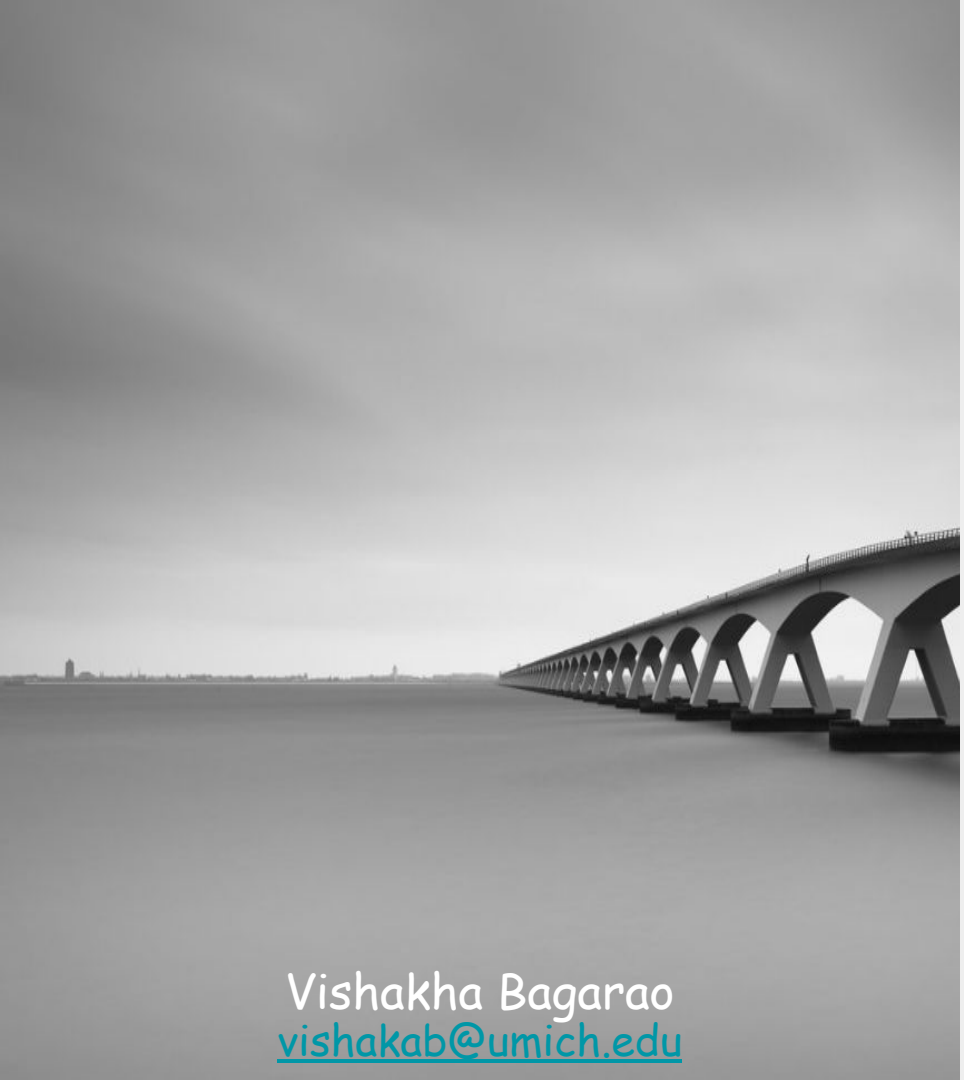
Arch 314- Structures I

Recitation 006

Vishakha Bagarao

6th Dec 2024



Vishakha Bagarao
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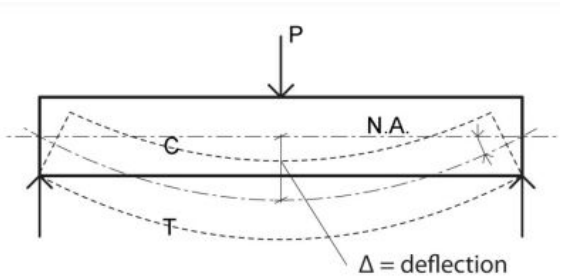
Contents

- Quick Recap
  - Deflection of Beam
    - Problem Set 14- Deflection of beams
- Lab 12- Deflection

Deflection in Beam:

- Deflection, a stiffness requirement, represents a change in the vertical position of a beam due to the applied loads.
- Factors that result in deflection- Load magnitude, beam-span length, moment of inertia of the beam cross-section, and the beam's modulus of elasticity.
- The change in lengths, top and bottom, results in the material straining. For a simple span with downward loading, the top is compressed and the bottom stretched.
- The curved shape of a deflected beam is called the elastic curve.

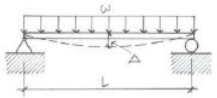
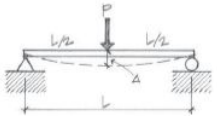
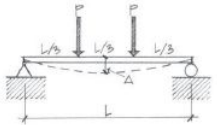
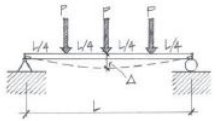
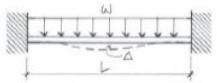
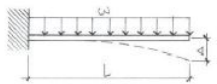
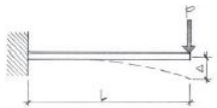


Axial Stiffness

$$\text{Stiffness} = \frac{EA}{L}$$

Flexural Stiffness

$$\text{Stiffness} = \frac{EI}{L}$$

Beam Load and Support	Actual Deflection[a]
 (a) Uniform load, simple span	$\Delta_{max} = \frac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \frac{PL^3}{48EI}$ (at the centerline)
 (c) Two equal concentrated loads at third points	$\Delta_{max} = \frac{23PL^3}{648EI} = \frac{PL^3}{28.2EI}$ (at the centerline)
 (d) Three equal concentrated loads at quarter points	$\Delta_{max} = \frac{PL^3}{20.1EI}$ (at the centerline)
 (e) Uniform load both ends fixed	$\Delta_{max} = \frac{\omega L^4}{384EI}$ (at the centerline)
 (f) Cantilever with uniform load	$\Delta_{max} = \frac{\omega L^4}{8EI}$ (at the free end)
 (g) Cantilever with concentrated load at the end	$\Delta_{max} = \frac{PL^3}{3EI}$ (at the free end)

Formulas:

- Step 1: Beam Reaction R1 (Symmetric)

$$\sum V = 0$$

- Step 2: Shear Values: V

  Draw a shear diagram

- Step 3: Moment Values: M

  Draw a moment diagram (shear areas)

- Step 4: Area under Moment diagram: MA

  Moment diagram areas

- Step 5: Slope (EI) : S

  S1= MA1+MA2+MA3

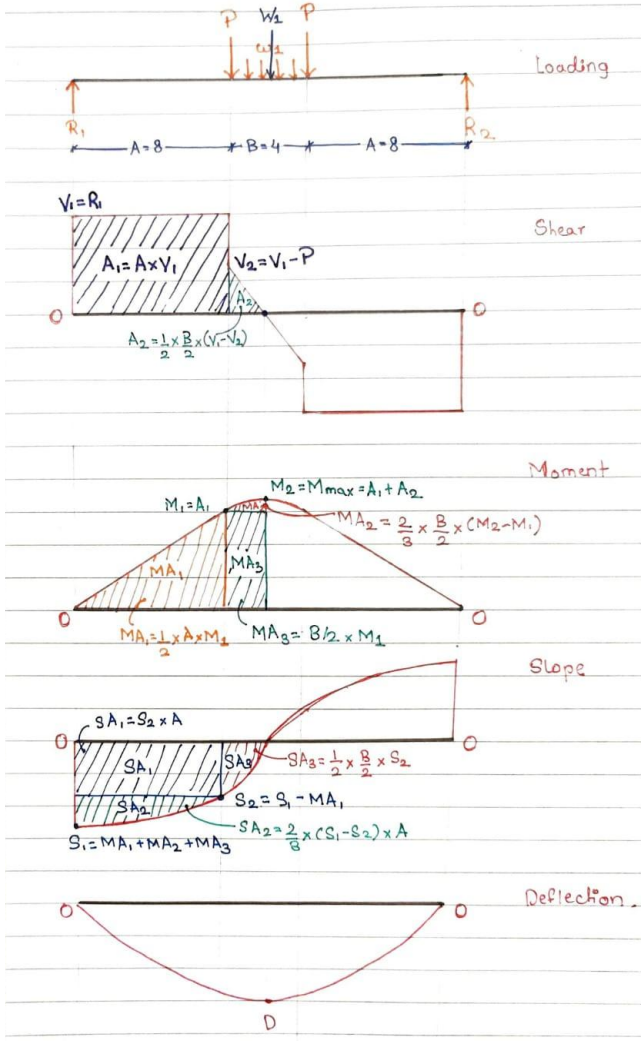
  S2= S1-MA1= MA2+MA3

- Step 6: Area under Slope (EI): SA

  SA3 is considered as a triangle

- Step 7: Cantilever deflection: (Inches)

$$Def = \frac{|D_{max}|}{E \times I} \times 12^3 \qquad |D_{max}| = SA_1 + SA_2 + SA_3$$



Problem Set 14

MICHIGAN Architecture — Structures Problems

Contact | Schedule | Lectures | Recitation | Bridges1 | Bridges2 | Problems

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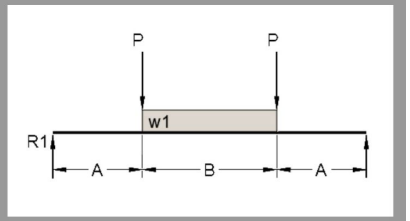
14. Beam Deflection

For the given simple span beam, use shear, moment, slope and deflection diagrams to determine the slope at each end, the deflection at points A and B distance from left, and the maximum deflection at the centerline. Remember to divide out EI to get deflection in inches. Be sure to correct errors at each step to maintain accuracy.



DATASET: 1 -2- -3-

Length A	8 FT
Length B	4 FT
Point Load P	8 KIPS
Uniform Load w	4 KLF
Modulus of Elasticity	29000 KSI
Moment of Inertia	310 IN^4

#	Question	Your Response	Correct Answer	Score
1	R1 (+ = upward)	16 KIPS	16 KIPS	5
2	Shear at reaction (V1)	16 KIPS	16 KIPS	5
3	Shear at point load (V2)	8 KIPS	8 KIPS	5
4	Moment at point load (M1)	128 KIP-FT	128 KIP-FT	5
5	Moment at center line (M2)	136 KIP-FT	136 KIP-FT	5
6	Area under moment diagram (MA1)	512 KIP-FT^2	512 KIP-FT^2	5
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8	Area under moment diagram (MA3)	256 KIP-FT^2	256 KIP-FT^2	5
9	Slope(EI) at reaction (S1). Give absolute value.	778.667 KIP-FT^2	778.667 KIP-FT^2	5
10	Slope(EI) at point load (S1). Give absolute value.	266.667 KIP-FT^2	266.667 KIP-FT^2	5
11	Area under slope(EI) diagram (SA1)	2133.34 KIP-FT^3	2133.33 KIP-FT^3	5
12	Area under slope(EI) diagram (SA2)	2730.67 KIP-FT^3	2730.67 KIP-FT^3	5
13	Area under slope(EI) diagram (SA3)	266.667 KIP-FT^3	266.667 KIP-FT^3	5
14	Centerline deflection. Give absolute value in INCHES.	0.986186 INCHES	0.986184 INCHES	5

Current Score: 70 / 70

Problem Menu

Logout

Beam Deflection.

#1 Beam reactions: R_1 (+ = upward ; − = downward)

$W_1 = w_1 \times B$
$= 4 \times 4$
$= 16$ KIPS

$\Sigma V = 0$
$0 = R_1 - P - W_1 - P + R_2$
$(\because$ Symmetrical, $R_1 = R_2)$
$= 2R_1 - 8 - 16 - 8$
$\therefore R_1 = 16$ KIPS

#2, #3 Shear values : V_1 ; V_2

#2 Shear at reaction (V_1)

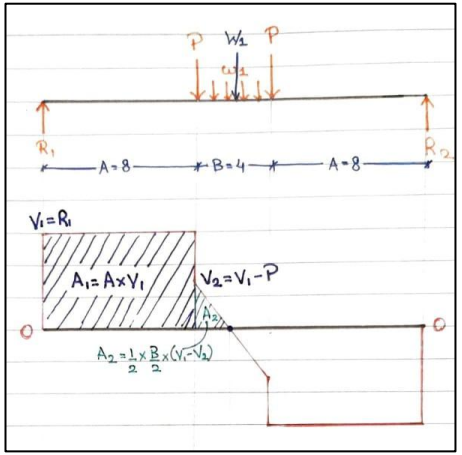
$\because R_1$ pushes beam upwards at reaction.

$\therefore V_1 = R_1 = 16$ KIPS

#3 Shear at point load (V_2)

$\because P$ is pushing beam downwards.

$V_2 = R_1 - P = V_1 - P$
$= 16 - 8$
$\therefore V_2 = 8$ KIPS.



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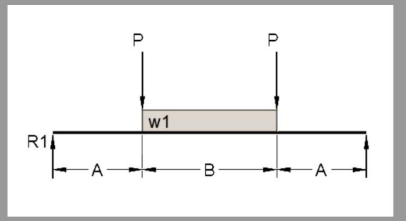

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#4, #5. Moment : M_1 ; M_2.

#4 Moment at point load : M_1

$M_1 = A_1 \cdot$

$= A \times Y_1$ ... (Area of rectangle)

$= 8 \times 16$

$\therefore M_1 = 128$ KIP-FT

Center Line

#5. Moment at ~~point load~~ : M_2

$M_2 = A_1 + A_2$

$= A_1 + (1/2 \times B/2 \times (Y_1 - Y_2))$ ... (Area of □ + Δ)

$= 128 + (1/2 \times 4/2 \times (16-8))$

$= 128 + 8$

$\therefore M_2 = 136$ KIP-FT

#6, 7, 8 Area under moment diagram: MA_1 ; MA_2 ; MA_3

#6 MA_1

$MA_1 = 1/2 \times A \times M_1$ ... (Area of Δ)

$= 1/2 \times 8 \times 128$

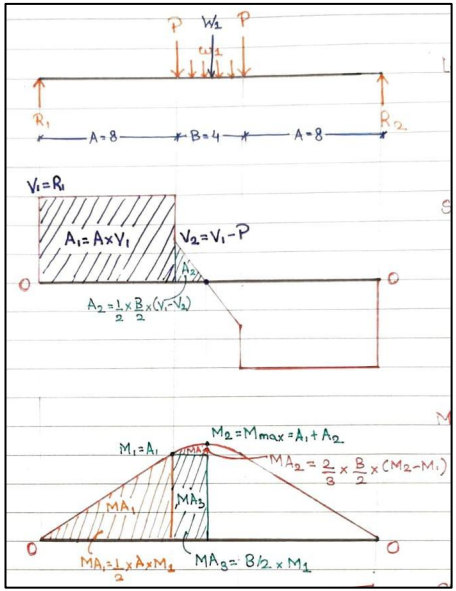
$\therefore MA_1 = 512.$ KIP FT²

#9 MA_3

$MA_3 = (B/2 \times M_1)$ ... (Area of □)

$= 2 \times 128$

$\therefore MA_3 = 256$ KIP FT²



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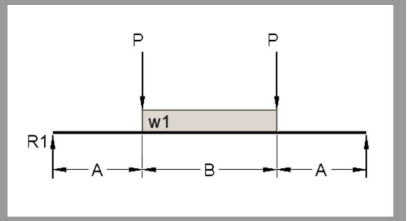

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#7. MA_2

$$MA_2 = \frac{2}{3} \times \frac{B}{2} \times (M_2 - M_1)$$

$$= \frac{2}{3} \times \frac{4}{2} \times (136 - 128)$$

$$= \frac{2}{3} \times 8$$

$$\therefore MA_2 = 10.667 \text{ KIP-FT}^2$$

#10, #9 Slope EI : S_1 ; S_2

#9 Slope (EI) at reaction (S_1)

∵ Maximum slope occurs at end of the beam. $PL^2/16EI$

$$S_1 = MA_1 + MA_2 + MA_3$$

$$= 512 + 10.6667 + 256$$

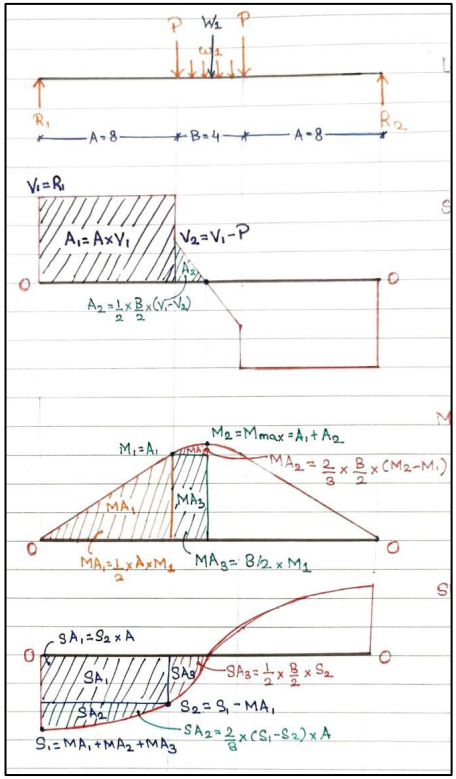
$$\therefore S_1 = 778.6667 \text{ KIP-FT}^2$$

#10 Slope (EI) at point load (S_2)

$$S_2 = S_1 - MA_1$$

$$= 778.6667 - 512$$

$$\therefore S_2 = 266.6667 \text{ KIP-FT}^2$$



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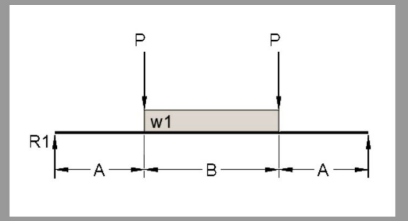

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#11, 12, 13. Area under slope (EI) diagram ; SA_1 ; SA_2 ; SA_3

#11. Area under slope (EI) diagram (SA_1)

$SA_1 = S_2 \times A$

$= 266.6667 \times 8$

$\therefore SA_1 = 2133.34 \ KIP\text{-}FT^3$

#12. Area under slope (EI) diagram (SA_2)

$SA_2 = \frac{2}{3} \times (S_1 - S_2) \times A$

$= \frac{2}{3} \times (778.6667 - 266.6667) \times 8$

$\therefore SA_2 = 2730.67 \ KIP\text{-}FT^3$

#13. Area under slope (EI) diagram (SA_3)

$SA_3 = 1/2 \times B/2 \times S_2$

$= 1/2 \times 4/2 \times 266.6667$

$\therefore SA_3 = 266.6667 \ KIP\text{-}FT^3$

#14. Centerline deflection. Give absolute value in INCHES

$\text{Deflection} = \frac{|Dmax|}{E \times I} \times 12^3$ ... ($12^3 \rightarrow$ convert to inches)

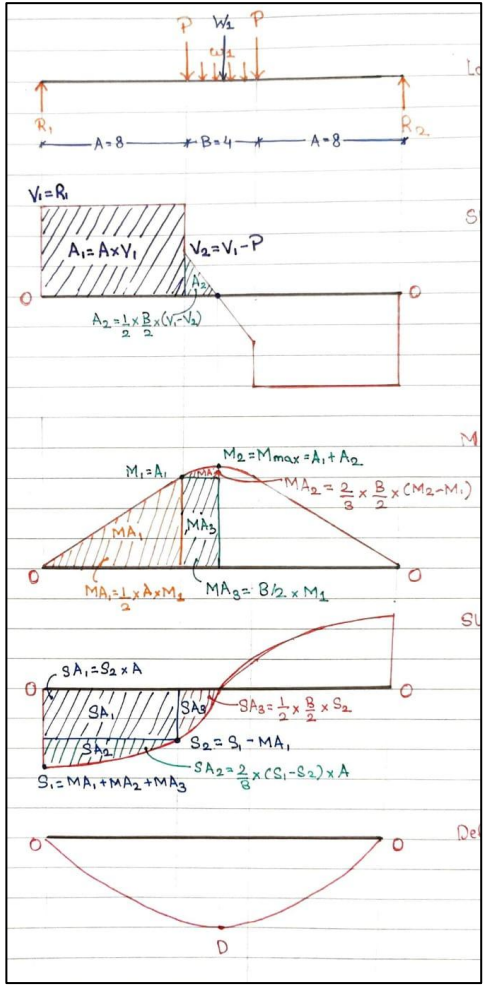
$\left[\frac{KIP\text{-}FT^3}{\frac{KIP}{in^2} \times in^4} \times \frac{in^3}{FT^3} = in \right]$ ... Unit calculation

$\because |Dmax| = |SA_1 + SA_2 + SA_3|$

$\therefore \text{Deflection} = \frac{|SA_1 + SA_2 + SA_3|}{29000 \times 310} \times 12^3$

$= \frac{|2133.34 + 2698.67 + 266.6667|}{29000 \times 310} \times 12^3$

$\therefore \text{Deflection} = 0.986186 \ \text{Inches}$



Lab 12 Deflection

Structures II

Arch 314

Name 1 _____

Name 2 _____

Name 3 _____

Deflection

Description

This project uses observation and calculation to understand how a cantilever member deflects under load.

Goals

To observe the bending behavior of a cantilever through physical modeling.
To find the deflection using the diagram method.
To verify the deflection using beam equations.

Procedure

1. Hold the 1/16"x1/2" basswood stick flatwise on the 2x4 support as shown.
2. Load first the free end, and measure the deflection against the graph paper (small squares = 0.1 inch).
3. Repeat the procedure for a load at the half point and at both points.
4. For each load measure and record a deflection.
5. Use the diagram method to calculate the deflection for the point load at the end.
6. Finally, calculate the deflection for the end load case with the equation below.

Basswood Properties

E = 1,650,000. psi
I_x = 0.0000102 in^4
P_1 = 0.035 lbs.
L = 10.5 in



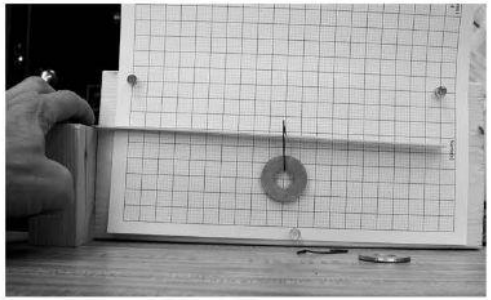
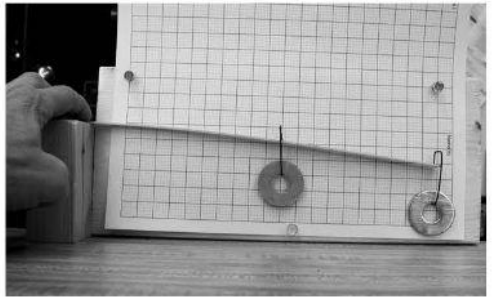

Equations:

$$I = \frac{bd^3}{12} \qquad \delta = \frac{Pl^3}{3EI}$$

Due

During recitation



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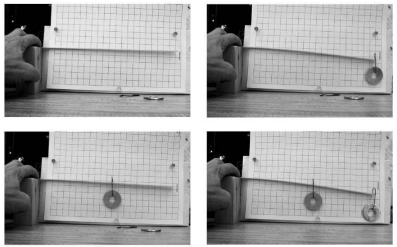

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$$I = \frac{bd^3}{12} \qquad \delta = \frac{Pl^3}{3EI}$$

Due
During recitation

Cantilever Beams

- Fixed end has maximum moment, but zero slope and deflection.
- Free end has maximum slope and deflection, but zero moment.
- Slope is either downward (-) or upward (+) depending on which end is fixed.
- Shear sign also depends on which end is fixed.
- Moment is always negative for gravity loads.
- Deflection is always negative with maximum at the free end for gravity loads.