TAUBMAN COLLEGE
ARCHITECTURE & URBAN PLANNING
UNIVERSITY OF MICHIGAN

Arch314
STRUCTURES I

Fall 2024
Recitation

FACULTY: Prof. Peter von Bülow
Mohsen Vatandoost

Welcome to Recitation session 11/15

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Office: Room 3122
hours:
Wed: 11:30 – 14:30
Mon, Fri: 11:30 - 13:30
walk-ins welcome!

Please feel free to ask questions.

Welcome to Recitation session 11/15

Outline:

- Quick **Recap** of the week
- Provide the solution for the assignment **(Homework 12)**
- Answering student's questions
- Lab: **Moment Diagrams**
- **Bridge** project (How to prepare the Final Report)/

Please feel free to ask questions.

Contact: Mohsenv@umich.edu

Recap of the week

Integral

Derivation

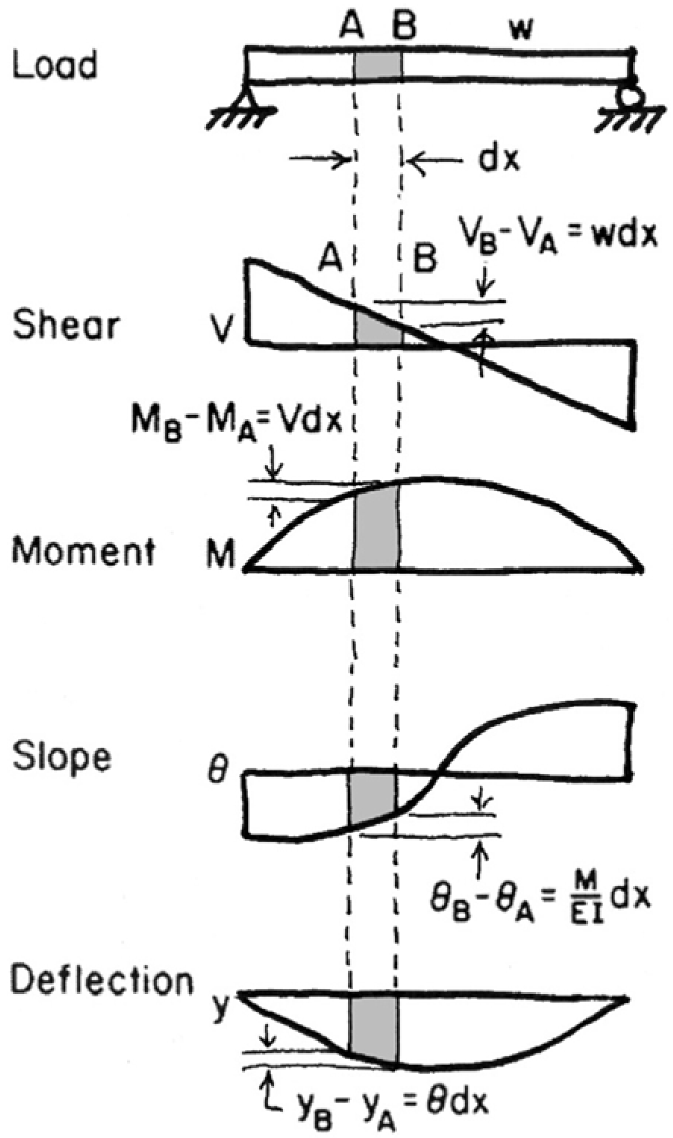
w

$V = \int w\,dx$

$M = \int V\,dx$

$\theta = \int \frac{M}{EI}\,dx$

$y = \int \theta\,dx$



Recap of the week

1. Equilibrium Method - example

$-w$

$$V = \int w\,dx$$

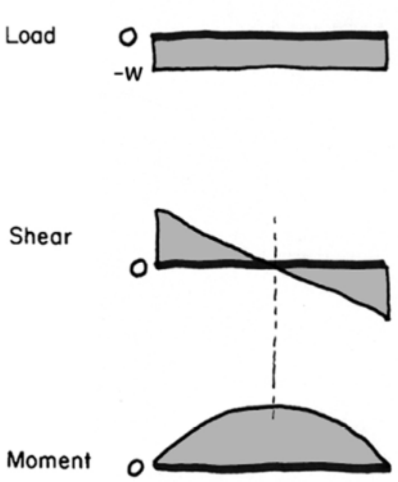
$$V = -wx + C$$

$$V = -wx + \frac{wl}{2}$$

$$M = \int V\,dx$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x + C$$

$$M = -\frac{w}{2}x^2 + \frac{wl}{2}x$$



Methods to Determine Values of Shear and Moment

1. Equilibrium Method
   - Select a point along the beam
   - Cut a section and draw the FBD
   - Solve for the internal shear and moment forces at the section
2. Integration of Equations
   - Write the equation of the load function
   - Integrate load equation to get shear equation
   - Solve integration constant (use end reaction)
   - Integrate shear equation to get moment equation
   - Solve integration constant (use point with zero moment, e.g. end point)
3. Semi-graphical Method
   - Draw load diagram and solve end reactions with equilibrium equations.
   - Start at left and construct the shear diagram using point loads and areas on load diagram
   - Calculate areas of shear diagram to find change in value on moment diagram
   - Find points of zero moment to begin moment diagram, e.g. end points
4. Superposition of Equations
   - Break the loading into standard cases
   - Use given equations to solve shear and moment for each case
   - Add the cases to get combined values of original loading

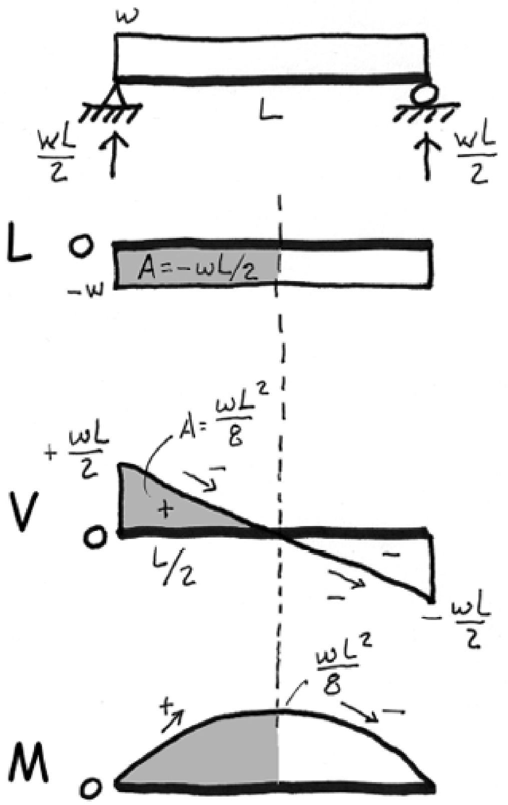
Recap of the week

3. Shear and Moment by Semi-graphical Method – diagram relationships

By recognizing the diagrammatic relationships between curves and their derivatives and integrals, shear and moment diagrams can be constructed based on areas and slopes of those curves.

Moving from Upper to Lower Diagrams:

- The area between any two points on the upper diagram is equal to the change in value between same points on the lower diagram.
- The degree of the curve increases by one for each diagram.
- The value on the upper diagram is equal to the slope of the lower diagram.
- Where the upper diagram crosses 0 on the axis, the lower diagram is at a maximum or minimum.
- Points of inflection or “contraflexure” (between + and – curvature) on the elastic curve (deflected shape) are points of zero moment.

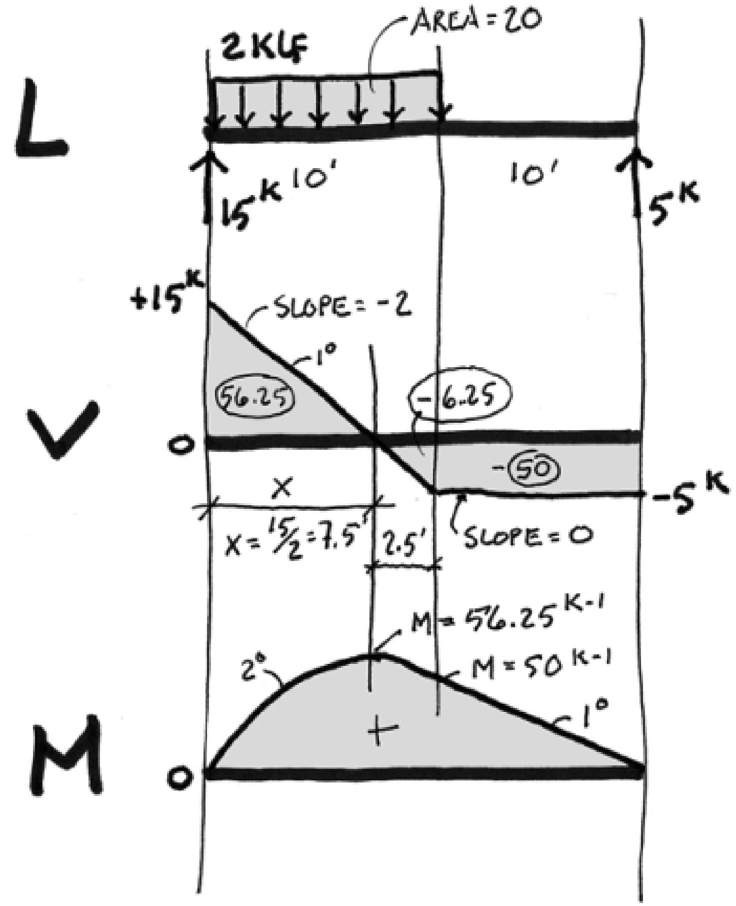


Recap of the week

3. Semi-graphical Method

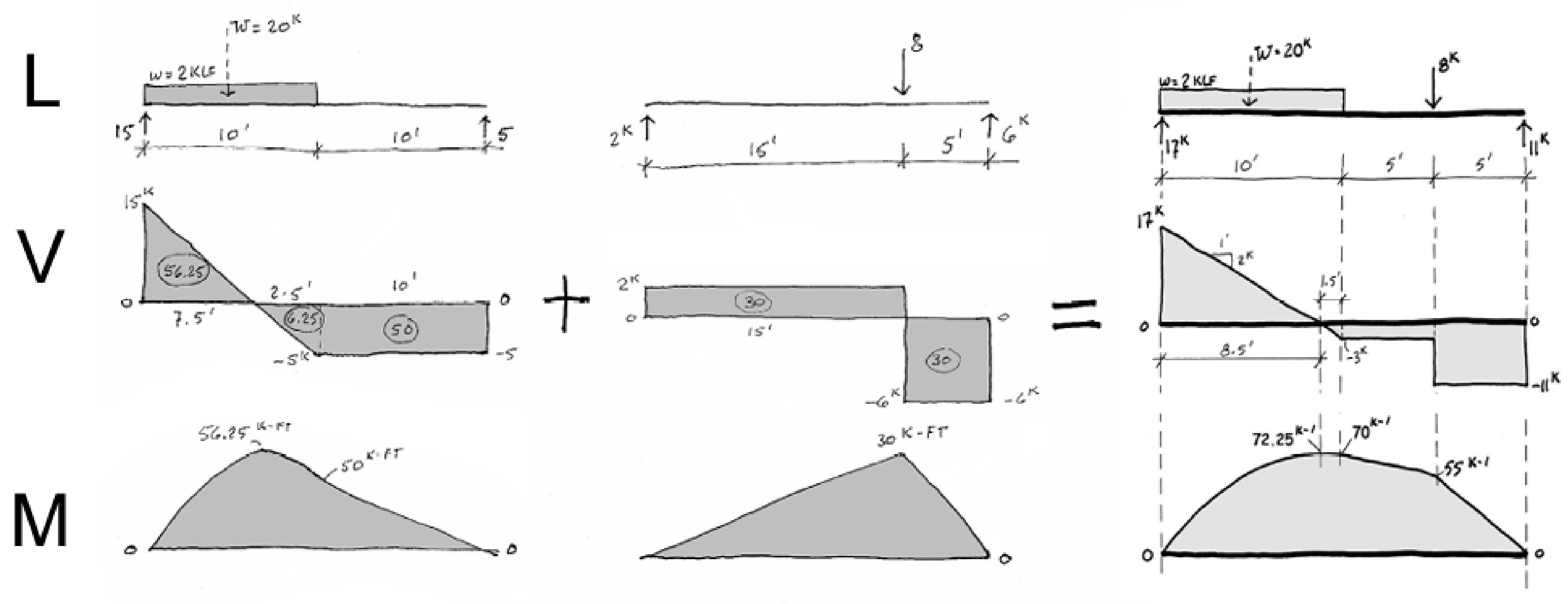
Procedure:

1. Find end reactions
2. Start at left end of V-Diagram and "apply" load from left to right
3. Calculate areas of V-Diagram
4. Find max. and min. values on M-Diagram using V-Diagram areas between axis crossings.
5. Check slope and + or - values



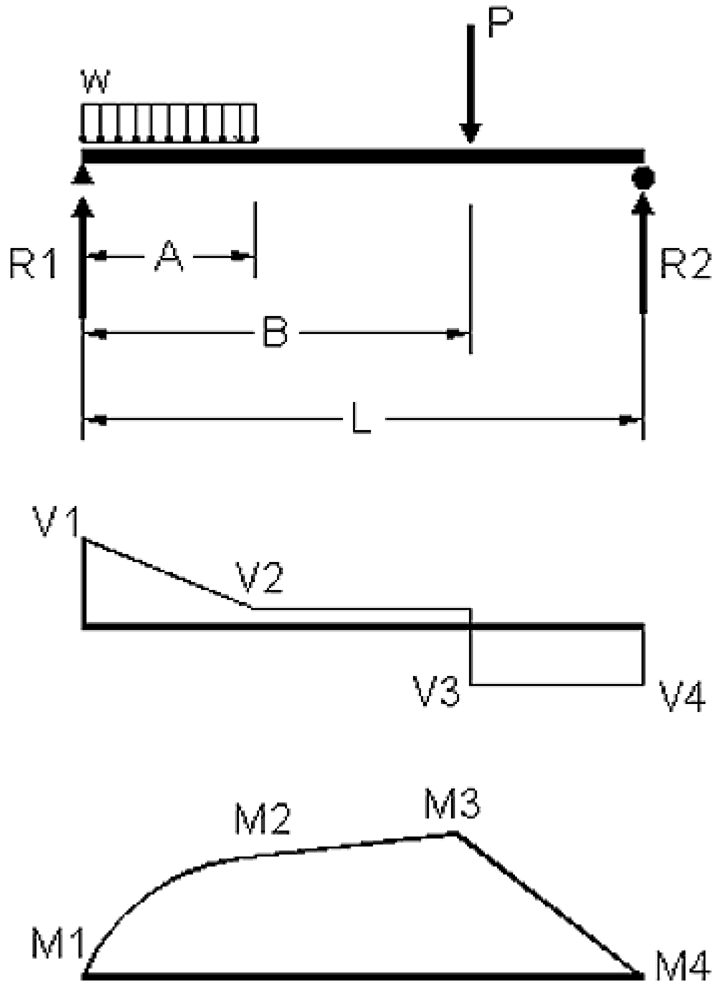
Recap of the week

4 . Semi-graphical Method - Superposition



Provide the solution for the assignment – **HW12**

- Problem:



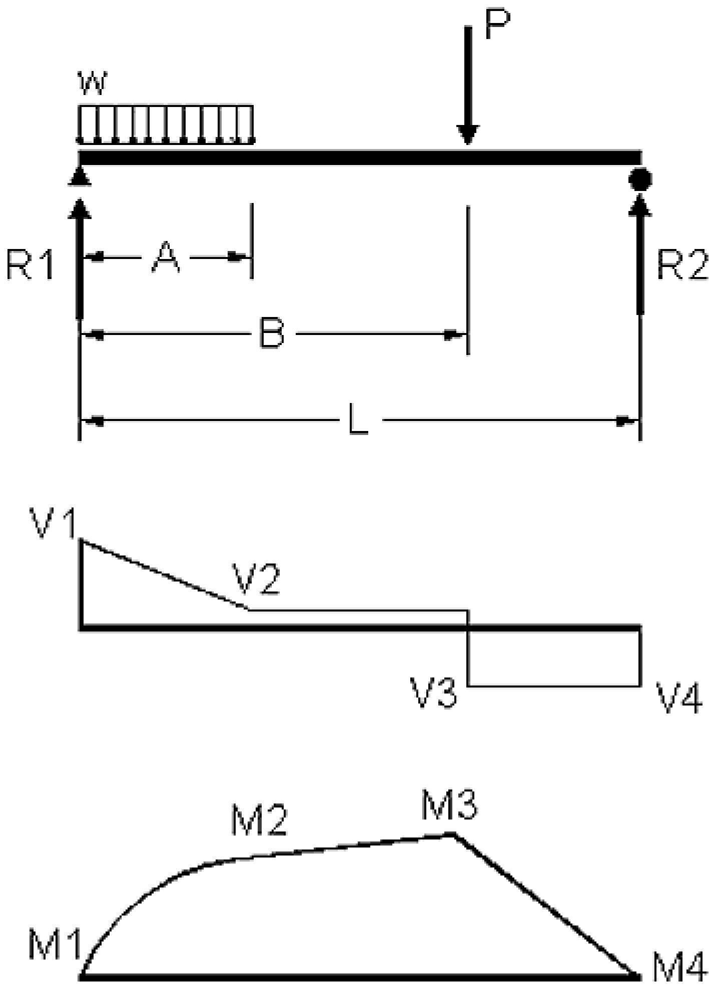
14. Shear and Moment Diagrams

Calculate end reactions and construct the shear & moment diagrams for the loading shown.

DATASET: 1 -2- -3-

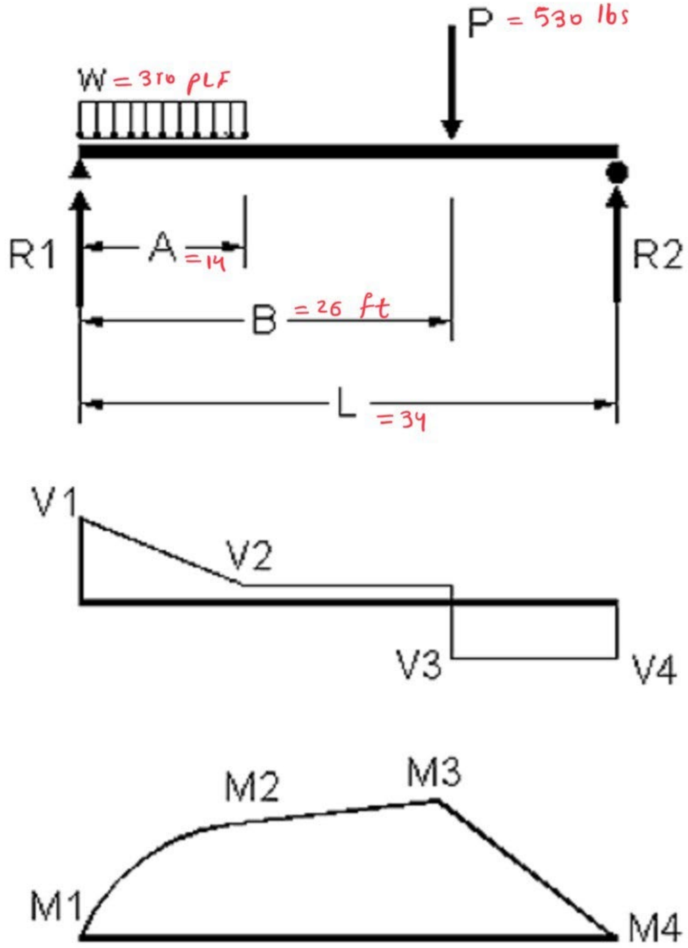
Total Span L	34 FT
Length A	14 FT
Length B	26 FT
Uniform Load on Length A (w)	310 PLF
Point Load (P)	530 LBS

Provide the solution for the assignment – **HW12**

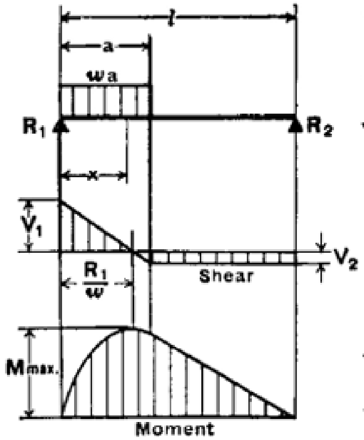


#	Question	Your Response	
1	Left Reaction (R1) (+ is upward; - is downward)		LBS
2	Right Reaction (R2) (+ is upward; - is downward)		LBS
3	Peak Shear value at R1 (V1) (use + or - sign)		LBS
4	Moment value at R1 (M1)		FT-LBS
5	Shear value at A distance from R1 (V2) (use + or - sign)		LBS
6	Moment value at A dist. from R1 (M2 tension on bottom is +)		FT-LBS
7	Peak Shear value at B distance from R1 (V3) (use + or - sign)		LBS
8	Moment value at B dist. from R1 (M3 tension on bottom is +)		FT-LBS
9	Peak Shear value at R2 (V4) (use + or - sign)		LBS
10	Moment value at R2 (M4)		FT-LBS
11	Maximum Moment (tension on bottom is +)		FT-LBS
12	Distance from Left to Max. Moment in (decimal)		FT

Provide the solution for the assignment – **HW12**

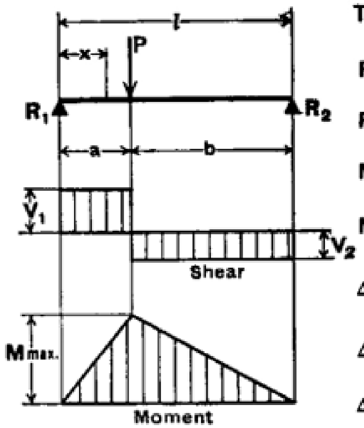


5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



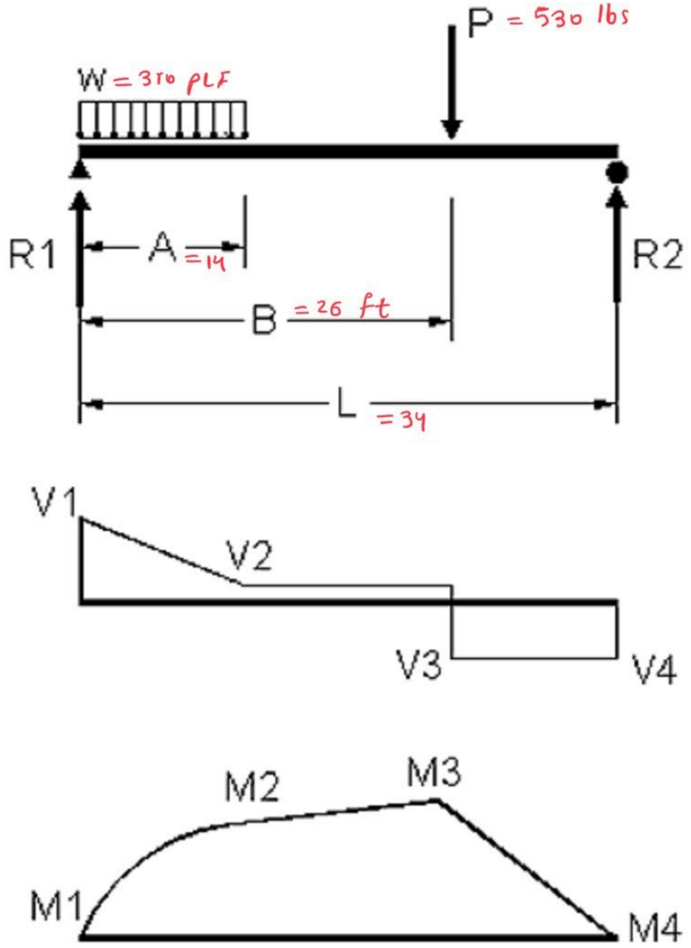
$R_1 = V_1$ max.		$= \frac{wa}{2l}(2l - a)$
$R_2 = V_2$		$= \frac{wa^2}{2l}$
V_x	(when $x < a$)	$= R_1 - wx$
M max.	(at $x = \frac{R_1}{w}$)	$= \frac{R_1^2}{2w}$
M_x	(when $x < a$)	$= R_1 x - \frac{wx^2}{2}$
M_x	(when $x > a$)	$= R_2 (l - x)$
Δx	(when $x < a$)	$= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$
Δx	(when $x > a$)	$= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load		$= \frac{8Pab}{l^2}$
$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb}{l}$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa}{l}$
M max.	(at point of load)	$= \frac{Pab}{l}$
M_x	(when $x < a$)	$= \frac{Pbx}{l}$
Δmax.	(at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)	$= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
Δa	(at point of load)	$= \frac{Pa^2b^2}{3EIl}$
Δx	(when $x < a$)	$= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

Provide the solution for the assignment – **HW12**



uniform load:

$$R_{1u} = \frac{wa}{2l}(2l - a) = \frac{(310)(14)}{2(34)}(2(34) - 14) = 3446.47 \text{ lbs}$$

Concentrated load:

$$R_{1p} = \frac{pb}{l} = \frac{(530)(34-26)}{34} = 124.70$$

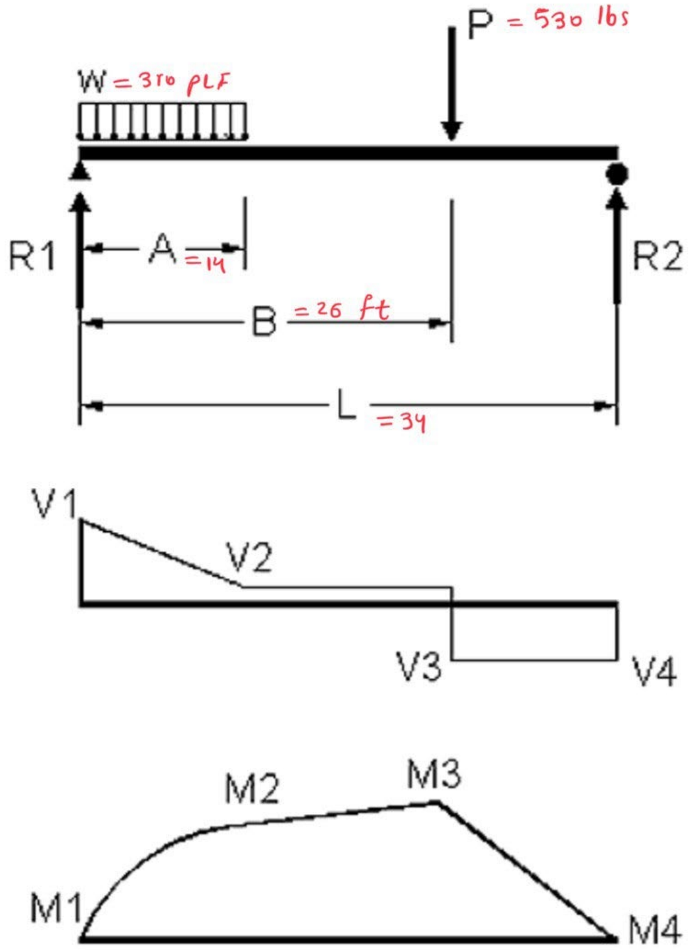
$$R_1 = R_{1u} + R_{1p} = 3446.47 + 124.70 = \boxed{3571.17} \text{ lbs}$$

$$R_{2u} = \frac{wa^2}{2L} = \frac{(310)(14)^2}{2(34)} = 893.53$$

$$R_{2p} = \frac{pa}{l} = \frac{(530)(26)}{34} = 405.29$$

$$R_2 = R_{2u} + R_{2p} = 893.53 + 405.29 = \boxed{1298.82} \text{ lbs}$$

Provide the solution for the assignment – **HW12**



peak shear value at R_1 (V_1)

uniform load:

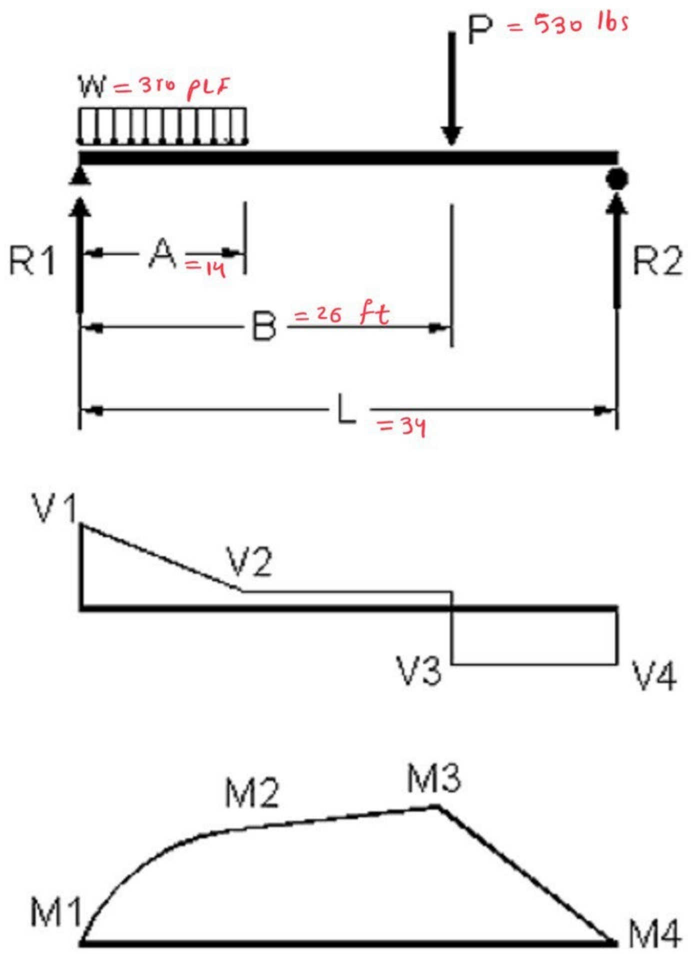
$V_x = R_1 - wx$

V_1 : $x = 0 \longrightarrow V_1 = R_1 = 3571.17$

shear value at the distance $A = 14$: (V_2)

$V_x = R_1 - wx = 3571.17 - (310)(14) = \boxed{-768.83}$ lbs

Provide the solution for the assignment – **HW12**



Moment value at A = 14:

uniform load:

M_u (when $x > a$) $= R_{2u}(l - x) = 893.53\,(34 - 14)$

$= 17870.6$ ft-lbs

or

M_u (when $x < a$) $= R_{1u}x - \dfrac{wx^2}{2} = 3446.47\,(14) - \dfrac{310\,(14)^2}{2}$

$= 17870.58$

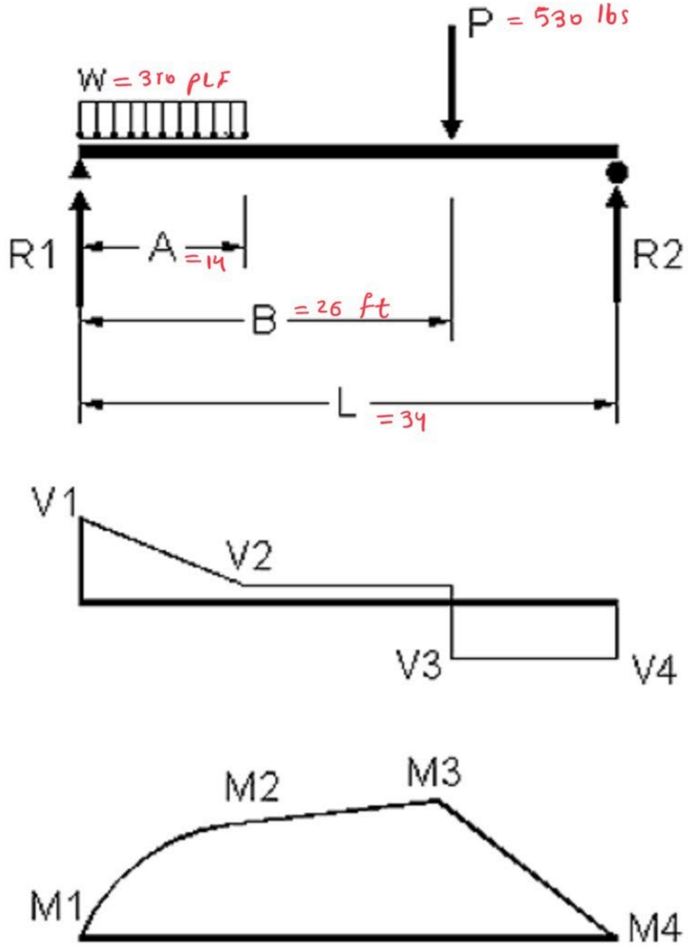
Concentrated load:

M_u (when $x < a$) $= \dfrac{Pbx}{l} = \dfrac{(530)(34-26)(14)}{34} = 1745.88$ ft-lbs

At A = 14 $\quad M_u = M_{uu} + M_{up} = 17870.6 + 1745.88$

$= \boxed{19616.48}$ ft-lbs

Provide the solution for the assignment – **HW12**



At $x = 26$: Shear

point load:

$$V_u = V_2 = -\frac{wa^2}{2L} = -\frac{310\,(14)^2}{2(34)} = -893.53 \text{ lbs}$$

$$V_p = -\frac{Pa}{L} = -\frac{(580)(26)}{34} = -443.529 \text{ lbs}$$

$$V_3 = V_u + V_p = -893.53 - 443.53 = \boxed{-1337.0} \text{ lbs}$$

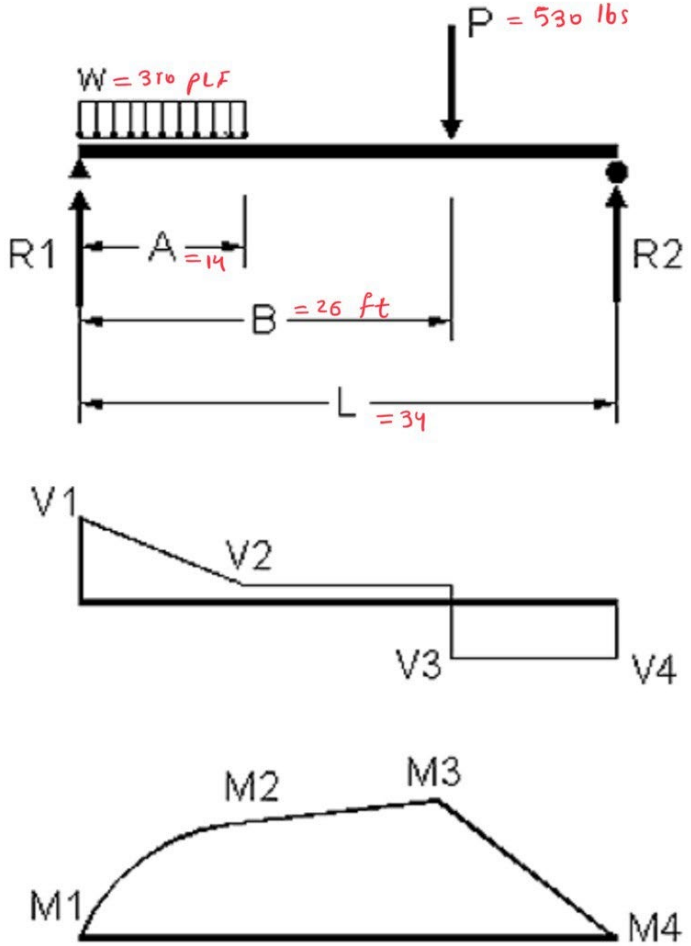
There is another way!

Note :

$$V_3 = V_2 - P$$

$$V_3 = -768.83 - 530 = -1298.83$$

Provide the solution for the assignment – **HW12**



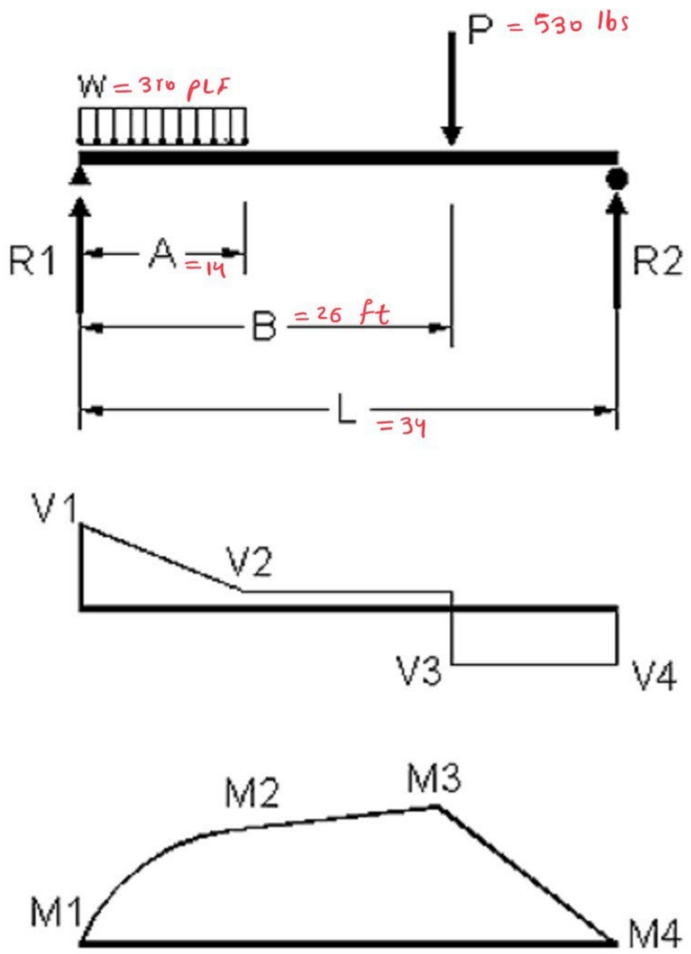
At $x = 26$: moment

M_u (when $x > a$) $= R_{2u}(L - x) = 893.53(34 - 26)$
$= 7148.29$ ft-lbs

M_p (at point of load) $= \dfrac{Pab}{L} = \dfrac{(530)(26)(34-26)}{34}$
$= 3242.35$ ft-lbs

$M_3 = M_u + M_p = 7148.29 + 3242.35 = 10390.59$ ft-lbs

Provide the solution for the assignment – **HW12**



peak shear value at $R_2 (V_4)$:

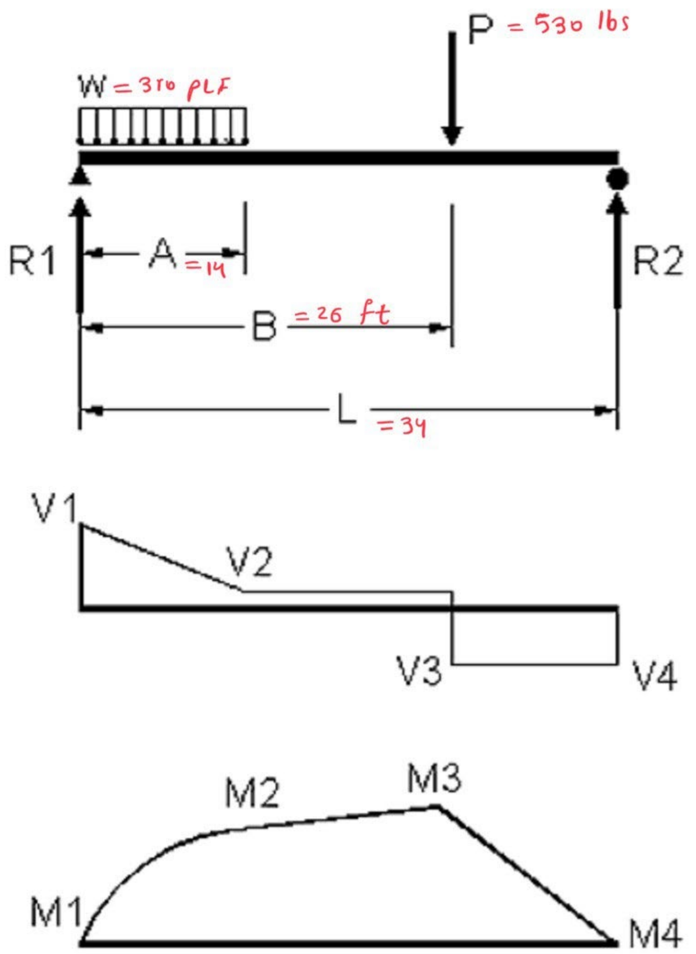
$= -R_2 = -1298.82$

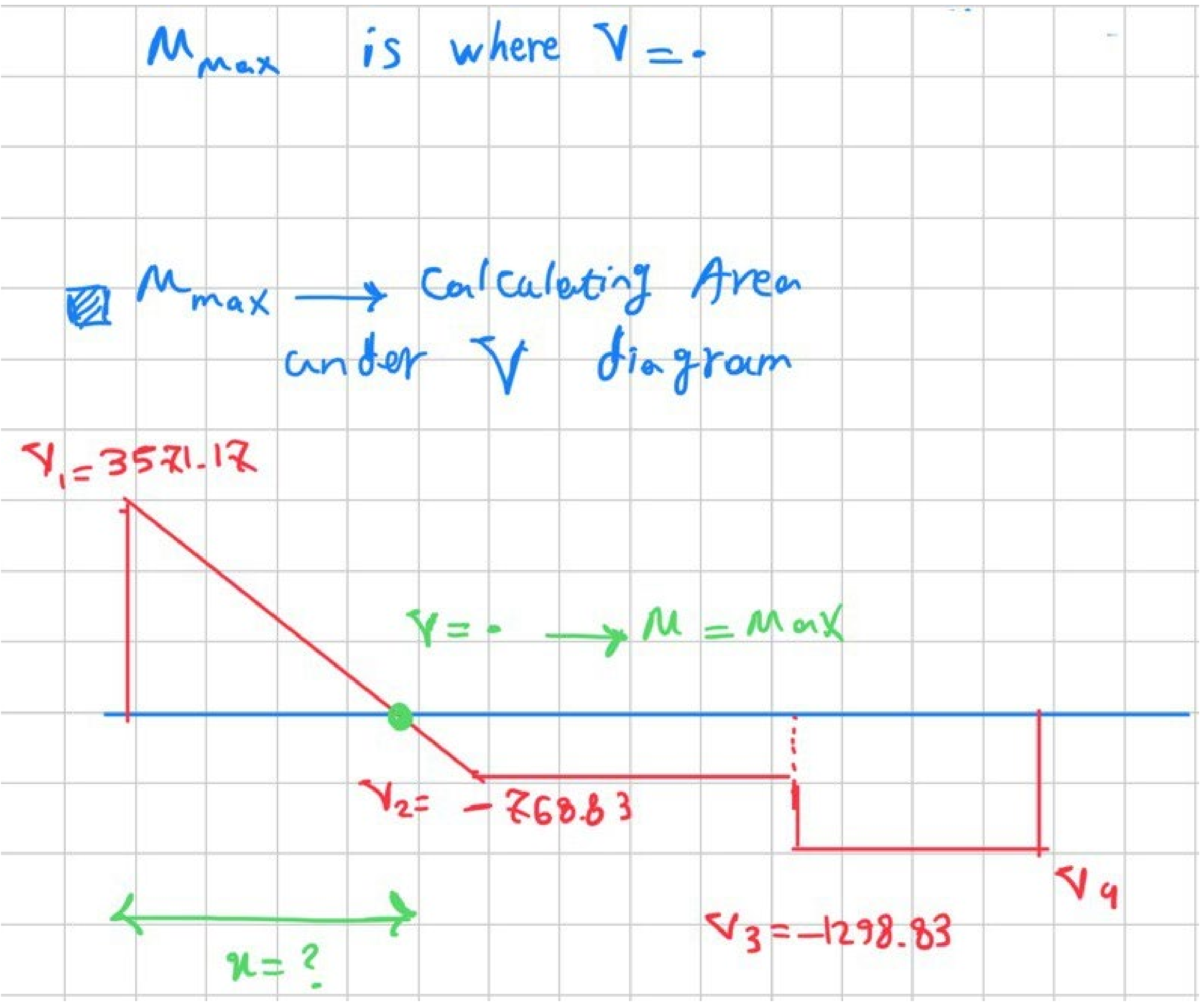
moment value at $R_2 (M_4)$: $x = L = 34$

$M_u = R_2 (L - x) = .$ $\longrightarrow$ $M = .$

$M_p = .$

Provide the solution for the assignment – **HW12**





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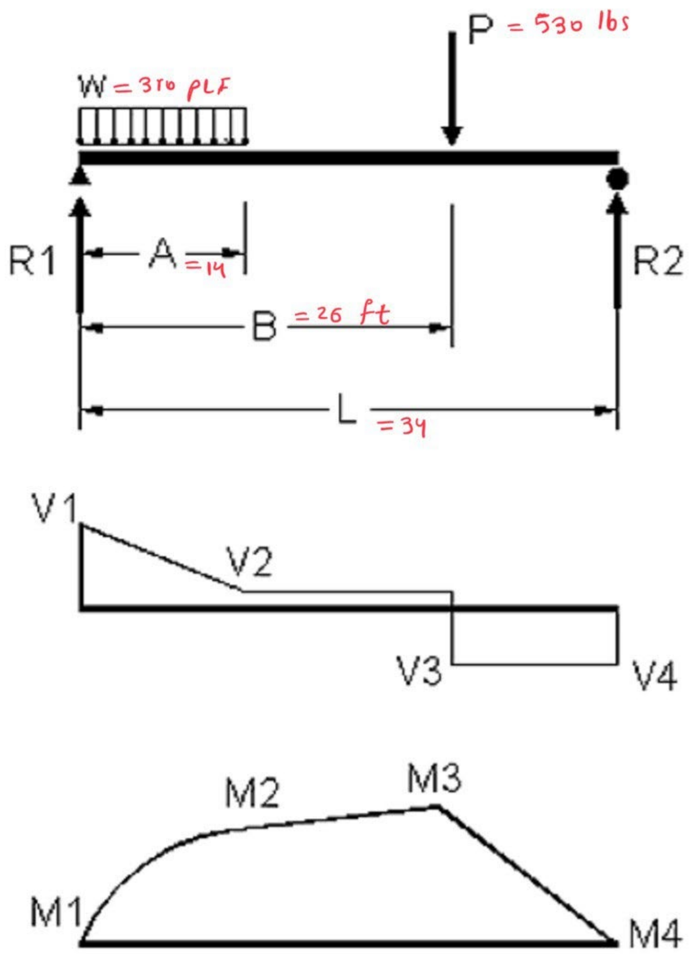


derivate moment formula and equal to zero

$x < a$

$$M = R_1 x - \frac{w x^2}{2} + \frac{P b x}{l}$$

$$M = 3446.47\,x - \frac{310(x)^2}{2} + \frac{530(34-26)\,x}{34}$$

$$M = 3446.47\,x - \frac{310\,x^2}{2} + 124.70\,x$$

$$\frac{dM}{dx} = 3446.47 - 310\,x + 124.70$$

$$\rightarrow \boxed{x = 11.52}\ \text{ft}$$

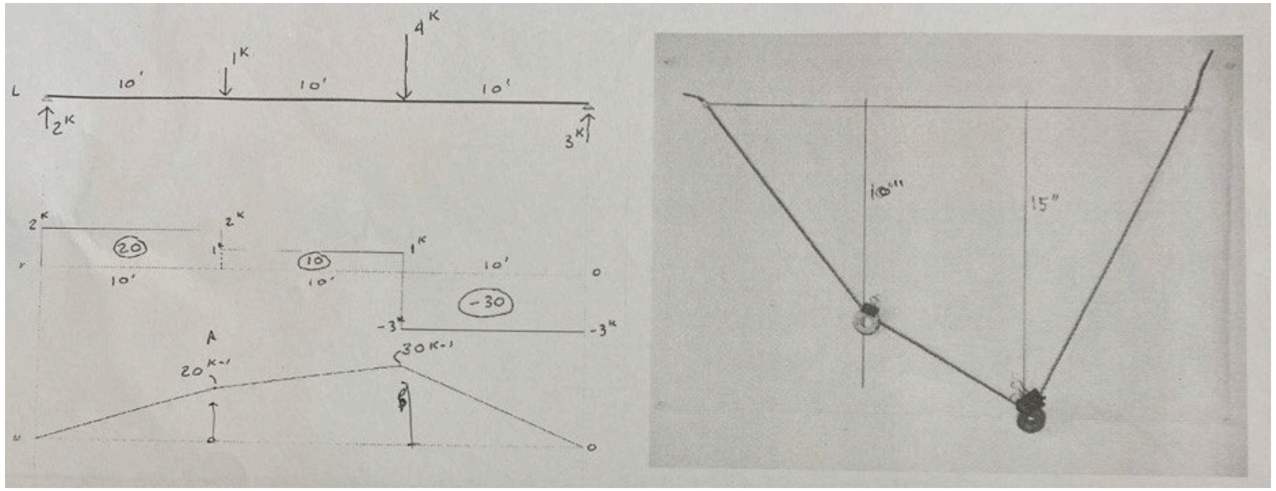
M at $x = 11.52 \longrightarrow$

$$M_{max} = 3446.47(11.52) - \frac{310(11.52)^2}{2} + 124.70(11.52)$$

$$= 39703.33 - 20570.112 + 1436.544$$

$$= \boxed{20569.762}\ \text{ft.lbs}$$

Lab: Moment Diagram



Description

This project compares a calculated moment diagram with one constructed with a weighted string.

Goals

To measure the distances to the weights on a catenary model.
To compare the scaled height of the string model with the calculate heights on the diagram.

Procedure

1. Check the shear and moment diagrams for the beam below.
2. Place the same load on the string and measure the deflection at the weights.
3. Compare the string model with the moment diagram and determine the scale factors.

Bridge Project_ Final Report

- Pay Particular attention to the Tally sheet and requirements. (We Don't want you to lose points!)
- Look at the examples provided in the course website.
- It is not finished yet! Prepare your report Properly.

PRELIMINARY REPORT (re-submit original)	40
TESTING	60
FINAL REPORT REQUIREMENTS	150

Thank you.

Any question?

Please feel free to ask questions.

Contact: Mohsenv@umich.edu