Arch 314- Structures I

Recitation 006

Vishakha Bagarao

8th Nov 2024

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Contents

- Bridge Report
- Quick Recap
    - Moment of Inertia
        - Problem Set 11- Moment of Inertia
- Lab 09- Moment of Inertia

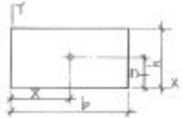
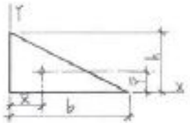
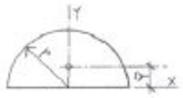
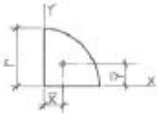
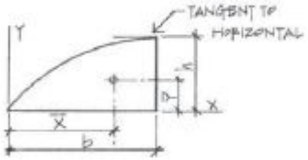
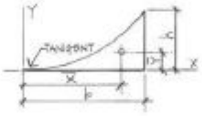
Final Bridge Report Rubrics

- Due- 25th Nov

FINAL REPORT REQUIREMENTS	150	
Preliminary Design Development	**20**	
How initial (preliminary) bridge design was developed	4	
How initial (preliminary) member sizes were chosen	4	
Why bridge design was or was not adjusted from preliminary design	4	
Why member sizes were or were not adjusted from preliminary design	4	
Discussion of how pre-analysis of initial bridge impacted the final design	4	
Revised Bridge Design Analysis	**50**	
Internal axial force calculations/modeling (with proper design loading indicated) (Dr. Frame acceptable)	10	
Derivation of member cross-sectional areas from axial forces	10	
Member size selection from available stock	4	
Est. weight calculation of bridge - including members, glue & fasteners	6	
Method of joints/sections calculation for at least 1 joint (@ reaction is usually easiest based on truss geometry, but could be done elsewhere)	10	
Member crushing calculations/check (show work) using $F'_c=P/A$	4	
Prediction of capacity of bridge and mode of failure	6	
Illustration of Tested (Revised from Preliminary) Design	**20**	
Cross-section of bridge	4	
Elevation(s) of bridge	4	
Dimensions and units labeled in elevation and cross-section	4	
Member sizes labeled (with dimensions)	4	
Member stresses labeled (with units)	4	
Testing Results	**30**	
Weight and height of bridge	5	
Capacity of bridge	5	
Observations of testing	6	
Description of mode of failure	5	
Images of failure	5	
Following the guidelines	4	
Post-Testing Analysis	**30**	
Comparison of testing with predicted capacity and modes of failure	10	
Discussion of discrepancies between results	10	
Suggested improvements for future designs with reasoning discussed	10	
FINAL GRADE	**250**	

Centroid

- Center of gravity (C.G.), or center of mass, refers to masses or weights idealized as a single point at which the weight could be held and be in balance in all directions.
- Centroid refers to the geometric center of lines, areas, and volumes. Beam and column design utilizes the centroid of cross-sectional areas for computing other section properties.
- Composite areas are made up of simpler areas.
  $A = \Sigma \Delta A$

Shape	Drawing	$\bar{x}$	$\bar{y}$	Area
Rectangle		$b/2$	$h/2$	bh
Triangle		$b/3$	$h/3$	$bh/2$
Semicircle		0	$4r/3\pi$	$\pi r^2/2$
Quarter Circle		$4r/3\pi$	$4r/3\pi$	$\pi r^2/4$
Parabolic Segment		$5b/8$	$2h/5$	$2bh/3$
Complement of a Parabolic Segment		$3b/4$	$3h/10$	$bh/3$

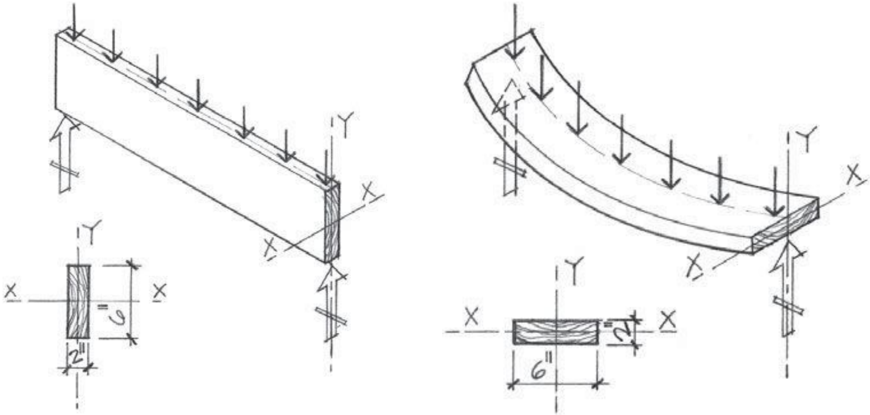
Moment of Inertia of an area

- The moment of area about an axis equals the algebraic sum of the moments of its component areas about the same axis.

$$\bar{x} = \frac{\Sigma x \Delta A}{A}; \qquad \bar{y} = \frac{\Sigma y \Delta A}{A}$$

- The moment of an area is defined as the product of the area multiplied by the perpendicular distance from the moment axis to the centroid of the area.

- The moment of inertia is a measure of cross sectional stiffness.



Moment of Inertia of an area

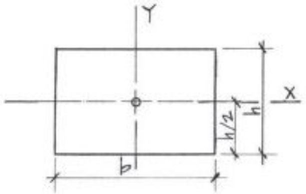
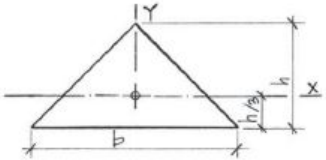
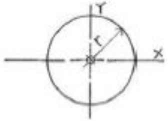
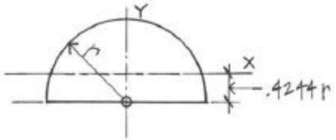
- Moment of inertia (I) is a factor that quantifies the effect of the beam or column's cross sectional shape in resisting bending, deflection, and buckling.

$$I_x = \Sigma y^2 \Delta A; \quad I_y = \Sigma x^2 \Delta A;$$

- When cross-sectional shapes are not simple geometric shapes but a composite of two or more simple shapes combined, the parallel axis theorem is used for determining the moment of inertia about any axis parallel to the centroidal axis

$$I_x = \Sigma I_{x_c} + \Sigma A d_y^2;$$

$$I_y = \Sigma I_{y_c} + \Sigma A d_x^2$$

Shape	Moment of Inertia (I_x)
	$I_x = \frac{bh^3}{12}$
	$I_x = \frac{bh^3}{36}$
	$I_x = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$
	$I_x = \frac{\pi(D^4 - d^4)}{64}$
	$I_x = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = 0.11r^4$

Moment of Inertia:

- **Q1-3.** Moment of Inertia of shape about its own centroid: Ixx

$$Ixx = \frac{1}{12} \times b \times h^3$$

- **Q4.** Summation of x-x Moments of Inertia of all shapes :SUM Ixx

- **Q5-7.** Distance from the centroid of particular shape to the centroid of the whole shape

$$d_{c1} = |d_1 - y|$$

- **Q8-10.** 2nd Moment of area 1 about centroid of whole shape x dist. to centroid: I2

$$I_2 = A \times dc1^2$$

Moment of Inertia:

- **Q11.** Summation of moments of areas times distances to centroid: SUM I2

- **Q12.** Moment of Inertia about the x-x axis for the whole shape: Ix

$$Ix = \Sigma Ixx + \Sigma I_2$$

- **Q13-15.** y-y Moment of Inertia of shape 1 about its own centroid: Iyy

$$Iyy = \frac{1}{12} \times b \times h^3$$

- **Q16.** Summation of y-y Moments of Inertia of all shapes: SUM Iyy

- **Q17.** Moment of Inertia about the y-y axis for the whole shape: Iy

Since, the Centroid of all the shapes is on one line, therefore d=0; $Ad^2=0$.
Therefore, no need of Parallel Axis Theorem. Iy= SUM Iyy + 0 = SUM Iyy

Problem Set 11

MICHIGAN Architecture

Structures Problems

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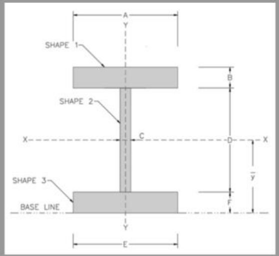
Logged in as: Vishakha Bagarao

11. Moment of Inertia

Use the Parallel Axis Theorem to find the moments of inertia about both the x-x and y-y axes of the compound section.



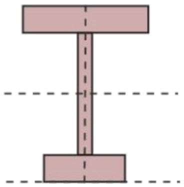
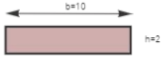
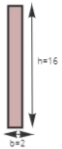
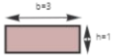
DATASET: 1 -2- -3-

Dimension A	10 IN
Dimension B	2 IN
Dimension C	2 IN
Dimension D	16 IN
Dimension E	3 IN
Dimension F	1 IN

#	Question	Your Response	Correct Answer	Score
1	x-x Moment of Inertia of shape 1 about its own centroid (Ixx)	6.6667 IN^4	6.66667 IN^4	5
2	x-x Moment of Inertia of shape 2 about its own centroid (Ixx)	682.667 IN^4	682.667 IN^4	5
3	x-x Moment of Inertia of shape 3 about its own centroid (Ixx)	0.25 IN^4	0.25 IN^4	5
4	Summation of x-x Moments of Inertia of all shapes (SUM Ixx)	689.583 IN^4	689.583 IN^4	5
5	Distance from the centroid of shape 1 to the centroid of the whole shape	6.1909 IN	6.19091 IN	5
6	Distance from the centroid of shape 2 to the centroid of the whole shape	2.80909 IN	2.80909 IN	5
7	Distance from the centroid of shape 3 to the centroid of the whole shape	11.3091 IN	11.3091 IN	5
8	2nd Moment of area 1 about centroid of whole shape x dist. to centroid (Ad^2)	766.547 IN^4	766.547 IN^4	5
9	2nd Moment of area 2 about centroid of whole shape x dist. to centroid (Ad^2)	252.512 IN^4	252.512 IN^4	5
10	2nd Moment of area 3 about centroid of whole shape x dist. to centroid (Ad^2)	383.687 IN^4	383.687 IN^4	5
11	Summation of moments of areas times distances to centroid (SUM Ad^2)	1402.75 IN^4	1402.75 IN^4	5
12	Moment of Inertia about the x-x axis for the whole shape (Ix)	2092.33 IN^4	2092.33 IN^4	5
13	y-y Moment of Inertia of shape 1 about its own centroid (Iyy)	166.667 IN^4	166.667 IN^4	5
14	y-y Moment of Inertia of shape 2 about its own centroid (Iyy)	10.6667 IN^4	10.6667 IN^4	5
15	y-y Moment of Inertia of shape 3 about its own centroid (Iyy)	2.25 IN^4	2.25 IN^4	5
16	Summation of y-y Moments of Inertia of all shapes (SUM Iyy)	179.583 IN^4	179.583 IN^4	5
17	Moment of Inertia about the y-y axis for the whole shape (Iy)	179.583 IN^4	179.583 IN^4	5

Current Score: 85 / 85

Procedure: -Create a table with basic Data

	**Area (A=b x h)**	**Distance (d)**	**Area x Distance (Ad)**	
Shape 1  b=10 h=2	10 x 2 **= 20**	B/2 +D+F **= 18**	Ad1= A x d =20 x 18 **=360**	$y = \frac{\Sigma Ad}{\Sigma A}$
Shape 2  h=16 b=2	2 x 16 **= 32**	D/2 +F **= 9**	Ad2= A x d =32 x 9 **=288**	**y= 649.5 / 55** **y= 11.80909**
Shape 3  b=3 h=1	3 x 1 **= 3**	F/2 **= 0.5**	Ad3= A x d =3 x 0.5 **=1.5**	
	∑A=55	Note: Refer Problem Set for dimensions A,B,C,D,E, and F.	**∑Ad=649.5**	

Problem Set 11

Michigan Architecture — Structures Problems

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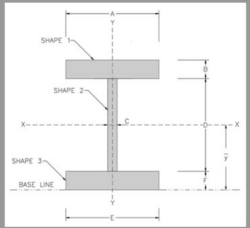

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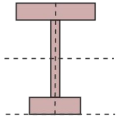
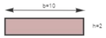
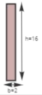
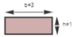
Current Score: 85 / 85

Problem Menu

Logout

Procedure:

	Area (A=b x h)	Distance (d)	Area x Distance (Ad)		$Ixx = \frac{1}{12} \times b \times h^3$	dc= |d-y|	$I_2 = A \times dc1^2$	Parallel Axis Theorem
Shape 1 	10 x 2 = **20**	B/2 +D+F = **18**	Ad1= A x d =20 x 18 **=360**	$y = \frac{\Sigma Ad}{\Sigma A}$	Q1 $Ixx = \frac{10 \times 2^3}{12}$ **=6.6667**	Q5 dc=|18-11.809| **=6.1909**	Q8 I2= 20 x 6.1909^2 **=766.5471**	
Shape 2 	2 x 16 = **32**	D/2 +F = **9**	Ad2= A x d =32 x 9 **=288**	**y= 649.5 / 55** **y= 11.80909**	Q2 $Ixx = \frac{2 \times 16^3}{12}$ **=682.6667**	Q6 dc=|9-11.809| **=2.8090**	Q9 I2= 32 x 2.8090^2 **=252.51173**	Q12 **IX= ∑XX + ∑I2** =689.5833+1402.7454 **=2092.33**
Shape 3 	3 x 1 = **3**	F/2 = **0.5**	Ad3= A x d =3 x 0.5 **=1.5**		Q3 $Ixx = \frac{3 \times 1^3}{12}$ **=0.25**	Q7 dc=|0.5-11.809| **=11.3090**	Q10 I2= 3 x 11.3090^2 **=383.6866**	
	∑A=55	Note: Refer Problem Set for dimensions A,B,C,D,E, and F.	**∑Ad=649.5**		Q4 **∑Ixx=689.5833**		Q11 **∑I2=1402.7454**	

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Problem Set 11

MICHIGAN Architecture

Structures Problems

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11. Moment of Inertia

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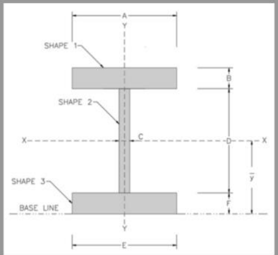

DATASET: 1 -2- -3-

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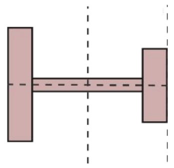
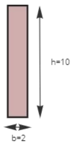
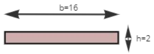
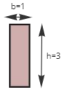
Current Score: 85 / 85

Procedure:

	Area (A=b x h)	Distance (d)	Area x Distance (Ad)	$Iyy = \frac{1}{12} \times b \times h^3$	Parallel Axis Theorem
Shape 1 	10 x 2= 20	0	0	Q13 $Iyy = \frac{2 \times 10^3}{12}$ **= 166.6667**	Since, the Centroid of all the shapes is on one line, d=0; Ad2=0. Therefore, Iy= SUM Iyy
Shape 2 	2 x 16 = 32	0	0	Q14 $Iyy = \frac{16 \times 2^3}{12}$ **=10.6667**	Q17 **Iy= ∑yy + ∑I2** **Iy= ∑yy + 0** **Iy= ∑yy** **Iy=179.5833**
Shape 3 	3 x 1 = 3	0	0	Q15 $Iyy = \frac{1 \times 3^3}{12}$ **= 2.25**	
	∑Areas=55		**∑Ad=0**	Q16 **∑Iyy=179.5833**	

Lab 09 Moment of Inertia

Structures I — Name 1 ____________________

Arch 314 — Name 2 ____________________

Name 3 ____________________

Moment of Inertia

Description

This project uses observation and calculation to investigate the moment of inertia.

Goals

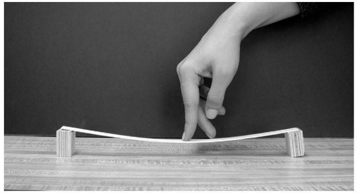
To observe the strong and weak axis moment of inertia through physical testing.
To calculate the strong and weak axis moment of inertia for a given section.
To compare the physical sense with the calculated values.

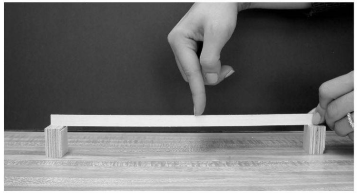
Procedure

1. Span the given 1/16" x 1/2" basswood stick flatwise between two supports.
2. Load the 'beam' at mid span with your finger to cause about 1/2" deflection.
3. Now rotate the stick 90° so that it is on the narrow edge, and hold it in place.
4. Again with your finger apply about the same load as before, and notice how much stiffer the beam has become.
5. Now calculate the moment of inertia for both orientations – flatwise and on edge.
6. Compare the two numbers and observe how they relate to the actual stiffness you felt with your finger.

Moment of Inertia
of a rectangle

$$I = \frac{bd^3}{12}$$





Due

During recitation