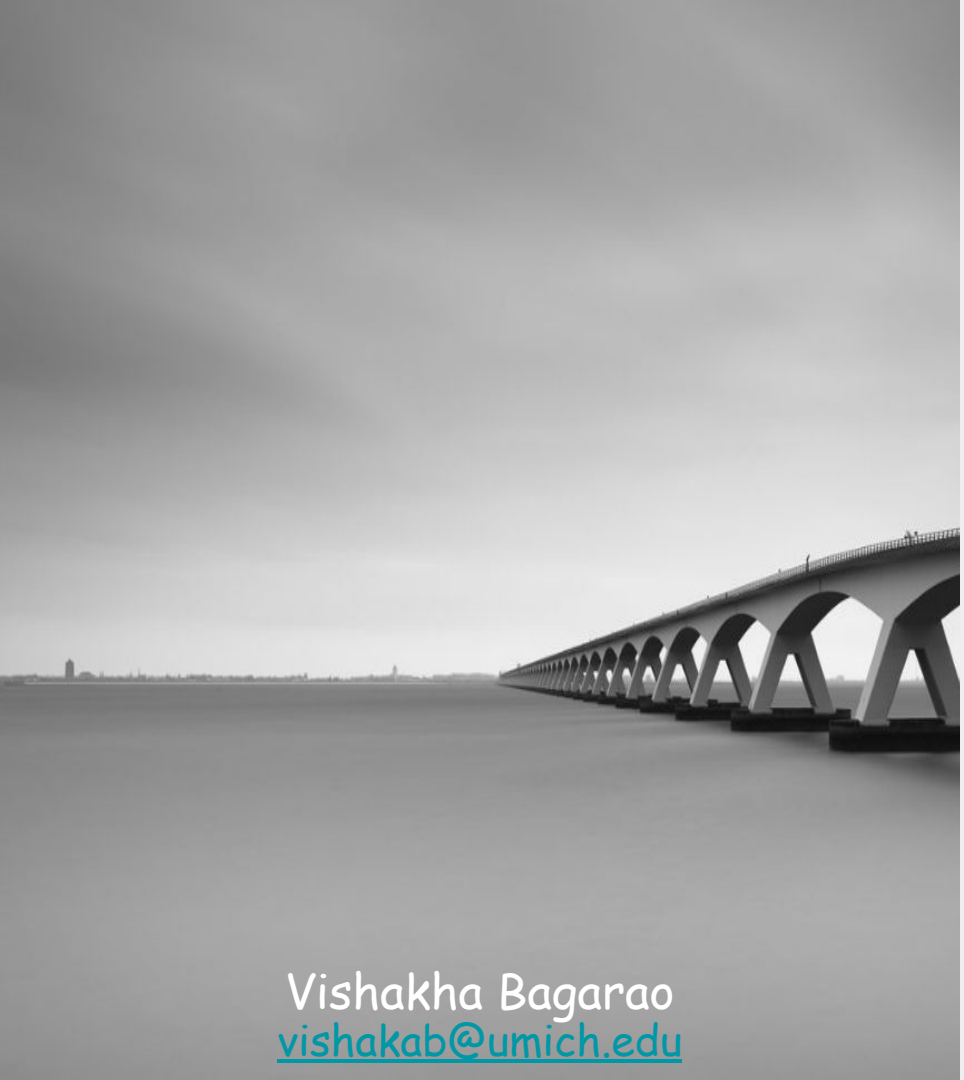
Arch 314- Structures I

Recitation 006

Vishakha Bagarao

18th Oct 2024



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  - Three Hinged Arches
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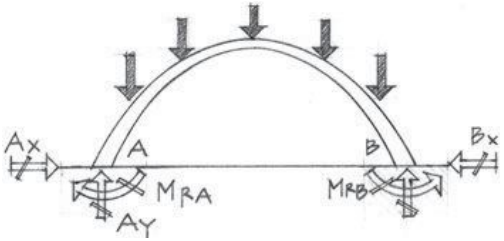
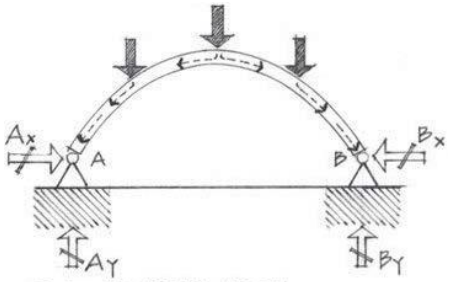
Three Hinged Arches

- Arches are a structural type suitable for spanning large distances.
- Forces developed within an arch are primarily compressive, with relatively small bending moments.

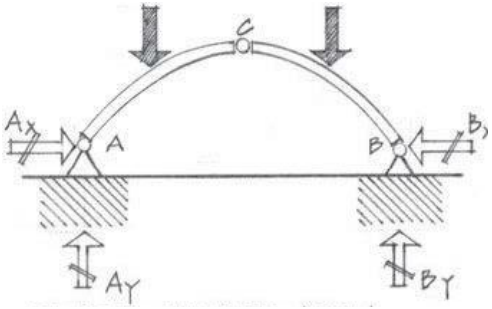
- The 3-Hinged Arch has a “hinge” at each pinned support plus one more internally.
  The internal hinge provides one additional statics equation to be written since the moment at C is known (MC = 0).
  Therefore, Three Hinged Arches are statically determinate systems.



Fixed Arch



Two Hinged Arch

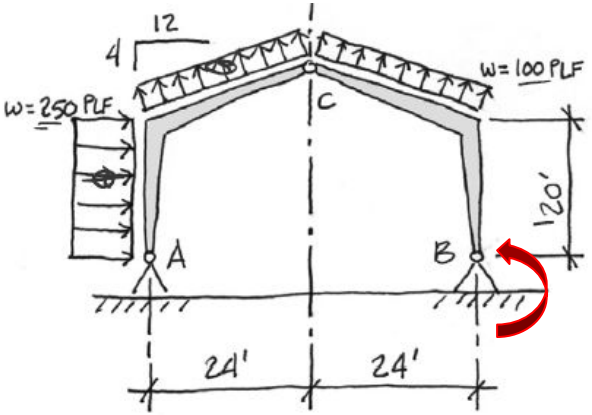


Three Hinged Arch

Three Hinged Arches:

Step 01:
Determine all external loads.

- Find resultants of distributed loads if any (e.g. wind, snow, dead load)

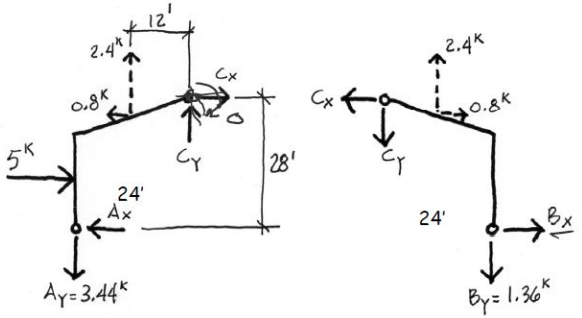


Step 02:
Calculate vertical end reactions. (Ay & By)

- Calculate Ay & By by using the diagram of the whole structure.
- Sum moments at each reaction.
- ΣM = 0
- ΣFy = 0

Step 03:
Draw FBD

- Draw an FBD of each side of the arch split at the hinge.

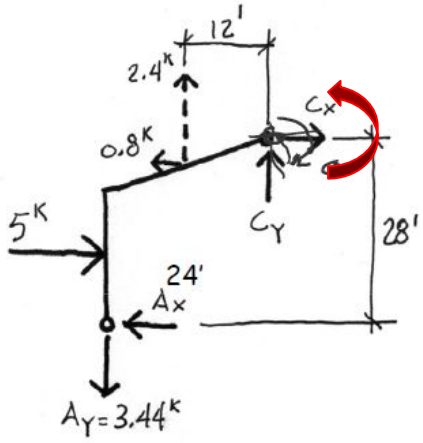


Three Hinged Arches:

Step 04:

Calculate horizontal end reactions. (Ax & Bx)

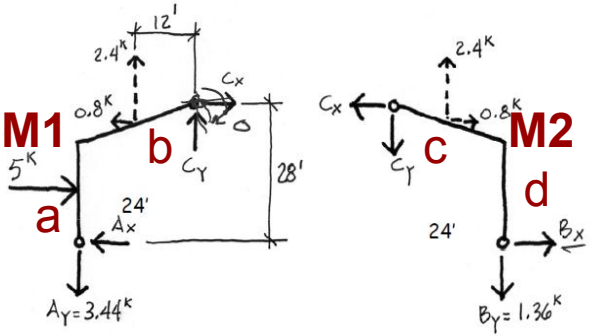
- Summing moments at the internal hinge on an FBD of half of the structure to find the horizontal forces.
- $\Sigma M = 0$
- $\Sigma Fx = 0$



Step 05:

Find internal moments.

- Cut additional FBDs (e.g., at the knees)
- $\Sigma M1@_{\text{left knee}} = 0$
- $\Sigma M2@_{\text{right knee}} = 0$



Step 06:

Find axial force in member:

- For member a: $Fa = \sqrt{Fax^2 + Fay^2}$

Problem Set 08

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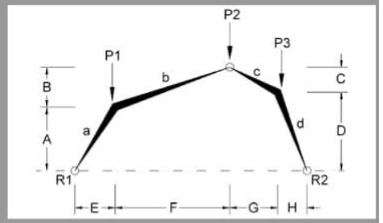
8. Three Hinged Arches

For the three-hinged arch shown, determine horizontal and vertical components of each reaction, and the moments at the knees.



DATASET: 2 -1- -3-

Height A	17 FT
Height B	5 FT
Height C	15 FT
Height D	7 FT
Length E	9 FT
Length F	9 FT
Length G	22 FT
Length H	4 FT
Force P1	7 KIPS
Force P2	7 KIPS
Force P3	12 KIPS

#	Question	Your Response	Correct Answer	Score
1	HORIZONTAL component of R1 (+ = to the right)	5.969 KIPS	5.96901 KIPS	5
2	VERTICAL component of R1 (+ = upward)	10.7955 KIPS	10.7955 KIPS	5
3	HORIZONTAL component of R2 (+ = to the right)	-5.969 KIPS	-5.96901 KIPS	5
4	VERTICAL component of R2 (+ = upward)	15.2045 KIPS	15.2045 KIPS	5
5	Moment at M1 (+ = tension inside)	4.3135 KIP-FT	4.31405 KIP-FT	5
6	Moment at M2 (+ = tension inside)	19.035 KIP-FT	19.0351 KIP-FT	5
7	Axial force in member "a" (+ is compression)	12.3358 KIPS	12.3358 KIPS	5
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Problem Menu

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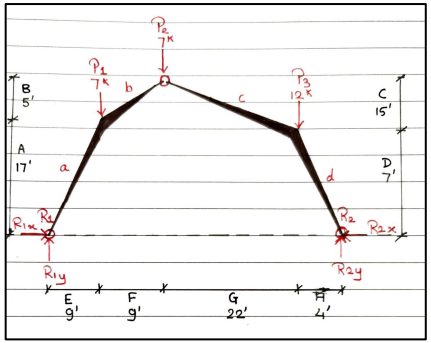


2,4. Vertical component of end reactions: (R_{1x}, R_{2x}).

$$\Sigma M_{R_2} = R_{1x}(0) + R_{1y}(E+F+G+H) - P_1(F+G+H) - P_2(G+H) - P_3(H) + R_{2x}(0) + R_{2y}(0).$$

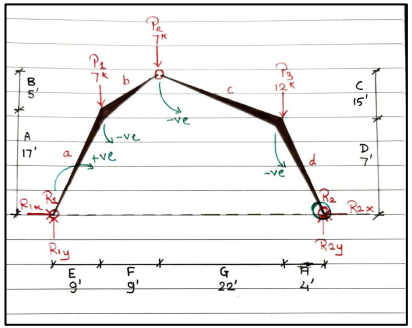
$$0 = 44R_{1y} - 7(9+22+4) - 7(22+4) - 12(4)$$

$$\therefore R_{1y} = 10.7955 \text{ KIPS} (\uparrow)$$

$$\Sigma F_y = R_{1y} - P_1 - P_3 - P_2 + R_{2y}$$

$$0 = 10.7955 - 7 - 12 - 7 + R_{2y}$$

$$\therefore R_{2y} = 15.2045 \text{ KIPS} (\uparrow)$$



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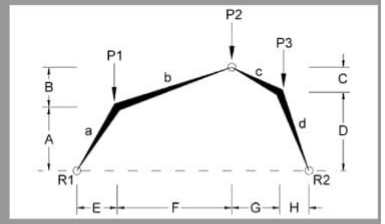
For the three-hinged arch shown, determine horizontal and vertical components of each reaction, and the moments at the knees.



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1, 3. Horizontal component of end reactions: (R_{1y}, R_{2y}):

$$\Sigma M_{P_2} = -R_{1x}(A+B) + R_{1y}(E+F) - P_1(F)$$

$$0 = -22R_{1x} + 10.7955(18) - 7(9)$$

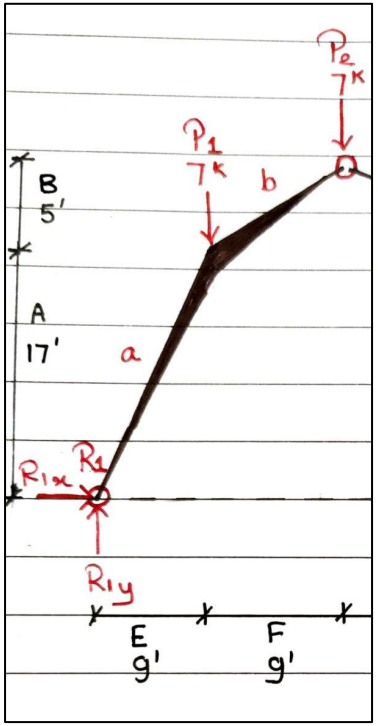
$$R_{1x} = \frac{131.319}{22}$$

$$\therefore R_{1x} = 5.969 \text{ KIPS} \quad (\rightarrow, +)$$

$$\Sigma F_x = R_{1x} - R_{2x} = 0$$

$$\therefore R_{2x} = 5.969 \text{ KIPS} \quad (\leftarrow, -)$$

$$\therefore R_{2x} = -5.969 \text{ KIPS}$$



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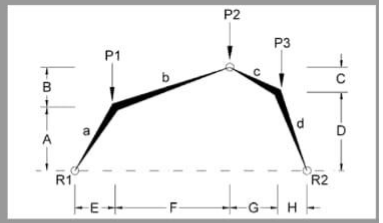

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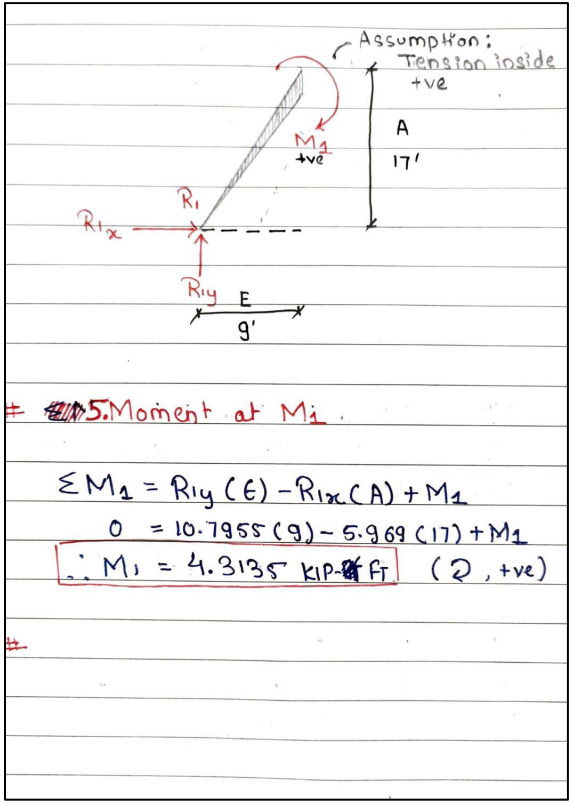

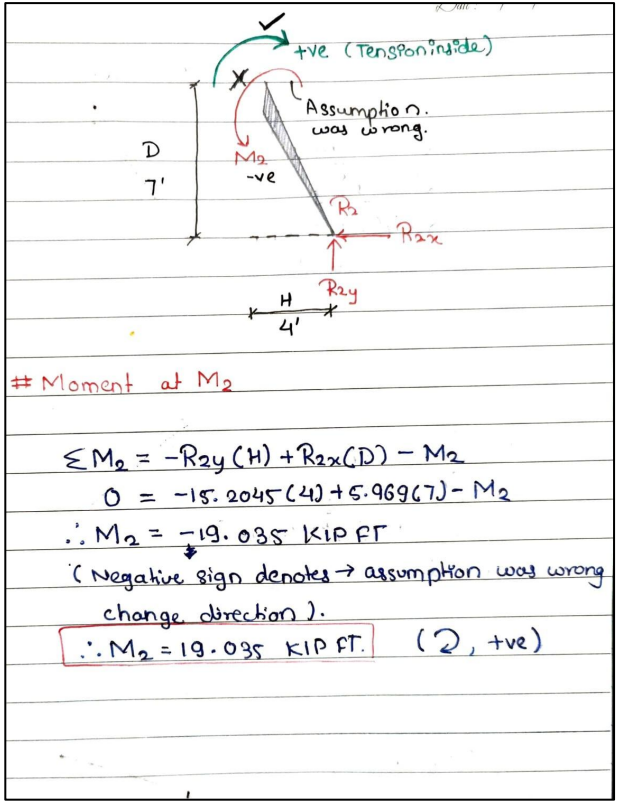

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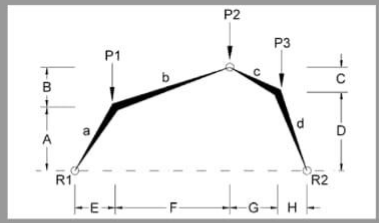

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7. Axial Force in member "a".

$F_a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{R_{1x}^2 + R_{1y}^2}$

$= \sqrt{5.969^2 + 10.79^2}$

$\therefore F_a = 12.3358$ KIPS

10. Axial Force in member "d".

$F_d = \sqrt{d_x^2 + d_y^2}$

$= \sqrt{R_{2x}^2 + R_{2y}^2}$

$= \sqrt{(-5.969)^2 + (15.2045)^2}$

$\therefore F_d = 16.3342$ KIPS.

8. Axial Force in member "b" @ P1

$\Sigma F_y = R_{1y} - b_y - P_1 = 0$

$b_y = 10.7955 - 7$

$\therefore b_y = 3.7955$ KIPS (↓)

MATRIKAS

$\Sigma F_x = a_x - b_x = 0$

$\therefore b_x = 5.969$ KIPS (←)

$F_b = \sqrt{b_x^2 + b_y^2}$

$= \sqrt{(5.969)^2 + (3.7955)^2}$

$\therefore F_b = 7.0735$ KIPS

9. Axial force in member "c" @ P3.

$\Sigma F_y = R_{2y} - C_y - P_3$

$0 = 15.2045 - C_y - 12$

$\therefore C_y = 3.2045$ KIPS (↓)

$\Sigma F_x = C_x - R_{2x} = 0$

$\therefore C_x = R_{2x}$

$\therefore C_x = 5.969$ KIPS (→)

$F_c = \sqrt{C_x^2 + C_y^2}$

$= \sqrt{(5.969)^2 + (3.2045)^2}$

$\therefore F_c = 6.7748$ KIPS.

Lab 06: Three Hinged Arches

The arch may be visualized as a “cable” turned upside down, developing compressive stresses of the same magnitude as the tensile stresses in the cable.

Structures I
Arch 314

Name 1 _____
Name 2 _____
Name 3 _____

3-Hinged Arches

Description

This project finds the reactions and moments of a three-hinged arches.

Goals

To observe the end thrust behavior of a three-hinged arch.
To calculate the end reactions of the arch.
To calculate the moment at the knee.
To find the geometry of a catenary arch.

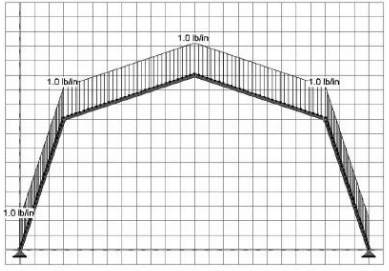
Procedure

Part 1

1. Adjust the 3-hinged arch model on graph paper to have a 9” span.
2. Copy the geometry onto graph paper to determine the dimensions – the member lengths should be about 4” each.
3. Assume a uniform vertical load of 1 pound / inch on the length of each member (like a selfweight). Find and locate the resultant forces on your drawing.
4. Calculate the end reactions.
5. Calculate the peak moment at the knee.

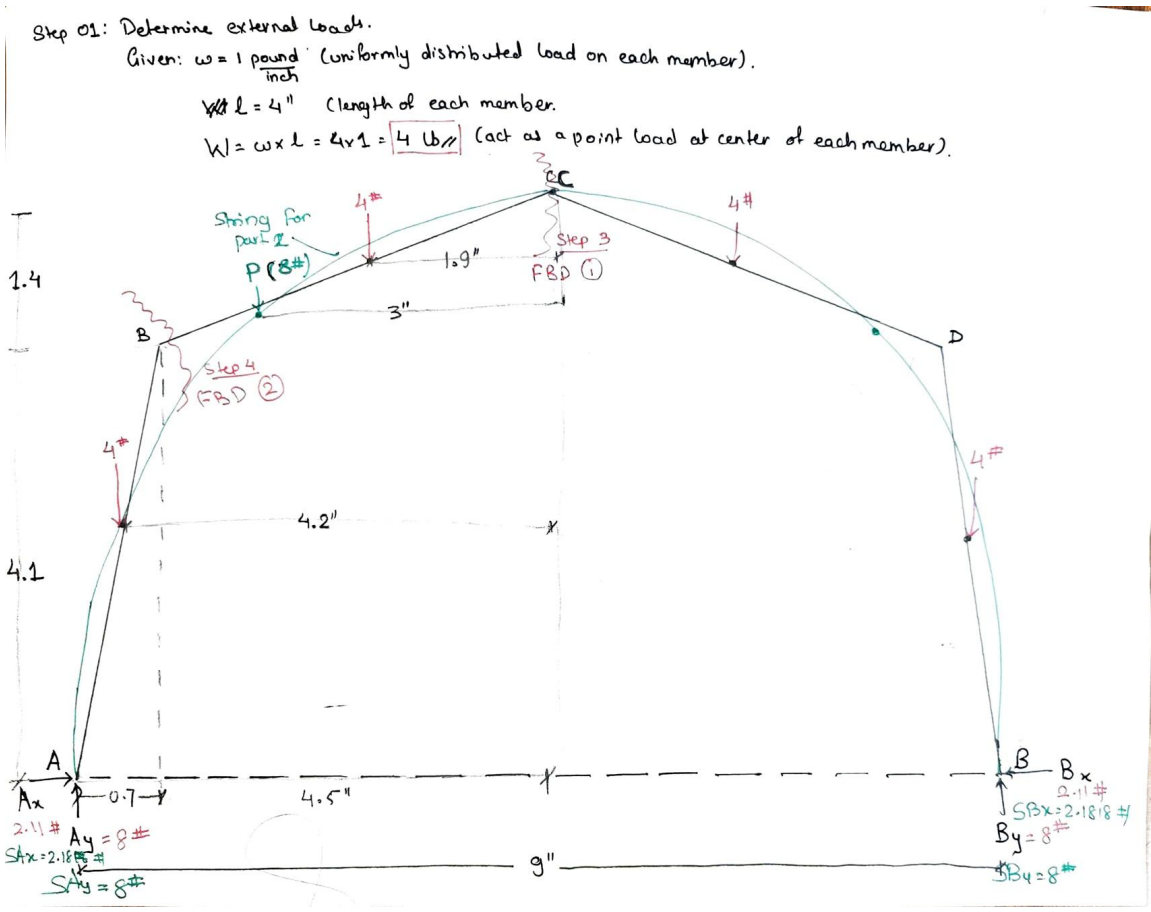
Part 2

6. Next use the string to find a funicular shape with the same span.
7. Copy the new geometry onto the graph paper (overlaid on the original arch) to determine the dimensions.
8. Segment the arch into four symmetric sections.
9. Calculate the end reactions.
10. Calculate the peak moment at the knee.



Due

During recitation



Step 02: End Reactions:

By symmetry $A_y = B_y$

Total forces acting downwards = 4+4+4+4 = 16#

$\Sigma F_y = A_y + B_y - 16 = 0$

$A_y + B_y = 16 \quad (\because A_y = B_y)$

$\therefore A_y = B_y = 8\#$.

Step 03: FBD 1 (left half).

$\Sigma M_C = 8(4.5) - 4(4.2) - 4(1.9) - A_x(5.5) = 0$

$\therefore A_x = 2.11\#$ $\quad \therefore B_x = 2.11\#$

Step 04: Peak moment at knee

FBD 2

$\Sigma M_D = -4(0.35) + 8(0.7) - 2.1(4.1) + M$

$\therefore M = 4.41$ pounds ft.

Step 07: Peak moment at the knee

FBD 2

$\Sigma M_D = 8(0.7) - 2.1818(4.1) + M$

$\therefore M = 3.345$ pounds ft.

PART 02: follow same steps with string.

Step 05: Determine external loads (P).

(measure the length of the string $= 8''$) $\therefore l = 8''$

$P = w \times l = 8 \times 1 = 8\#$

Step 06: FBD 1 (left half).

$\Sigma M_C = 8(4.5) - 8(3) - SA_x(5.5) = 0$

$\therefore SA_x = 2.1818\#$ $\quad \therefore SB_x = 2.1818\#$