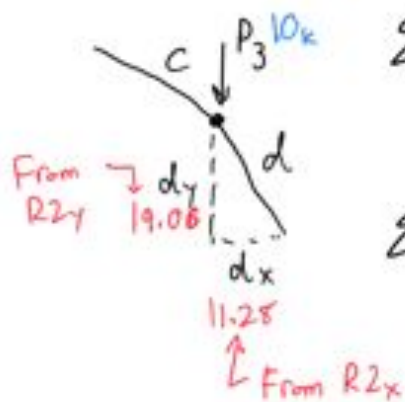
member d)



$$d = \sqrt{11.28^2 + 19.06^2} = 22.15 \text{ kips}$$

#10, Forces are towards joint so it is in compression

member c)



$$\sum F_x = 0 = dx - C_x$$


$$C_x = 11.28$$

$$\sum F_y = 0 = dy - P_3 - C_y$$

$$C_y = 19.06_k - 10_k$$

$$C_y = 9.06_k$$

$$C = \sqrt{11.28^2 + 9.06^2} = 14.466 \text{ kips}$$

#9, Forces are towards 
joint so it is in compression

LAB

