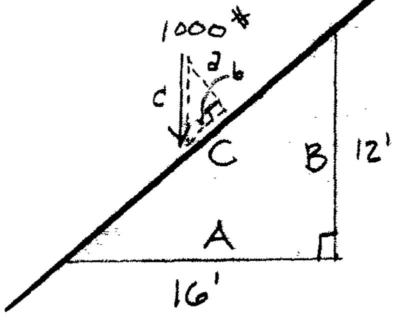
ARCH 331 | 2-1 (2, 3) | AUG 26, 1991 PvB

2-1 (2)

DETERMINE THE MAGNITUDE OF THE COMPONENTS PARALLEL AND PERPENDICULAR TO THE SLOPE



SLOPE TRIANGLE :

$C = \sqrt{A^2 + B^2}$

$= \sqrt{16^2 + 12^2} = 20$

PARALLEL :

$\frac{c}{C} : \frac{b}{B} \qquad \frac{1000}{20} : \frac{b}{12}$

$b = \underline{600\#}$

CHECK

$\sqrt{800^2 + 600^2} = 1000$ ✓

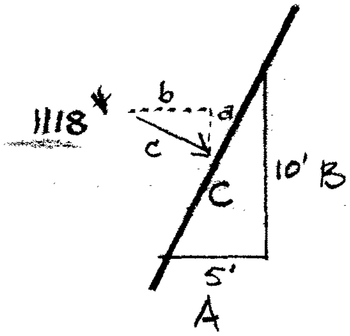
PERP. :

$\frac{c}{C} : \frac{a}{A} \qquad \frac{1000}{20} : \frac{a}{16}$

$a = \underline{800\#}$

2-1 (3)

DETERMINE THE MAGNITUDE OF THE HORIZONTAL AND VERTICAL COMPONENTS



SLOPE TRIANGLE

$C = \sqrt{A^2 + B^2}$

$= \sqrt{5^2 + 10^2}$

$= 11.18$

$\sqrt{500^2 + 1000^2} = 1118$ ✓

VERTICAL

$\frac{c}{C} : \frac{a}{A} \qquad \frac{1118}{11.18} : \frac{a}{5}$

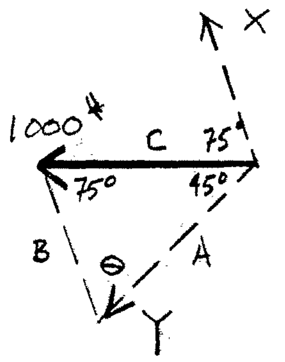
$a = \underline{500\#}$

HORIZONTAL

$\frac{c}{C} : \frac{b}{B} \qquad \frac{1118}{} : \frac{b}{10} \qquad \underline{b = 1000}\#$

ARCH 331 | 2-1(4d, 5c) | 26 AUG 91 | TUB

2-1(4-d)



DETERMINE THE MAGNITUDE OF THE COMPONENTS X AND Y.

$\theta = 180° - 75° - 45° = 60°$

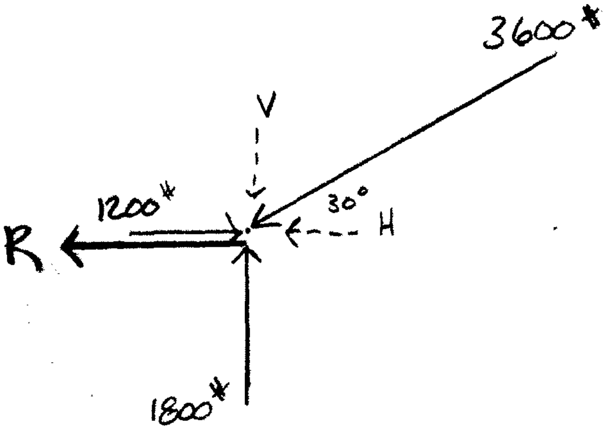
$$\frac{A}{\sin 75°} = \frac{B}{\sin 45°} = \frac{C}{\sin 60°} = \frac{1000}{.8660} = 1154.7$$

$A = \sin 75°(1154.7) = \underline{1115.4\#}$

$B = \sin 45(1154.7) = \underline{816.5\#}$

2-1(5c)

DETERMINE THE MAGNITUDE AND DIRECTION OF THE RESULTANT OF THE GIVEN FORCES



V COMP.

$\sin 30° = \frac{V}{3600}$

$V = 1800\# \downarrow$

H COMP.

$\cos 30° = \frac{H}{3600}$

$H = 3117.7\# \leftarrow$

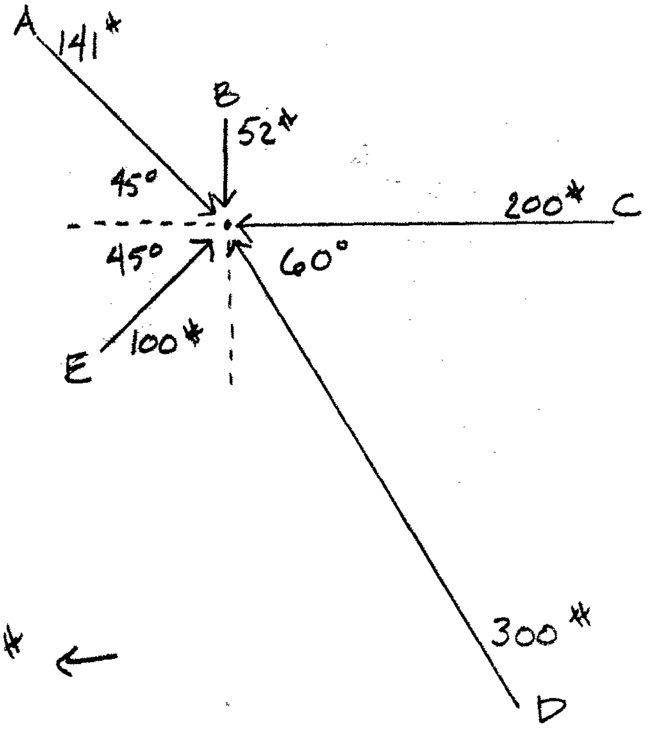
SUM COMPONENTS

$\Sigma V = 1800 - 1800 = 0$

$\Sigma H = 1200 - 3117.7 = \underline{1917.7\#} \leftarrow$ HORIZ

ARCH 331 | 2-2 (c) | 26 AUG 91 PVB

DETERMINE THE MAGNITUDE AND DIRECTION OF THE RESULTANT OF THE CONCURRENT FORCES SHOWN.



COMPONENTS OF EACH FORCE :

$A_H = \sin 45^\circ(141)$

$= 99.7^{\#} \rightarrow$

$A_V = \cos 45^\circ(141)$

$= 99.7^{\#} \downarrow$

$B_V = 52^{\#} \downarrow$ $\qquad B_H = 0$

$C_V = 0$ $\qquad C_H = 200^{\#} \leftarrow$

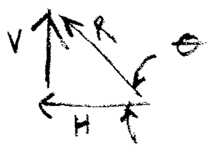
$D_V = \sin 60^\circ(300)$ $\qquad D_H = \cos 60^\circ(300)$

$= 259.8^{\#} \uparrow$ $\qquad = 150^{\#} \leftarrow$

$E_V = \sin 45(100)$ $\qquad E_H = \cos 45(100)$

$= 70.7 \uparrow$ $\qquad = 70.7 \rightarrow$

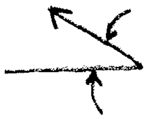
$\text{TOTAL}_V = 178.8 \uparrow$ $\qquad \text{TOTAL}_H = 179.6 \leftarrow$



$\text{RESULTANT} = \sqrt{178.8^2 + 179.6^2} = \underbrace{253.4^{\#}}$

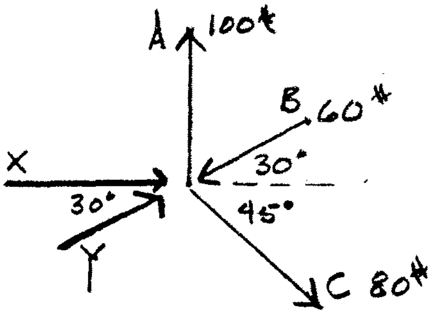
$\tan\theta = \dfrac{178.8}{179.6} = 0.9955$

$\theta = \underbrace{44.87^\circ}$

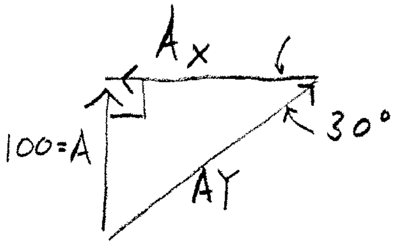


ARCH 331 PROB 2-3 (III) 26 AUG 91 PVB

WHAT FORCES ACTING ALONG LINES X AND Y WILL EQUILIBRATE THE OTHER FORCES?



FORCE A :



$$\frac{A}{\sin a} = \frac{100}{\sin 30} = 200$$

$$200 = \frac{Ax}{\sin 60}$$

$Ax = 173.2$ ←

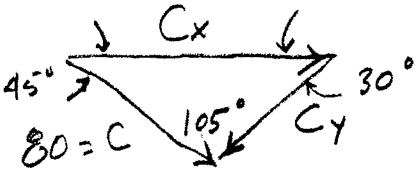
$$200 = \frac{AY}{\sin 90}$$

$AY = 200$ ↗

FORCE B : $By = 60$ ↙

$Bx = 0$

FORCE C :



$$\frac{C}{\sin 30^\circ} = \frac{80}{.5} = 160$$

$$160 = \frac{Cx}{\sin 105^\circ}$$

$Cx = 154.5$ →

$$160 = \frac{CY}{\sin 45}$$

$CY = 113.1$ ↙

RESULTANTS :

$X = 18.7$ ←
$Y = 26.9$ ↗

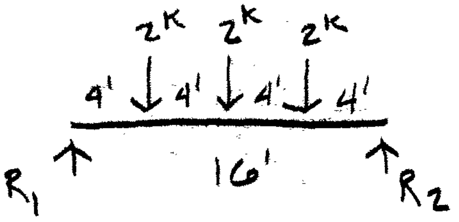
EQUILIBRANTS :

$X = 18.7$ →
$Y = 26.9$ ↙

ARCH 331 | PROB. 3-1(A,B) | 28 AUG 91 RJB

PARALLEL FORCES

DETERMINE THE MAGNITUDE OF THE REACTIONS R1 & R2



$\Sigma M_{R_1} = 0$

$2(4) + 2(8) + 2(12) - R_2(16) = 0$

$R_2(16) = 48$

$R_2 = \underline{3^K} \uparrow$

$\Sigma M_{R_2} = 0$

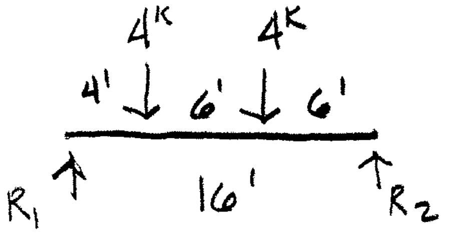
$R_1(16) - 2(12) - 2(8) - 2(2) = 0$

$R_1(16) = 48$

$R_1 = \underline{3^K} \uparrow$

CHECK $\Sigma F_V = 0$

$-2 - 2 - 2 + 3 + 3 = 0$ ✓



$\Sigma M_{R_1} = 0$

$4(4) + 4(10) - R_2(16) = 0$

$R_2(16) = 56$

$R_2 = \underline{3.5^K} \uparrow$

$\Sigma M_{R_2} = 0$

$R_1(16) - 4(12) - 4(6) = 0$

$R_1(16) = 72$

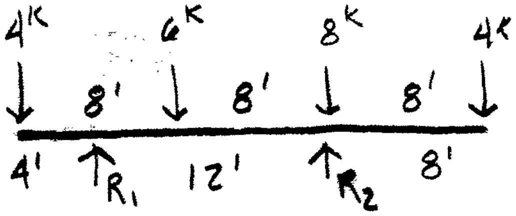
$R_1 = \underline{4.5^K} \uparrow$

CHECK $\Sigma F_V = 0$

$-4 - 4 + 3.5 + 4.5 = 0$ ✓

ARCH 331 | PROB 3-1 (D,E) | 27 Aug 91 PUB

(D)



$\Sigma M_{R_1} = 0$

$$-4(4) + 6(4) + 8(12) + 4(20) - R_2(12) = 0$$

$$R_2(12) = 184^k$$

$$R_2 = \underline{15.33}^k \uparrow$$

$\Sigma M_{R_2} = 0$

$$-4(16) + R_1(12) - 6(8) + 4(8) = 0$$

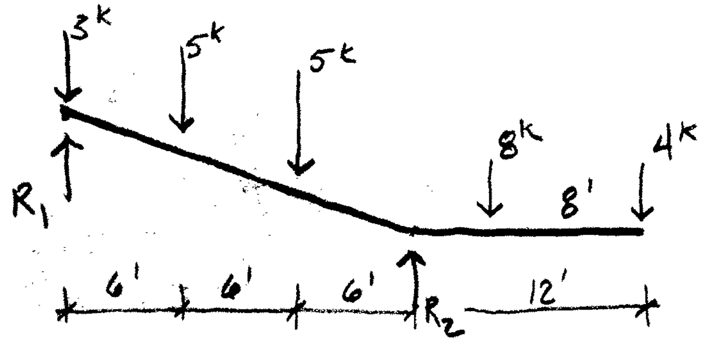
$$R_1(12) = 80$$

$$R_1 = \underline{6.67}^k \uparrow$$

CHECK $\Sigma F_V = 0$

$$-4 - 6 - 8 - 4 + 15.33 + 6.67 = 0 \quad \checkmark$$

(E)



$\Sigma M_{R_1} = 0$

$$5(6) + 5(12) - R_2(18) + 8(22) + 4(30) = 0$$

$$R_2(18) = 386$$

$$R_2 = \underline{21.44} \uparrow$$

$\Sigma M_{R_2} = 0$

$$R_1(18) - 3(18) - 5(12) - 5(6) + 8(4) + 4(12) = 0$$

$$R_1(18) = 64$$

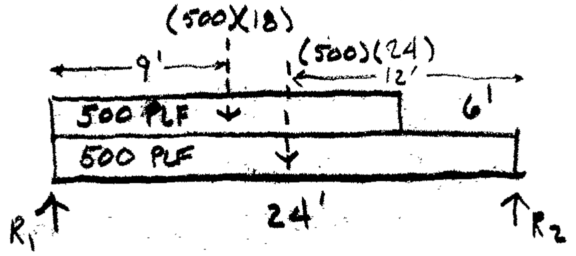
$$R_1 = \underline{3.56}^k \uparrow$$

CHECK $\Sigma F_V = 0$

$$-3 - 5 - 5 - 8 - 4 + 3.56 + 21.44 = 0 \quad \checkmark$$

ARCH 331 | PROB 3-1(M,N) | 27 AUG 91 | PVB

Ⓜ



$\Sigma M_{R_1} = 0$

$(500)(18)(9) + (500)(24)(12) - R_2(24) = 0$

$R_2(24) = 225\,000$

$R_2 = 9375\# \uparrow$

$\Sigma M_{R_2} = 0$

$R_1(24) - (500)(18)(15) - (500)(24)(12) = 0$

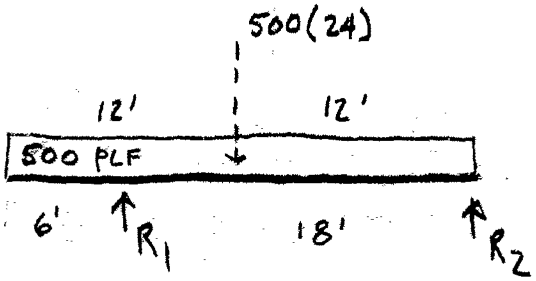
$R_1(24) = 279000$

$R_1 = 11625\# \uparrow$

CHECK $\Sigma F_V = 0$

$-500(18) - 500(24) + 9375 + 11625 = 0$ ✓

Ⓝ



$\Sigma M_{R_1} = 0$

$(500)(24)(6) - R_2(18) = 0$

$R_2(18) = 72000\#$

$R_2 = 4000\# \uparrow$

$\Sigma M_{R_2} = 0$

$R_1(18) - (500)(24)(12) = 0$

$R_1(18) = 144000$

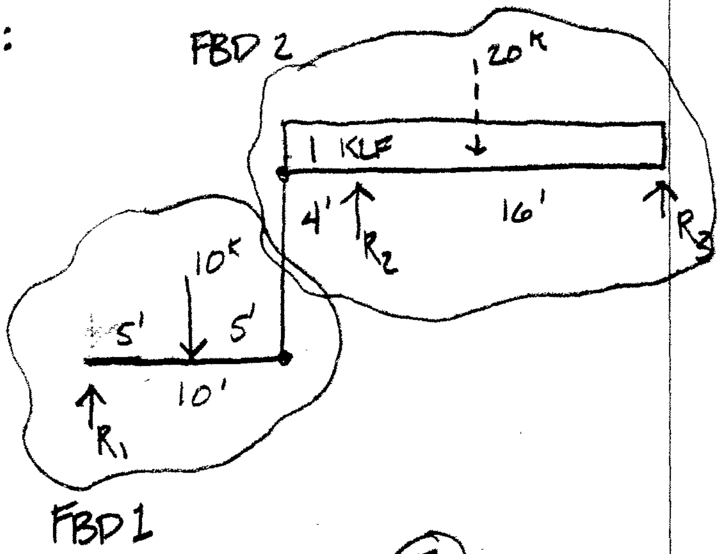
$R_1 = 8000\# \uparrow$

CHECK $\Sigma F_V = 0$

$-500(24) + 4000 + 8000 = 0$ ✓

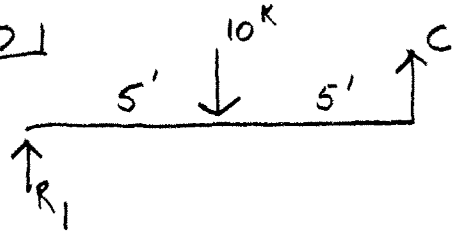
ARCH 331 | PROB 3-2(f) | 5 SEP 91 RVB

DETERMINE THE REACTIONS :



Ⓕ

FBD 1

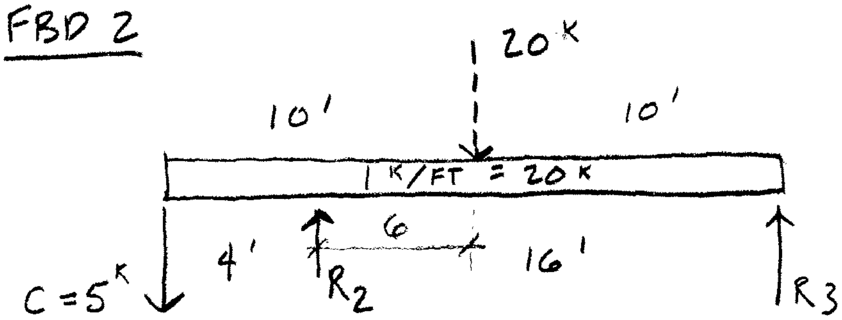


BY SYMETRY :

$\underline{R_1 = 5^K} \uparrow$

$C = 5^K$

FBD 2



$\Sigma M_{R_2} = 0$

$-5(4) + 20(6) - R_3(16) = 0$

$R_3(16) = 100$

$R_3 = \underline{6.25}^K \uparrow$

$\Sigma M_{R_3} = 0$

$-5(20) + R_2(16) - 20(10) = 0$

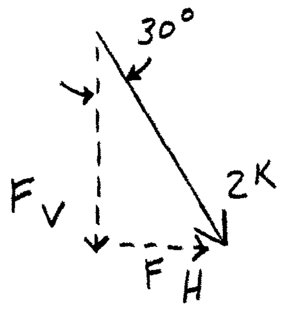
$R_2(16) = 300$

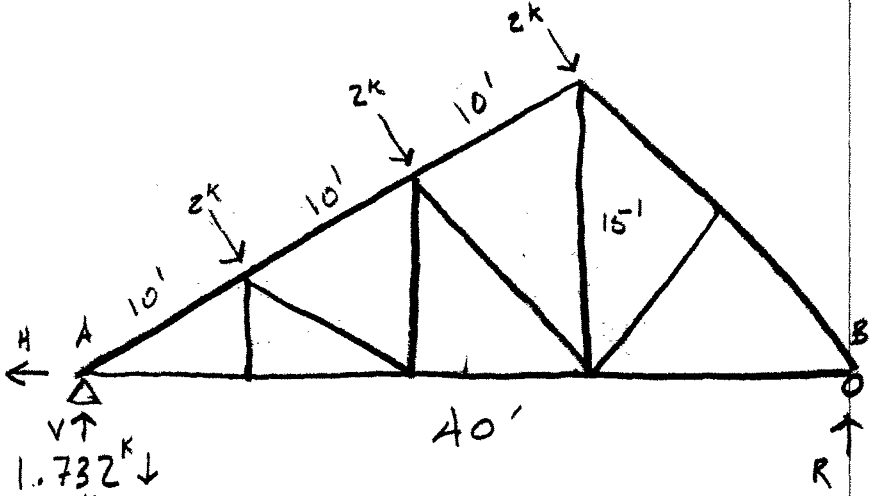
$R_2 = \underline{18.75}^K \uparrow$

CHECK $\Sigma F_V = 0$

$-10 - 1(20) + 5 + 6.25 + 18.75 = 0$ ✓

ARCH 331 | PROB 3-4(6) | 28 AUG 91 PvB





$F_V = \cos 30°(2) = 1.732^k \downarrow$
$F_H = \sin 30°(2) = 1.0^k \rightarrow$

$\Sigma F_H = 0$

$1 + 1 + 1 - H = 0$
$H = \underline{3^k} \leftarrow$

$\Sigma M_A = 0$

$2(10) + 2(20) + 2(30) - R(40) = 0$
$R(40) = 120$
$R = \underline{3^k} \uparrow$

$\Sigma F_V = 0$

$-1.732(3) + 3 + V = 0$
$V = \underline{2.196^k} \uparrow$

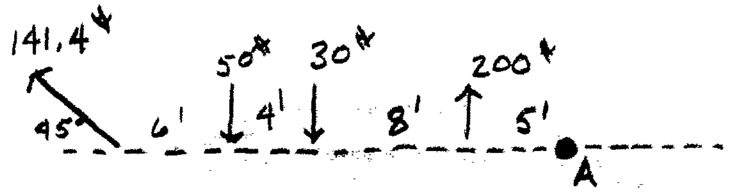
CHECK $\Sigma M_C = 0$

$3(15) - 2(20) - 2(10) - 3(14.02) + 2.196(25.98) = 0.007$
≈ 0 ✓

ARCH 331 | PROB 3-5(d) | 28 AUG 91 | PVB

RESULTANT OF NON-CONCURRENT FORCES

DETERMINE THE MAGNITUDE, DIRECTION AND POSITION OF THE RESULTANT



$$\bar{d} = \frac{\Sigma(F(d))}{\Sigma(F)} = \frac{\overset{(141.4\sin 45)}{100}(23) - 50(17) - 30(13) + 200(5)}{100 - 50 - 30 + 200}$$

$$= \frac{2060}{220}$$

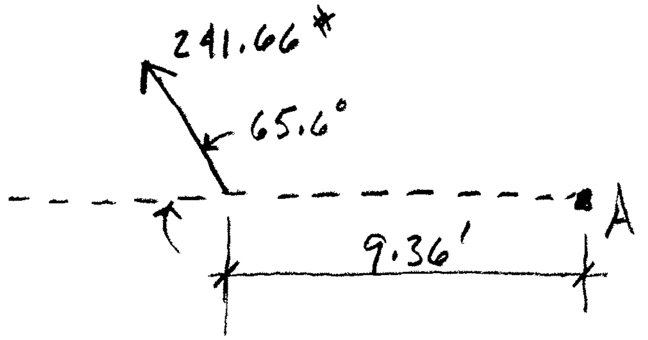
$$\bar{d} = \underline{9.36'} \text{ (TO THE LEFT OF A)}$$

$$\Sigma F_V = 100 - 50 - 30 + 200 = 220^{\#} \uparrow$$

$$\Sigma F_H = 100^{\#} \leftarrow$$

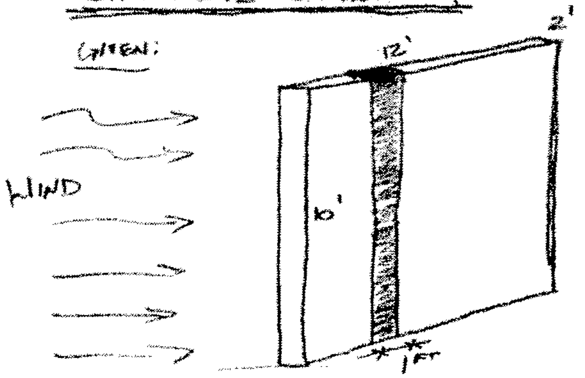
$$\text{RESULTANT} = \sqrt{100^2 + 220^2} = \underline{241.66^{\#}}$$

$$\text{DIRECTION} = \text{ARCTAN } \frac{220}{100} = \underline{65.6^\circ} \text{ FROM HORIZ.}$$



STRUCTURAL STABILITY

GIVEN:



CONCRETE DENSITY = 150 PCF
WIND LOAD = 20 PSF

$$\sum M_{OT} \leq \sum M_R$$

FIND: Ⓐ IS THE WALL STABLE?
Ⓑ UP TO WHAT WIND FORCE CAN THIS WALL WITHSTAND?

* WHEN ANALYZING A WALL, CONSIDER ONLY ONE LINEAR FOOT INSTEAD OF THE ENTIRE WALL (SO, IN THIS CASE DON'T USE 12' IN ANY CALCULATIONS.)

Ⓐ

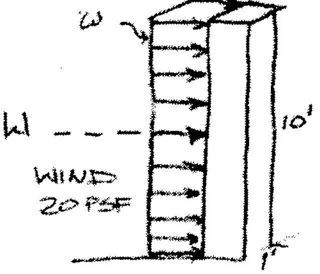
STEP 1: DETERMINE RESISTING FORCES, i.e. WEIGHT OF WALL.

FOR ONE LINEAR FOOT:

$$\text{WEIGHT OF WALL} = C = 150 \text{ PCF} \underbrace{(10')}_{\text{HEIGHT}} \underbrace{(2')}_{\text{THICKNESS OF WALL}} \overset{\text{LENGTH — ONE LINEAR FOOT (NOT 12 FT).}}{(1')}$$

$$C = 3000^{\text{lb}} \text{ FOR ONE LINEAR FOOT } \underset{\text{OR}}{\Longrightarrow} 3 \text{ KLF}$$

STEP 2: DETERMINE OVERTURNING FORCES, i.e. WIND LOAD.



FOR ONE LINEAR FOOT:

$$\text{WIND LOAD} = \omega = 20 \text{ PSF} \overset{\text{LENGTH — ONE LINEAR FOOT}}{(1')} = 20 \text{ PLF FOR ONE LINEAR FOOT}$$

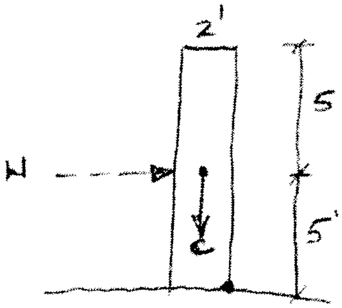
OR

$$W = 20\text{PSF}(1')(10') = 200^{\text{lb}} \text{ FOR ONE LINEAR FOOT.}$$

$\Downarrow$ OR

$$\text{SO } W = 200 \text{ PLF}$$

STEP 3: DETERMINE STRUCTURAL STABILITY BY COMPARING MOMENTS.



$W = 200$ lb.
$C = 3000$ lb
FROM STEPS 1 & 2.

$$M_R = 3000^{\text{lb}}\left(\frac{2'}{2}\right) = 3000^{\text{lbft}} = 3^{\text{kft}} \text{ FOR ONE LINEAR FOOT}$$

$$M_{OT} = 200^{\text{lb}}(5') = 1000^{\text{lbft}} = 1^{\text{kft}} \text{ FOR ONE LINEAR FOOT}$$

$$\sum M_{OT} \leq \sum M_R$$

$$1^{\text{kft}} \leq 3^{\text{kft}}$$ ✓ TRUE ∴ WALL DOESN'T BLOW OVER.

$$\boxed{\text{STABLE}}$$

STRUCTURAL STABILITY (CONT.)

Ⓑ APPROACH THIS PART BY GOING THE OPPOSITE DIRECTION YOU TOOK IN PART Ⓐ, BY STARTING W/ THE MOMENT EQUATION.

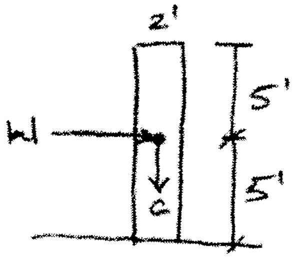
STEP 1: FIND THE MAXIMUM OVERTURNING MOMENT ALLOWED.

$M_{OT} \le M_R$ IF $M_R = 3^{kft}$ FOR ONE LINEAR FOOT. FROM PART Ⓐ.

$M_{OT} \le 3^{kft}$

$M_{OT} = 3^{kft}$ MAX.

STEP 2: FIND THE MAXIMUM OVERTURNING FORCE, i.e. WIND



$$M_{OT} = 3^{kft} = W(5')$$

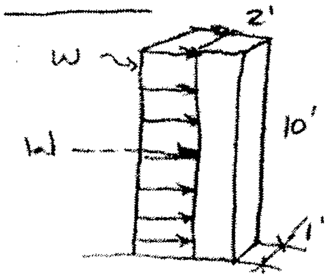
$$W = \frac{3000 \text{ lb ft}}{5 \text{ ft}} = 600^{lb} \text{ FOR ONE LINEAR FOOT}$$

⇓ OR

W = 600 PLF

KEEP IN MIND, THIS IS JUST THE POINT LOAD.

STEP 4: FIND THE MAXIMUM OVERTURNING DISTRIBUTED LOAD, i.e. WIND



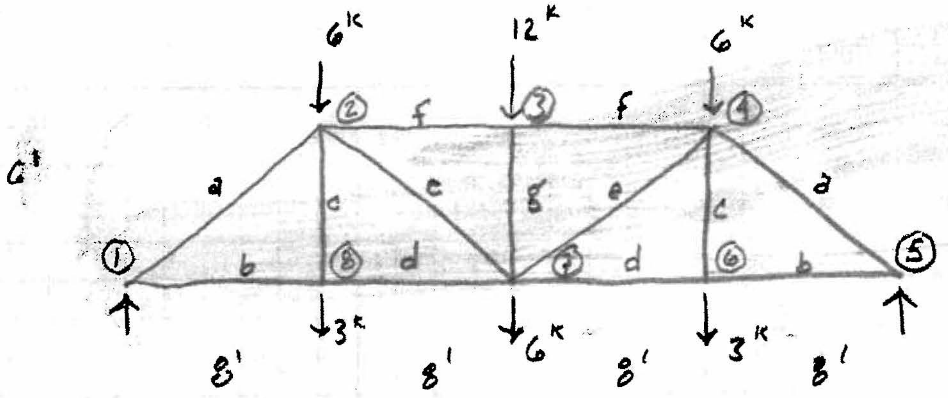
$W = 600^{lb}$

$$w = \frac{\text{FORCE}}{\text{SURFACE AREA}} = \frac{600^{lb}}{10'(1')} = 60 \text{ PSF}$$

HEIGHT ↗ ↖ LENGTH – ONE LINEAR FOOT.

SO, $\boxed{w = 60 \text{ PSF}}$, THREE TIMES THE AMOUNT OF THE PREVIOUS LOADING CONDITION IN PART Ⓐ

SOLVE FOR ALL MEMBERS BY METHOD OF JOINTS



REACTIONS :

$\Sigma M_A = 0 = 6(8) + 3(8) + 12(16) + 6(16) + 6(24) + 3(24) - B(32) = 0$

$B(32) = 576$

$\underline{B = 18^k \uparrow}$

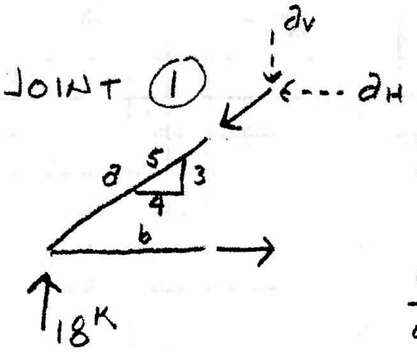
$\Sigma M_B = 0 = A(32) - 9(24) - 18(16) - 9(8) = 0$

$A(32) = 576$

$\underline{A = 18^K \uparrow}$

CHECK $F_V = 0 = 18 + 18 - 6 - 3 - 12 - 6 - 6 - 3 = 0$ ✓ OK

JOINT ①



$\Sigma F_V = 0 = 18 - a_V$

$a_V = 18^K$

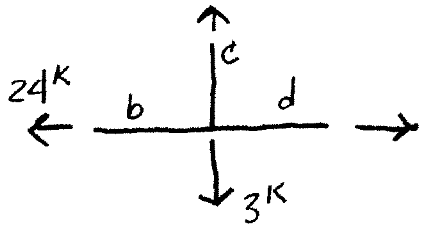
$\frac{a_H}{a_V} : \frac{4}{3} \quad \therefore \quad a_H = 24^K$

$a = \sqrt{18^2 + 24^2} = \underline{30^K C}$

$\Sigma F_H = 0 = b - 24 = 0$

$\underline{b = 24^K T}$

JOINT ⑧



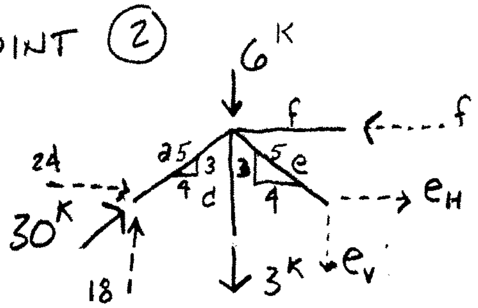
$\Sigma F_H = 0 = -24 + d$

$\underline{d = 24^K \ T}$

$\Sigma F_V = 0 = -3 + c$

$\underline{c = 3^K \ T}$

JOINT ②



$\Sigma F_V = 0 = -6 + 18 - 3 - e_V$

$e_V = 9^K$

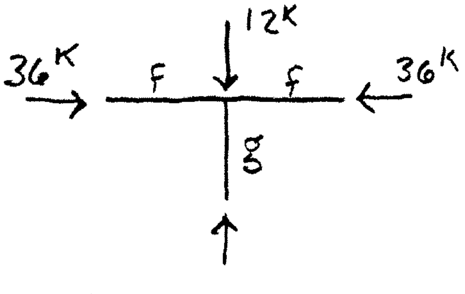
$\frac{3}{5} \quad \frac{9}{e} \quad \therefore \quad \underline{e = 15^K \ T}$

$\frac{3}{4} \quad \frac{9}{e_H} \quad \therefore \quad e_H = 12^K$

$\Sigma F_H = 24 + 12 - f = 0$

$\underline{f = 36^K \ C}$

JOINT ③



$\Sigma F_V = 0 = -12 + g = 0$

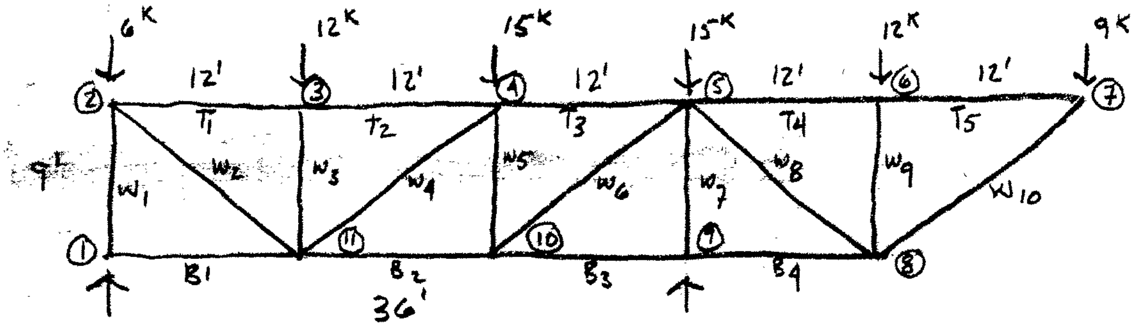
$\underline{g = 12^K \ C}$

SUMMARY

a = 30^K C
b = 24^K T
c = 3^K T
d = 24^K T
e = 15^K T
f = 36^K C
g = 12^K C

A = 18^K ↑
B = 18^K ↑

DETERMINE: REACTIONS A & B, ALL MEMBER FORCES



REACTIONS:

$\Sigma M_A = 0 = 12(12) + 15(24) + 15(36) + 12(48) + 60(9) - B(36) = 0$

$B(36) = 2160$

$\underline{B = 60^K \uparrow}$

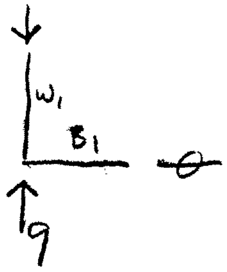
$\Sigma M_B = 0 = -6(36) - 12(24) - 15(12) + 12(12) + 9(24) + A(36) = 0$

$A(36) = 324$

$\underline{A = 9^K \uparrow}$

CHECK $\Sigma F_V = 0 = 60 + 9 - 6 - 12 - 15 - 15 - 12 - 9 = 0$ ✓

JOINT ①

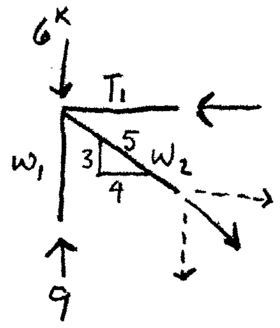


$\Sigma F_V = 0 = 9 - W_1$

$\underline{W_1 = 9^K C}$

$\Sigma F_H = 0 = \underline{B_1 = 0}$

JOINT ②



$\Sigma F_V = 0 = 9 - W_2V - 6$

$W_2V = 3^K$

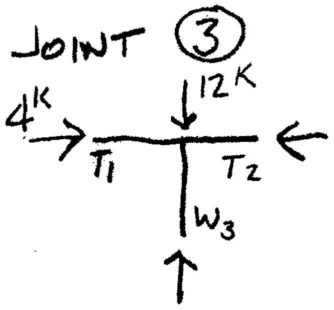
$\frac{W_2}{5} : \frac{3}{3} \therefore \underline{W_2 = 5^K T}$

$\frac{W_{2H}}{4} : \frac{3}{3} \therefore W_{2H} = 4^K$

$\Sigma F_H = 0 = 4 - T_1$

$\underline{T_1 = 4^K C}$

JOINT ③



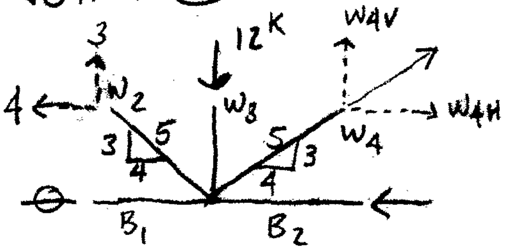
$\Sigma F_V = -12 + W_3 = 0$

$\underline{W_3 = 12^K C}$

$\Sigma F_H = 4 - T_2 = 0$

$\underline{T_2 = 4^K C}$

JOINT ⑪



$\Sigma F_V = +3 - 12 + W_{4V} = 0$

$W_{4V} = 9^K$

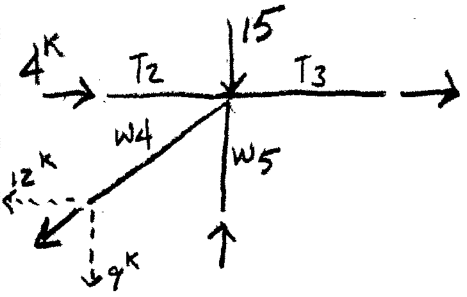
$\frac{W_{4V}}{3} : \frac{W_4}{5} \therefore \underline{W_4 = 15^K T}$

$\frac{W_{4H}}{4} : \frac{W_{4V}}{3} \therefore W_{4H} = 12^K$

$\Sigma F_H = -4 + 12 - B_2 = 0$

$\underline{B_2 = 8^K C}$

JOINT ④



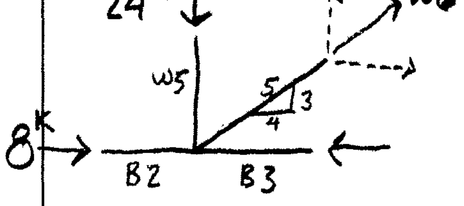
$\Sigma F_V = -15 - 9 + W_5 = 0$

$\underline{W_5 = 24^K C}$

$\Sigma F_H = 4 - 12 + T_3 = 0$

$\underline{T_3 = 8^K T}$

JOINT ⑩



$\Sigma F_V = -24 + W_{6V} = 0$

$W_{6V} = 24$

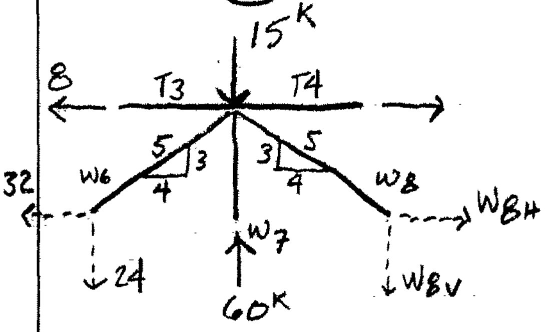
$\frac{24}{3} : \frac{W_6}{5} \therefore \underline{W_6 = 40^K T}$

$\frac{24}{3} \quad \frac{W_{6H}}{4} \therefore W_{6H} = 32$

$\Sigma F_H = 8 + 32 - B3 = 0$

$\underline{B3 = 40^K C}$

JOINT ⑤ (WORK ⑨ FIRST)



$\Sigma F_V = 0 = 60 - 15 - 24 - W8_V = 0$

$W8_V = 21$

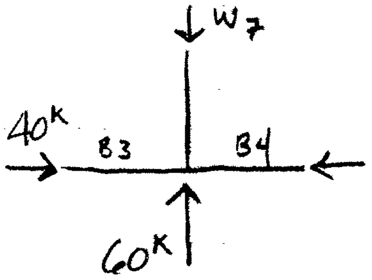
$\frac{W8_V}{3} \cdot \frac{W8}{5} \therefore W8 = 35^K T$

$\frac{W8_V}{3} \; \frac{W8_H}{4} \therefore W8_H = 28$

$\Sigma F_H = -8 + T4 - 32 + 28 = 0$

$T4 = 12^K T$

JOINT ⑨



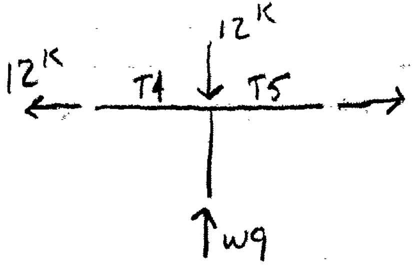
$\Sigma F_V = 60 - W_7 = 0$

$W_7 = 60^K C$

$\Sigma F_H = 40 - B4 = 0$

$B4 = 40^K C$

JOINT ⑥



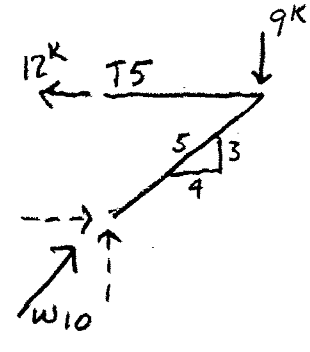
$\Sigma F_V = 0 = -12 + W9$

$W9 = 12^K C$

$\Sigma F_H = 0 = 12 - T5 = 0$

$T5 = 12^K T$

JOINT ⑦



$\Sigma F_V = -9 + W10_V = 0$

$W10_V = 9$

$\Sigma F_H = -12 + W10_H = 0$

$W10_H = 12$

$W_{10} = \sqrt{9^2 + 12^2} = 15$

$W_{10} = 15^K C$

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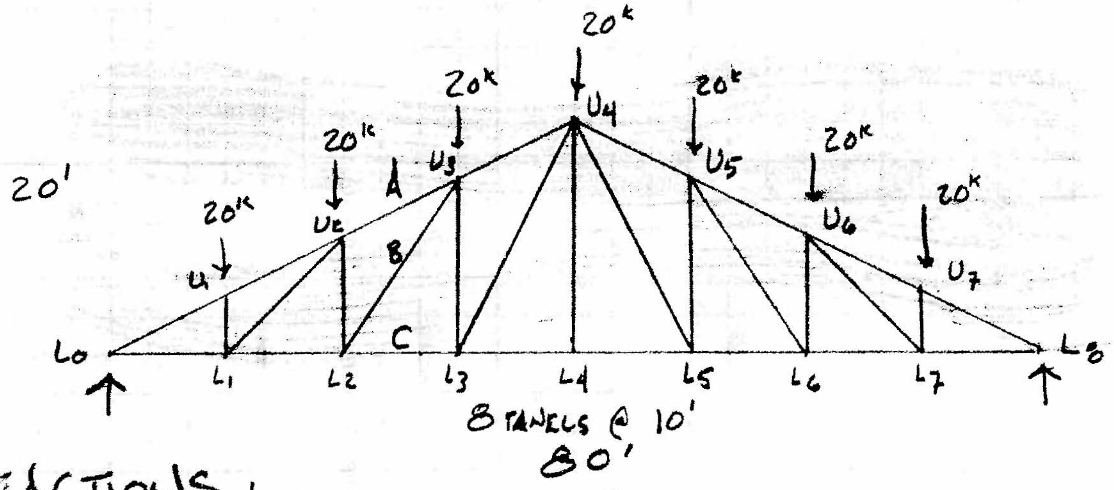
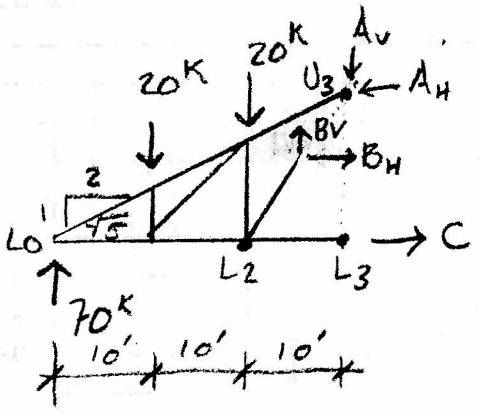


REACTIONS:
BY SYMETRY $R_1 = 70^K = R_2$

FBD ①



$\Sigma M_{U_3} = 0 = 70(30) - 20(20) - 20(10) - C(15) = 0$

$C(15) = 1500$

$\underline{C = 100^K} \; \textcircled{T}$

$\Sigma M_{L2} = 0 = 70(20) - 20(10) - A_H(10) = 0$

$A_H(10) = 1200$

$A_H = 120^K \leftarrow$ $\quad \frac{A}{\sqrt5} : \frac{A_H}{2}$ $\quad \underline{A = 134.2^K} \; \textcircled{C}$

$\Sigma M_{L_0} = 0 = 20(10) + 20(20) - B_V(20) = 0$

$B_V(20) = 600$

$B_V = 30 \uparrow$ $\quad \frac{B}{\sqrt{13}} \quad \frac{B_V}{3}$ $\quad \underline{B = 36.06^K} \; \textcircled{T}$

CHECK $\Sigma F_V = 0 = 70 - 20 - 20 + 30 - 60 = 0$ ✓

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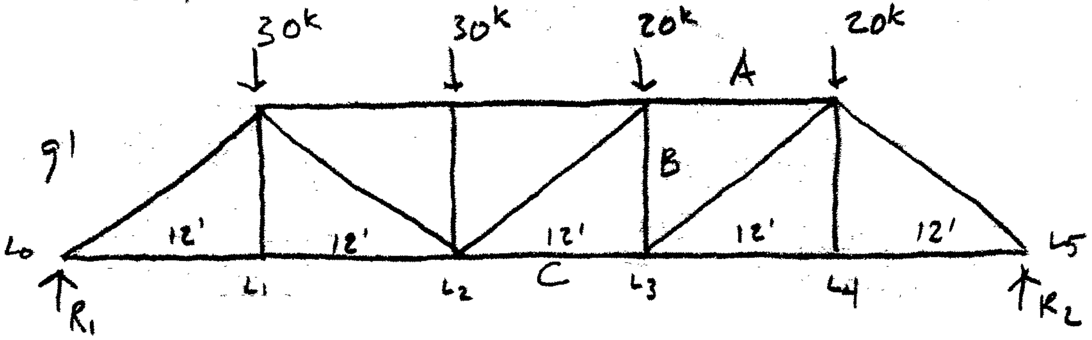
SOLVE FOR C, B, A



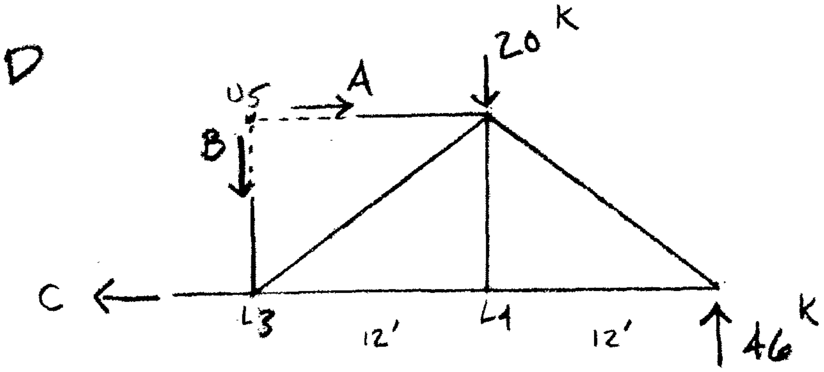
REACTIONS:

$$\Sigma M_{L_0} = 0 = 30(12) + 30(24) + 20(36) + 20(48) + R_2(60)$$
$$R_2(60) = 2760$$
$$R_2 = 46^k \uparrow$$

$$\Sigma M_{L_5} = 0 = R_1(60) - 30(48) - 30(36) - 20(24) - 20(12)$$
$$R_1(60) = 3240$$
$$R_1 = 54 \uparrow$$

CHECK $\Sigma F_V = 0 = 46 + 54 - 30 - 30 - 20 - 20 = 0$

FBD



$$\Sigma M_{L_3} = 0 = -46(24) + A(9) + 20(12)$$
$$A = 96.0^k \textcircled{C}$$

$$\Sigma M_{U5} = 0 = -46(24) + C(9) + 20(12)$$
$$C = 96.0^k \textcircled{T}$$

$$\Sigma F_V = 0 = -20 + 46 - B = 0$$
$$B = 26^k \textcircled{C}$$

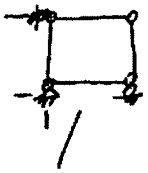
CHECK $\Sigma M_{L_5} = 0 = -26(24) + 96.0(9) - 20(12) = 0$

NOTES ON TRUSSES

①

Truss 10/06/04

Determinate and Indeterminate

Structure Characteristics	Externally Determinate/Indeterminate (For supports)	Internally Determinate/Indeterminate (For truss itself)
j = No. of joints m = Actual No. of bars. r_a = Actual No. of support Component. r = Min. No. of support component for stability ($r=3$ for truss) k = Min. No. of bars for stability	$r_a < r$ unstable $r_a = r$ determinate $r_a > r$ Indeterminate  $\left.\begin{matrix} r_a = 4 \\ r = 3 \end{matrix}\right\\}$ $r_a > r$. Thus, Externally indeterminate One extra support component.	$\boxed{k = 2j - r} \leftarrow r=3$ $m < k$ Unstable $m = k$ determinate $m > k$ Indeterminate  $\left.\begin{matrix} k = 2(4)-3=5 \\ m = 4 \end{matrix}\right\\}$ $m<k$ Internally Unstable ↓  $\left.\begin{matrix} k=5 \\ m=5 \end{matrix}\right\\}$ $m=k$ Internally determinate ↓  $\left.\begin{matrix} k=5 \\ m=6 \end{matrix}\right\\}$ $m>k$ Internally indeterminate

- Procedures of Solving Truss Problem.

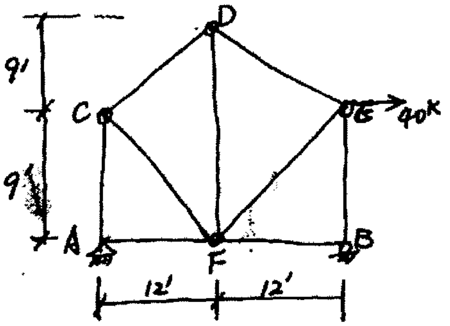
  (1) Determine determinancy/indeterminancy of truss

  (2) Find Reactions (exterior force)

  (3) Determine zero bars.

  (4) Start Calculation with the joints with the least unknowns

  (5) Check the results.



Find force along each bar.

Solution:

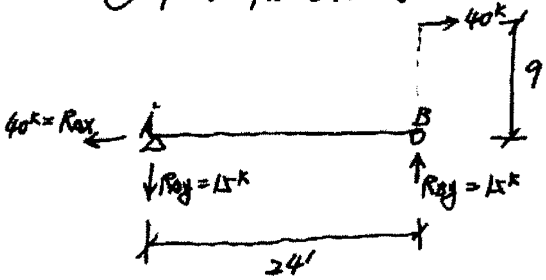
① Determine determinancy/indeterminancy of truss.

Externally: $r_a = r = 3$ determinate (no extra supports)

Internally: $k = 2j - r = 2(6) - 3 = 9$, $m = 9$ } → determinate (no extra bars)

---

② Find Reactions



$\Sigma F_x = 0 \Rightarrow R_{Ax} = 40^k$ (←)

$\Sigma M_A = 0$ ↻+ $\Rightarrow 40^k(9') - 24' R_{By} = 0$

$R_{By} = 15^k$ (↑)

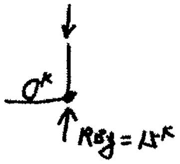
$\Sigma F_y = 0$ ↑+ $\Rightarrow R_{Ay} + R_{By} = 0$

$R_{Ay} = -15^k$ (↓)

---

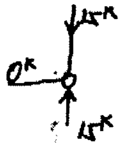
③ Determine 0 bars.

Joint B.



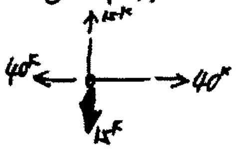
④ Start Calculation with the joints with the least Unknowns

(1) Joint B

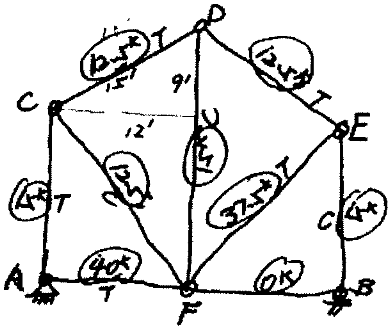


1 unknown

Joint A



2 unknowns



(Figure 1)

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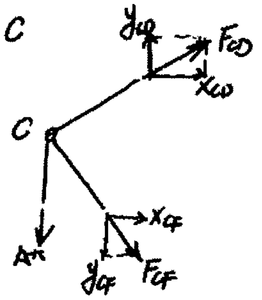
(2) Joint C

FBD:



Two unknowns: F_{CD}, F_{CF}

Two equations: $\Sigma F_x = 0$, $\Sigma F_y = 0$

$$\frac{F_{CD}}{15'} = \frac{X_{CD}}{12'} = \frac{y_{CD}}{9'} \Longrightarrow X_{CD} = 0.8F_{CD},\ y_{CD} = 0.6F_{CD}$$

$$\frac{F_{CF}}{15'} = \frac{X_{CF}}{12'} = \frac{y_{CF}}{9'} \Longrightarrow X_{CF} = 0.8F_{CF},\ y_{CF} = 0.6F_{CF}$$

$\Sigma F_x = 0 \qquad X_{CD} + X_{CF} = 0 \Longrightarrow X_{CD} = -X_{CF} \Longrightarrow F_{CD} = -F_{CF}$

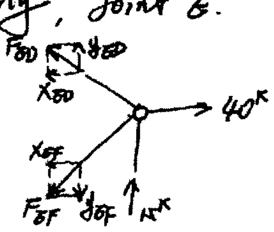
$\Sigma F_y = 0 \uparrow^{+} \qquad y_{CD} - y_{CF} - 15^k = 0$

$0.6F_{CD} - 0.6F_{CF} - 15^k = 0$

$0.6F_{CD} - 0.6(-F_{CD}) - 15^k = 0 \Longrightarrow F_{CD} = 12.5^k \nearrow \swarrow \quad F_{CF} = -12.5^k \nwarrow$

Similarly, Joint E.

FBD:



Two unknowns: F_{ED}, F_{EF}

Two equation: $\Sigma F_x = 0$, $\Sigma F_y = 0$

Continued.

Joint E.

$$\frac{F_{ED}}{15'} = \frac{X_{ED}}{12'} = \frac{y_{ED}}{9'} \Longrightarrow X_{ED} = 0.8F_{ED},\ y_{ED} = 0.6F_{ED}$$

$$\frac{F_{EF}}{15'} = \frac{X_{EF}}{12'} = \frac{y_{EF}}{9'} \Longrightarrow X_{EF} = 0.8F_{EF},\ y_{EF} = 0.6F_{EF}$$

↙ assumed positive direction.

$\Sigma F_x = 0 \overset{+}{\leftarrow}$

$$X_{ED} + X_{EF} - 40^k = 0 \Longrightarrow X_{ED} = 40 - X_{EF}$$

$$\Downarrow$$

$$0.8F_{ED} = 40 - 0.8F_{EF}$$

$$F_{ED} = 50 - F_{EF}$$

$\Sigma F_y = 0 \overset{+}{\uparrow}$ assumed

$$y_{ED} - y_{EF} + 15^k = 0$$

$$0.6F_{ED} - 0.6F_{EF} + 15^k = 0$$

$$0.6(50 - F_{EF}) - 0.6F_{EF} + 15^k = 0$$

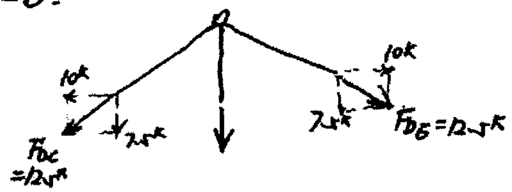
$$F_{EF} = 37.5^k \swarrow$$

$$F_{ED} = 50 - F_{EF} = 50 - 37.5^k = 12.5^k \nwarrow$$

---

5) Calculate the last joint and use it to check your all calculations.

FBD:



$\Sigma F_x = 10^k - 10^k = 0$, ok.

$\Sigma F_y = 0 \overset{+}{\uparrow}$

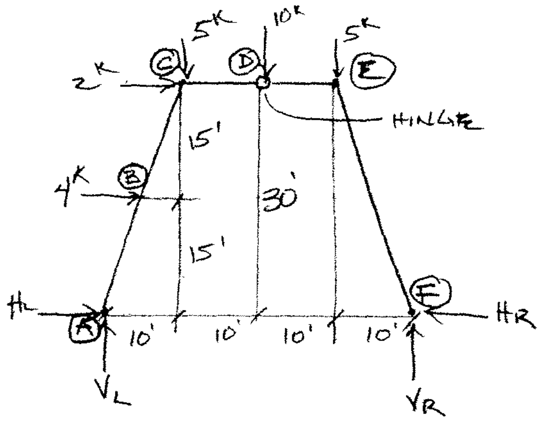
$$-y_{DC} - F_{DF} - y_{DE} = 0$$

$$-7.5 - F_{DF} - 7.5 = 0$$

$$F_{DF} = \underset{\downarrow}{-}15^k \uparrow$$

negative means the direction assumed is wrong.

See Figure 1 in Page 3 for the final results



(1) DETERMINE REACTIONS.

(2) DRAW BENDING MOMENT DIAGRAM W/ TENSION ON OUTSIDE OF FRAME.

(1) REACTIONS:

$$\Sigma M_L = 0 = 4^K(15') + 2^K(30') + 5^K(10') + 10^K(20') + 5^K(30') - V_R(40')$$

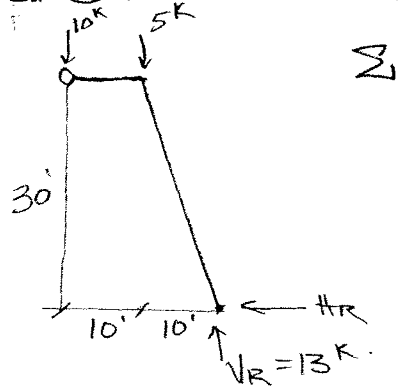
$$0 = 60 + 60 + 50 + 200 + 150 - 40V_R$$

$$40V_R = 520 \implies \underline{\underline{V_R = 13^K}}$$

$$\Sigma F_y = 0 = V_L + V_R - 5^K - 10^K - 5^K$$

$$0 = V_L + 13^K - 5^K - 10^K - 5^K \implies \underline{\underline{V_L = 7^K}}$$

FBD ① : RIGHT HALF OF FRAME



$$\Sigma M_{@HINGE} = 0 = 5^K(10') + H_R(30') - 13^K(20')$$

$$0 = 50 - 260 + 30H_R$$

$$30H_R = 210 \implies \underline{\underline{H_R = 7^K}}$$

$$\Sigma F_x = 0 = H_L + 4^K + 2^K - H_R$$

$$0 = H_L + 4^K + 2^K - 7^K \implies \underline{\underline{H_L = 1^K}}$$

SUMMARY:

- WHEN USING ΣF_x OR ΣF_y TO FIND THE REACTIONS - THE ENTIRE FRAME MUST BE CONSIDERED
- USING THE FACT THAT HINGES OF ZERO MOMENT, ΣM EQUATION CAN BE USED AT ANY HINGE.
- WHEN TAKING ΣM AT AN INTERNAL HINGE (i.e. NOT AT THE REACTIONS) ONLY ONE SIDE OF FRAME IS CONSIDERED - EITHER LEFT SIDE ONLY OR RIGHT SIDE OF HINGE ONLY.

(2) DRAW BENDING MOMENT DIAGRAM.

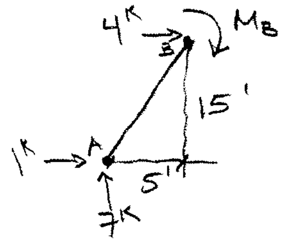
- FIRST FIND THE MOMENT AT EACH JOINT & POINTS OF EXTERNAL FORCES.

- ALREADY KNOW:

$M_A = 0$
$M_D = 0$
$M_F = 0$
} ALL HINGES

FOR REMAINING POINTS, ASSUME POSITIVE MOMENT IS CLOCKWISE. $\overset{+}{\curvearrowright} M$

FBD ②: FROM A → B



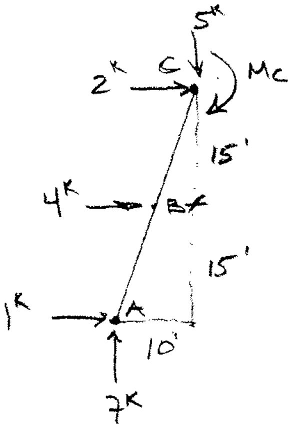
$$\Sigma M_{@B} = 0 = M_B + 7^k(5') - 1^k(15')$$

$$0 = M_B + 35 - 15 \implies \underline{\underline{M_B = -20^{k\,ft}}}$$

SO, SINCE ANSWER IS NEGATIVE, ASSUMED INCORRECT MOMENT DIRECTION.

$M_B = 20^{k\,ft} \curvearrowleft$ ∴ TENSION ON INSIDE

FBD ③: FROM A → C



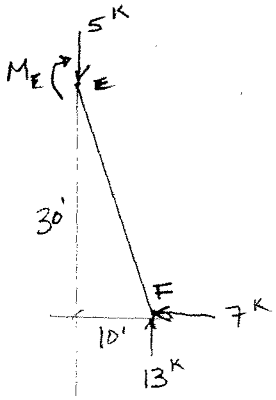
$$\Sigma M_{@C} = 0 = M_C + 7^k(10') - 1^k(30') - 4^k(15')$$

$$0 = M_C + 70 - 30 - 60$$

$\underline{\underline{M_C = 20^{k\,ft}}}$ ⟹ ASSUMED CORRECT DIRECTION.

∴ TENSION ON OUTSIDE

FBD ④: FROM E → F



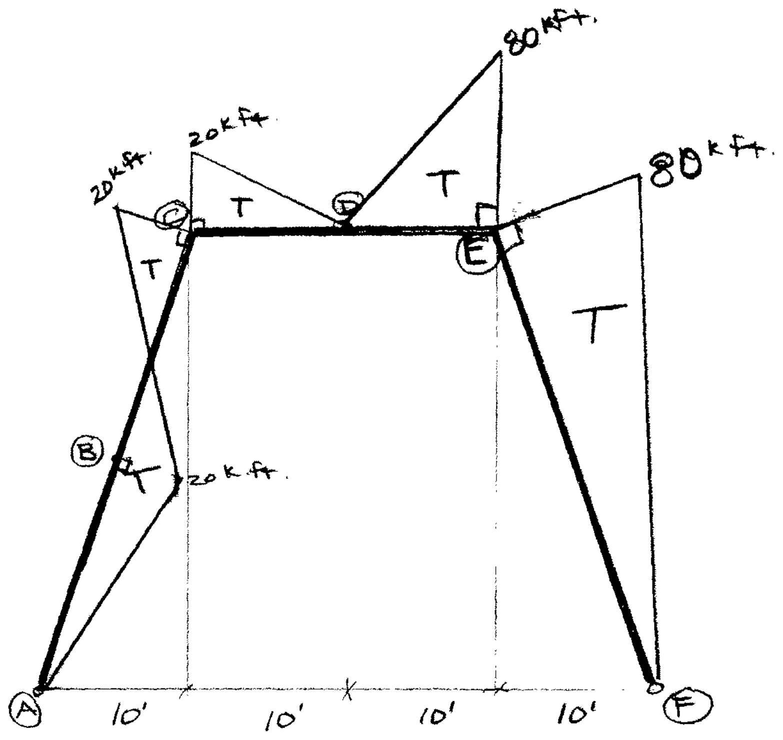
$$\Sigma M_{@E} = 0 = M_E + 7^k(30') - 13^k(10')$$

$$0 = M_E + 210 - 130$$

$$\underline{\underline{M_E = -80^{k\,ft}}}$$

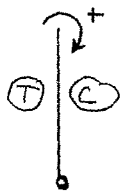
SO, SINCE ANSWER IS NEGATIVE, ASSUMED INCORRECT MOMENT DIRECTION.

$M_E = 80^{k\,ft} \curvearrowleft$ ∴ TENSION ON OUTSIDE



SUMMARY:

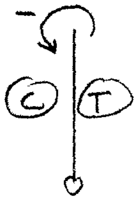
- WHEN FINDING A MOMENT AT A POINT ASSUME A POSITIVE MOMENT. i.e.

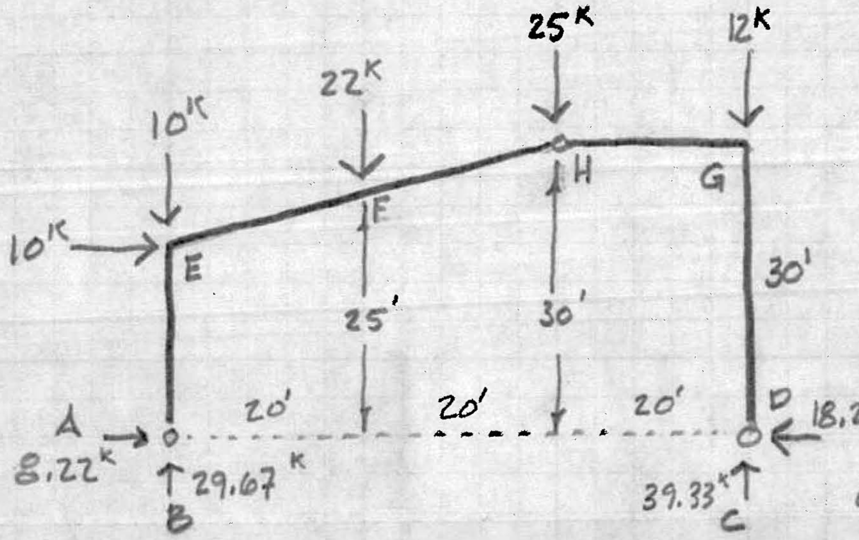


THIS GIVES YOU TENSION TO THE LEFT & COMPRESSION TO THE RIGHT

IF THE ANSWER COMES OUT POSITIVE, YOU ASSUMED CORRECTLY & TENSION IS TO THE LEFT.

IF THE ANSWER COMES OUT NEGATIVE, YOU ASSUMED INCORRECTLY & TENSION IS ACTUALLY TO THE RIGHT:





FIND REACTIONS:

$\Sigma M_A = 0 = 10(20) + 22(20) + 25(40) + 12(60) - C(60)$

$C(60) = 2360$

$C = 39.33^K$

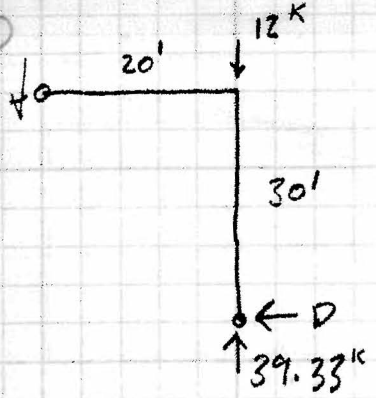
$\Sigma M_C = 0 = 10(20) - 10(60) - 22(40) - 25(20) + B(60)$

$B(60) = 1780$

$B = 29.67^K$

CHECK = $\Sigma F_V = 0$ ✓

FBD ①



FROM FBD ①

$\Sigma M_{HINGE} = 0 = 12(20) - 39.33(20) + D(30)$

$D(30) = 546.6$

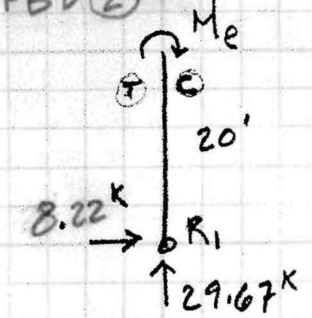
$D = 18.22^K$

FROM Ⓑ

$\Sigma F_H = 0 = A + 10 - 18.22$

$A = 8.22^k$

FBD ②

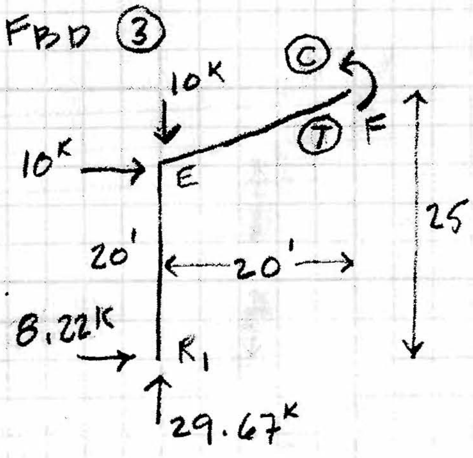


FROM FBD ②

$\Sigma M_E = 0 = -8.22(20) + M_e$

$M_e = 164.4^{K-'}$

FBD ③

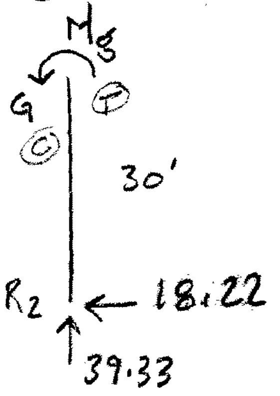


FROM FBD ③

$\Sigma M_F = 0 = 29.67(20) - 8.22(25) - 10(5) - 10(20) - M_f$

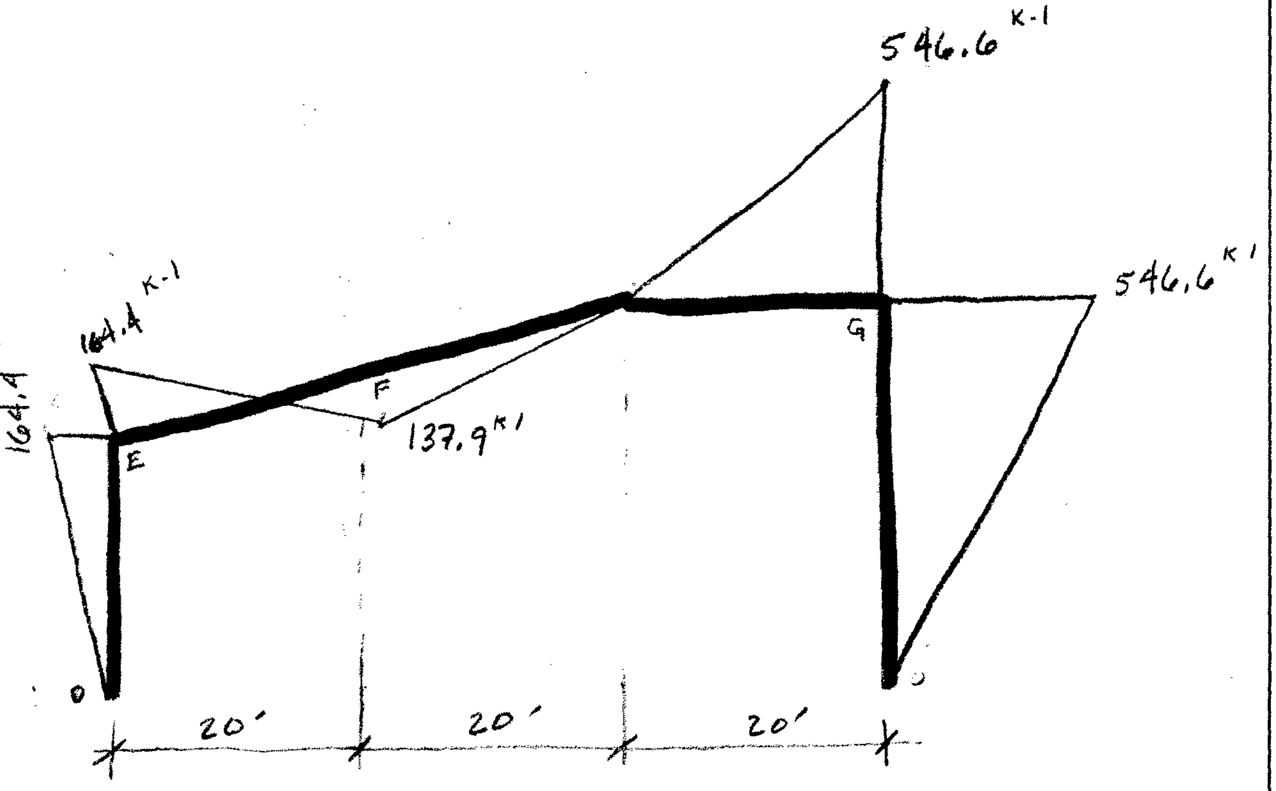
$M_f = 137.9^{K-'}$

FBD ④



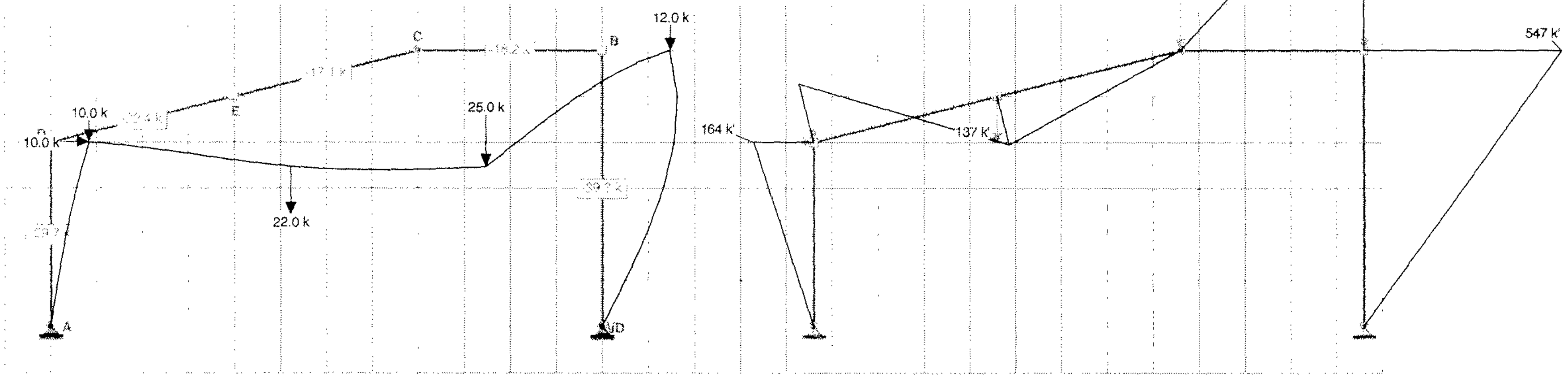
$$\Sigma M_G = 0 = 18.22(30) - M_g$$

$$M_g = 546.6^{K-1}$$

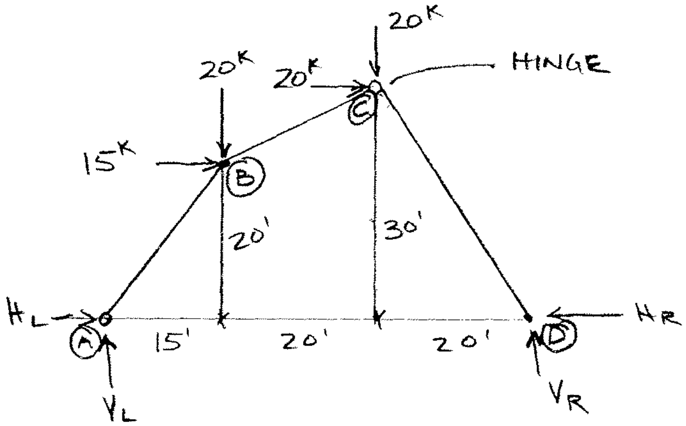


Load Combination: Scratch: 1.00S

Educational License



ARCH 314 PROB. 9-2 C NOV. 28, 2003 SR Y.



(1) FIND REACTIONS

(2) DRAW BENDING MOMENT DIAGRAM.

(1) REACTIONS:

$$\Sigma M_L = 0 = 15^k(20') + 20^k(15') + 20^k(30') + 20^k(35') - V_R(55')$$

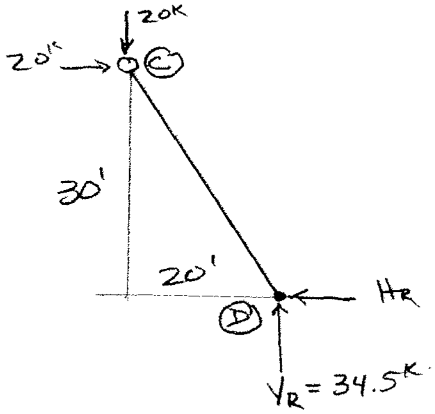
$$0 = 300 + 300 + 600 + 700 - 55V_R$$

$$55V_R = 1900 \Longrightarrow \underline{\underline{V_R = 34.5^k}} \uparrow$$

$$\Sigma F_y = 0 = V_L - 20^k - 20^k + V_R$$

$$0 = V_L - 20^k - 20^k + 34.5^k \Longrightarrow \underline{\underline{V_L = 5.5^k}} \uparrow$$

FBD ① : RIGHT HALF OF FRAME



$$\Sigma M_{@HINGE} = 0 = H_R(30') - 34.5^k(20')$$

$$0 = 30H_R - 690$$

$$30H_R = 690$$

$$\underline{\underline{H_R = 23^k}} \leftarrow$$

$$\Sigma F_x = 0 = H_L + 15^k + 20^k - H_R$$

$$0 = H_L + 15^k + 20^k - 23^k$$

$$\underline{\underline{H_L = -12^k}}$$

SINCE ANSWER IS NEGATIVE, ASSUMED INCORRECT DIRECTION.

$$\underline{\underline{H_L = 12^k}} \leftarrow$$

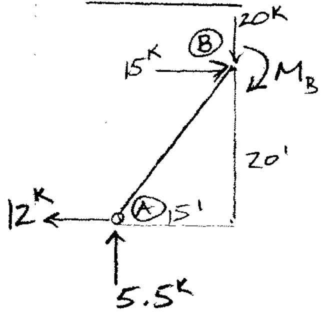
(2) DRAW BENDING MOMENT DIAGRAM

- FIRST FIND THE MOMENT AT EACH JOINT & POINTS OF EXTERNAL FORCES.

- ALREADY KNOW.

$$\left.\begin{aligned} M_A &= 0 \\ M_C &= 0 \\ M_D &= 0 \end{aligned}\right| \longrightarrow \text{ALL HINGES.}$$

FBD ②: FROM A → B.



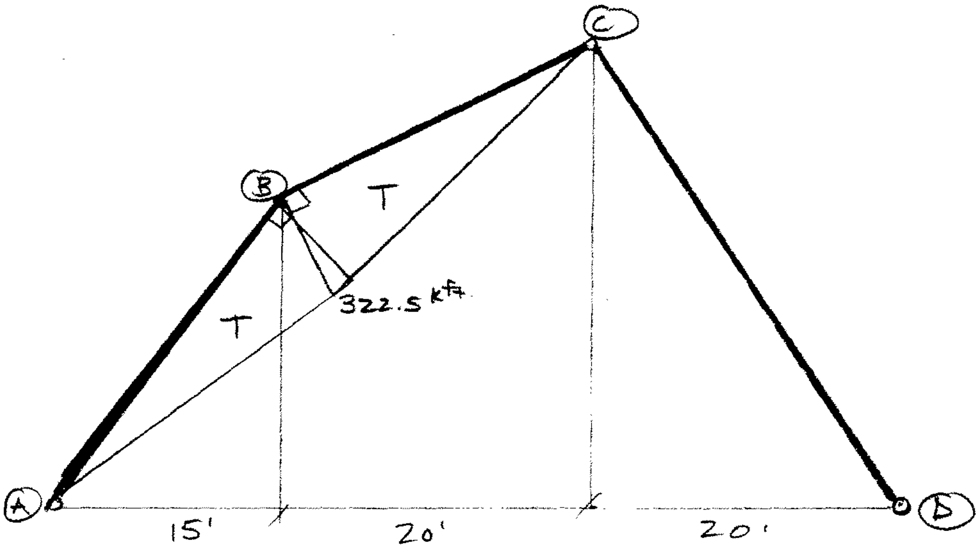
$$\Sigma M_{@B} = 0 = M_B + 5.5^K(15') + 12^K(20')$$

$$0 = M_B + 82.5 + 240$$

$$M_B = -322.5 \text{ k ft}$$

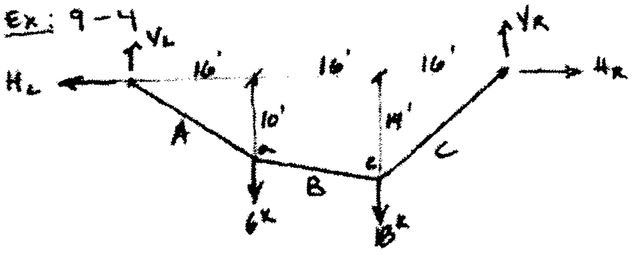
SINCE ANSWER IS NEGATIVE, ASSUMED INCORRECT MOMENT DIRECTION.

$$\underline{\underline{M_B = 322.5^{\text{k ft}}}} \circlearrowleft \quad \therefore \text{TENSION ON INSIDE}$$



CABLES

Ex: 9-4



FIND: MEMBER FORCES FOR A, B, C.

STEP 1: FIND VERTICAL REACTIONS – MUST USE EQUILIBRIUM EQUATIONS.

$$\Sigma M_{@L} = 0 = 6^k(16') + 18^k(32') - V_R(48') \Longrightarrow \boxed{V_R = 14^k}$$

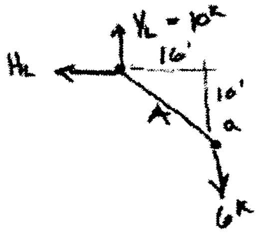
$$\Sigma F_y = 0 = V_L - 6^k - 18^k + \underset{14^k}{\cancel{V_R}} \Longrightarrow \boxed{V_L = 10^k}$$

STEP 2: FIND HORIZONTAL REACTIONS – TWO METHODS.

METHOD 1: EQUILIBRIUM EQUATIONS – YOU KNOW THAT THERE IS NO MOMENT ANYWHERE IN THE CABLE – TENSILE FORCES ONLY!

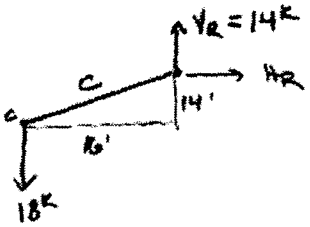
JUST LIKE METHOD OF SECTIONS IN TRUSSES, CUT CABLE & DRAW FBD:

FBD ①



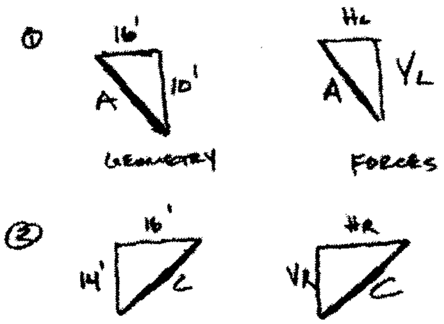
$$\Sigma M_{@a} = 0 = 10^k(16') - H_L(10') \Longrightarrow \boxed{H_L = 16^k}$$

FBD ②



$$\Sigma M_{@c} = 0 = H_R(14') - 14^k(16') \Longrightarrow \boxed{H_R = 16^k}$$

METHOD 2: GEOMETRY – SIMILAR TO EARLIER HOMEWORKS, USE SIMILAR TRIANGLES



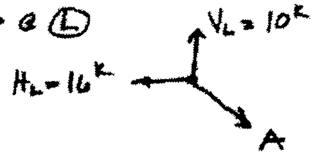
$$\frac{H_L}{16'} = \frac{10^k}{10'} \Longrightarrow \boxed{H_L = 16^k}$$

$$\frac{H_R}{16'} = \frac{14^k}{14'} \Longrightarrow \boxed{H_R = 16^k}$$

$\Sigma F_x = 0$ ✓

STEP 3: FIND MEMBER FORCES – USE FBD's SIMILAR TO METHOD OF JOINTS IN TRUSSES.

FBD @ Ⓛ



$$A = \sqrt{10^2 + 16^2} = 18.87^K$$

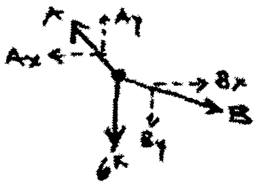
$A_x = 16^K$

$A_y = 10^K$

$$\boxed{A = 18.87^K}$$

TENSION

FBD @ ⓐ



$$\Sigma F_x = 0 = B_x - A_x$$
$$0 = B_x - 16^K \Longrightarrow$$

$B_x = 16^K$

$$\Sigma F_y = 0 = A_y - B_y - 6^K$$
$$0 = 10^K - B_y - 6^K \Longrightarrow$$

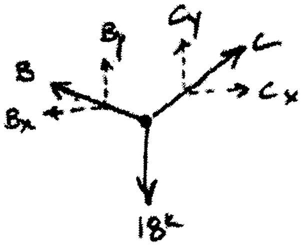
$B_y = 4^K$

$$B = \sqrt{16^2 + 4^2} = 16.5^K$$

$$\boxed{B = 16.5^K}$$

TENSION

FBD @ ⓒ



$$\Sigma F_x = 0 = C_x - B_x$$
$$0 = C_x - 16 \Longrightarrow$$

$C_x = 16^K$

$$\Sigma F_y = 0 = B_y - 18^K + C_y$$
$$0 = 4^K - 18^K + C_y \Longrightarrow$$

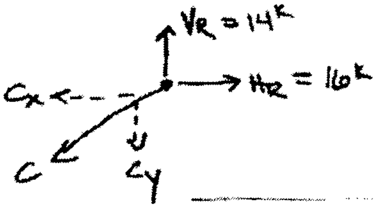
$C_y = 14^K$

$$C = \sqrt{16^2 + 14^2} = 21.3^K$$

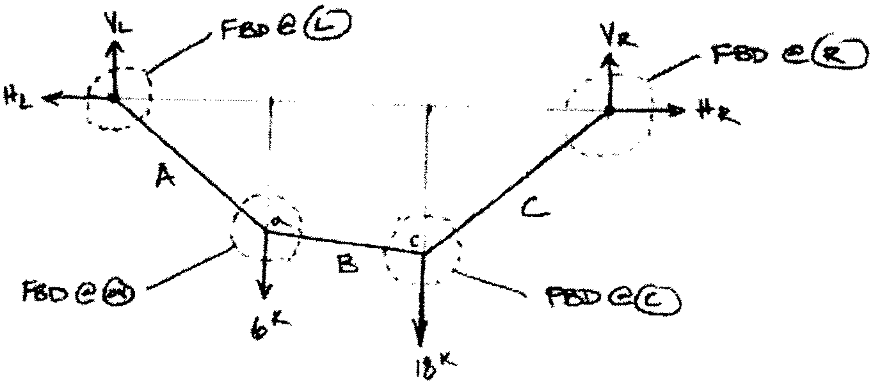
$$\boxed{C = 21.3^K}$$

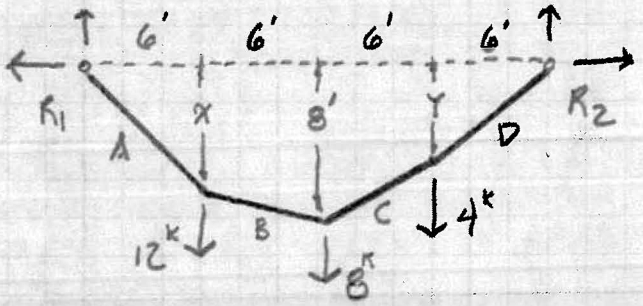
TENSION.

FBD @ Ⓡ CHECK



$$C = \sqrt{14^2 + 16^2} = 21.3^K$$ ✓ CHECKS.





REACTIONS :

$\Sigma M_{R_1} = 0 = 12(6) + 8(12) + 4(18) - R_2V(24)$

$R_2V(24) = 240$

$R_2V = 10$

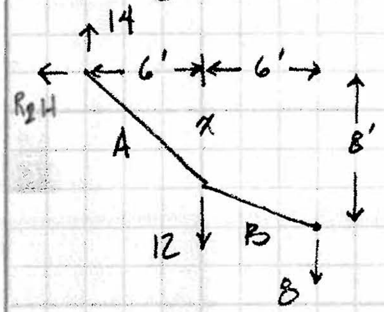
$\Sigma M_{R_2} = 0 = R_1V(24) - 12(18) - 8(12) - 4(6)$

$R_1V(24) = 336$

$R_1V = 14$

CHECK $\Sigma FV = 10 + 14 - 12 - 8 - 4 = 0$ ✓

FBD ①



FROM FBD ①

$\Sigma M_{\text{¢}} = 0 = -R_1H(8) + 14(12) - 12(6)$

$R_1H(8) = 96$

$R_1H = 12^K$

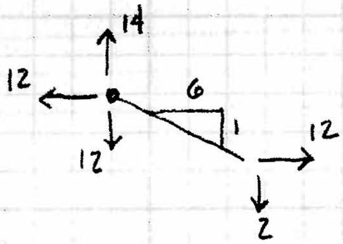
FIND x

$\frac{14}{x} \quad \frac{12}{6} \qquad x = 7'$

FIND FORCE IN A

$A = \sqrt{14^2 + 12^2} = 18.44^K$

FBD ②
(JOINT AT x)



FROM FBD ②

$\Sigma F_V = 0 = -12 - B_V + 14$

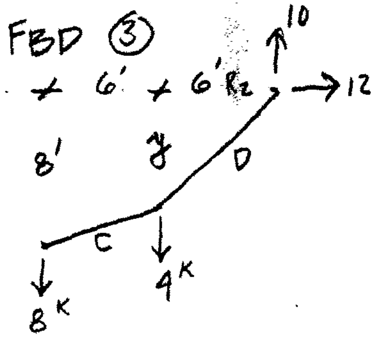
$B_V = 2$

$\Sigma F_H = 0 = -12 + B_H$

$B_H = 12$

$B = \sqrt{12^2 + 2^2} = 12.17^K$

FBD ③



FROM FBD ③

$$\Sigma M_{\not{c}} = 0 = 4(6) - 10(12) + R_2H(8)$$

$$R_2H(8) = 96$$

$$R_2H = 12^K$$

(WHOLE SYSTEM)
CHECK $\Sigma F_H = 0 = 12 - 12$ ✓

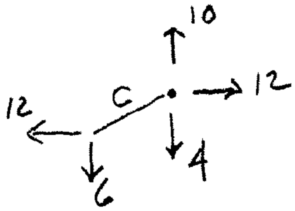
FIND y :

$\frac{10}{y}$ $\frac{12}{6}$ $y = 5'$

FIND FORCE IN D :

$$D = \sqrt{10^2 + 12^2} = 15.62^K$$

FBD ④
JOINT @ y



$$\Sigma F_H = 12 - C_H$$

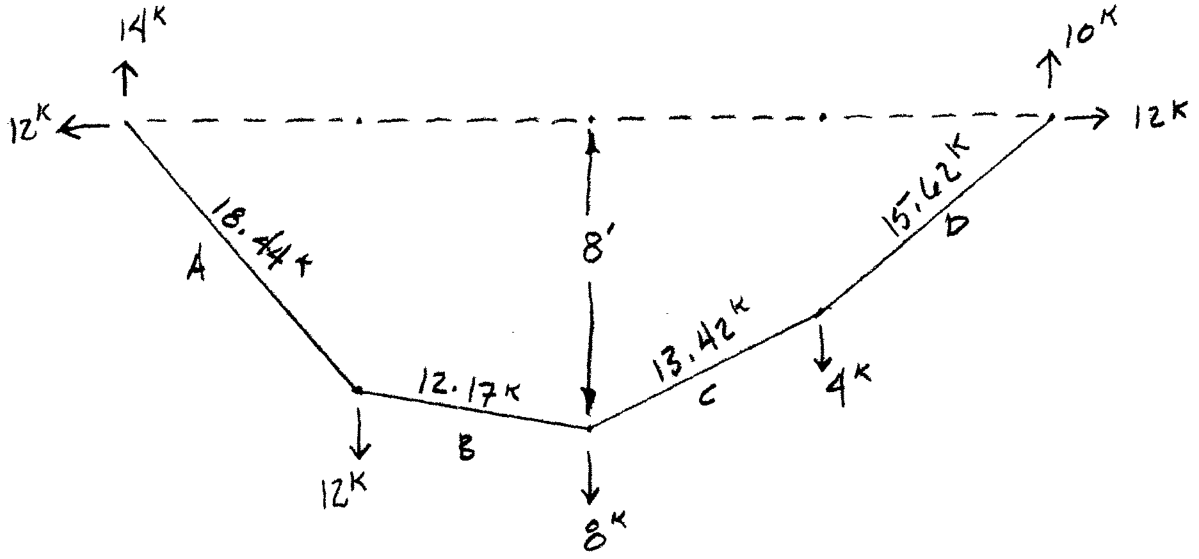
$$C_H = 12^K$$

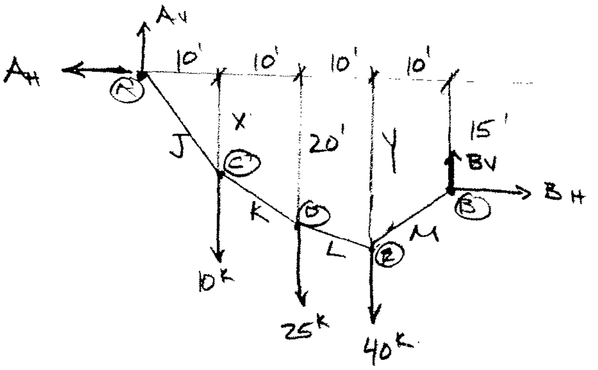
$$\Sigma F_V = 10 - 4 - C_V$$

$$C_V = 6^K$$

$$C = \sqrt{6^2 + 12^2} = 13.42^K$$

SUMMARY :





(1) FIND REACTIONS
(2) FIND DIMENSIONS X & Y.
(3) FIND MEMBER FORCES.

(1) REACTIONS: 4 UNKNOWNS → NEED 4 EQUATIONS

- SINCE CABLES HAVE ONLY TENSION FORCES, THERE ARE ZERO MOMENTS AT EVERY POINT.

$\Sigma F_y = 0 = A_V + B_V - 10^k - 25^k - 40^k \Longrightarrow A_V + B_V = 75^k$ $\underline{EQ.-1}$

$\Sigma M_A = 0 = 10^k(10') + 25^k(20') + 40^k(30') - B_V(40') - B_H(15')$

$40B_V = 1800 - 15B_H$ $\underline{EQ-2}$

$\Sigma M_D = 0 = -B_V(20') + B_H(5') + 40^k(10') \Longrightarrow 400 = 20B_V - 5B_H$
RIGHT SIDE ONLY
$\underline{EQ-3}$

$\Sigma F_x = 0 = B_H - A_H \Longrightarrow A_H = B_H$ $\underline{EQ-4}$

SUBSTITUTING EQ-2 INTO EQ-3:

EQ-2: $40B_V = 1800 - 15B_H$

$B_V = \frac{1800 - 15B_H}{40}$

EQ-3: $400 = 20B_V - 5B_H$

$400 = 20\left(\frac{1800}{40} - \frac{15B_H}{40}\right) - 5B_H$

$400 = 900 - 7.5B_H - 5B_H$

$12.5\,B_H = 500 \Longrightarrow \underline{\underline{B_H = 40^k}}$

SOLVING EQ-2 W/ KNOWN BH VALUE

EQ-2: $B_V = \frac{1800 - 15B_H}{40} = \frac{1800 - 15(40)}{40} \Longrightarrow \underline{\underline{B_V = 30^k}}$

SOLVING EQ-1 W/KNOWN BV VALUE.

EQ-1: $A_V + B_V = 75^k$

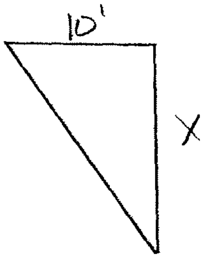
$A_V + 30 = 75 \Longrightarrow \underline{\underline{A_V = 45^k}}$

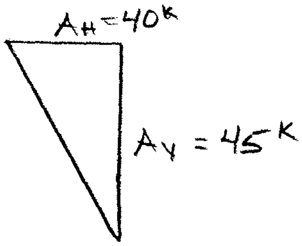
SOLVING EQ-4 W/ KNOWN BH VALUE.

EQ-4: $A_H = B_H \Longrightarrow \underline{\underline{A_H = 40^k}}$

(2) FIND DIMENSIONS X & Y:

USING SIMILAR TRIANGLES:

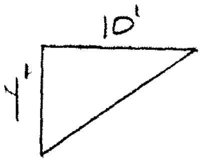


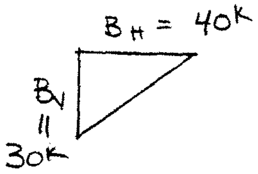


$$\frac{X}{45} = \frac{10}{40}$$

$$40X = 450$$

$$\underline{\underline{X = 11.25^{ft}}}$$

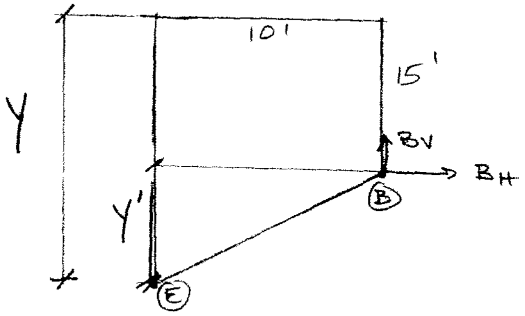




$$\frac{Y'}{30} = \frac{10}{40}$$

$$Y' = 7.5^{ft.}$$

BUT Y IS TOTAL DISTANCE:



$$\text{SO } Y = Y' + 15'$$

$$= 7.5 + 15 \implies \underline{\underline{Y = 22.5'}}$$

(3) FIND MEMBER FORCES.

MEMBER J IS RESULTANT OF REACTION FORCES:

$$J = \sqrt{A_V^2 + A_H^2} = \sqrt{45^2 + 40^2}$$

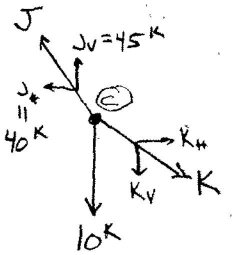
$$\underline{\underline{J = 60.2^K}}$$

MEMBER M IS RESULTANT OF REACTION FORCES:

$$M = \sqrt{B_Y^2 + B_H^2} = \sqrt{30^2 + 40^2}$$

$$\underline{\underline{M = 50^K}}$$

FBD JOINT ⓒ:



$$\Sigma F_x = 0 = K_H - 40^K \implies K_H = 40^K$$

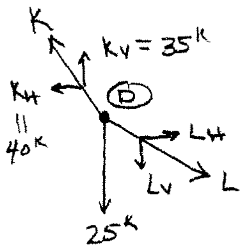
$$\Sigma F_y = 0 = 45^K - 10^K - K_Y \implies K_Y = 35^K$$

SO K IS THE RESULTANT OF THE FORCES K_H & K_V.

$$K = \sqrt{K_Y^2 + K_H^2} = \sqrt{35^2 + 40^2}$$

$$K = 53.2^K$$

FBD JOINT Ⓓ:



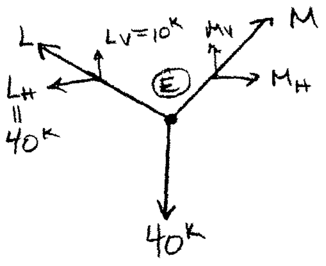
$\Sigma F_x = 0 = L_H - 40^k \Rightarrow L_H = 40^k$

$\Sigma F_y = 0 = 35^k - 25^k - L_V \Rightarrow L_V = 10^k$

So $L = \sqrt{L_V^2 + L_H^2} = \sqrt{10^2 + 40^2}$

$\underline{\underline{L = 41.2^k}}$

FBD JOINT Ⓔ:



$\Sigma F_x = 0 = M_H - 40^k \Rightarrow M_H = 40^k$

$\Sigma F_y = 0 = 10^k - 40^k + M_V \Rightarrow M_V = 30^k$

So $M = \sqrt{M_V^2 + M_H^2} = \sqrt{30^2 + 40^2}$

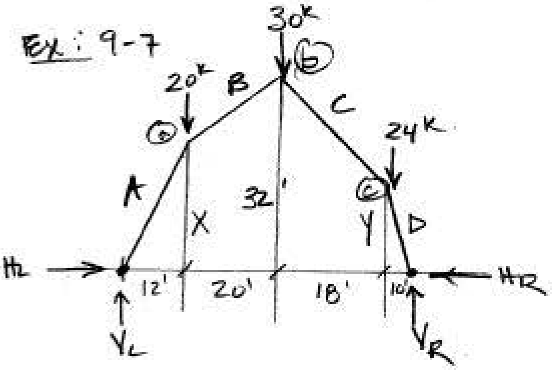
$\underline{\underline{M = 50^k}}$ ✓ CHECKS W/ ORIGINAL ANSWER ON PREVIOUS PAGE.

IDEAL COMPRESSION ARCHES.

EXACTLY SAME AS CABLES EXCEPT MIRROR IMAGE:

CABLES — TENSION ONLY
IDEAL ARCHES — COMPRESSION ONLY] BOTH HAVE NO MOMENTS ANYWHERE.

EX: 9-7



FIND: MEMBER FORCE: A, B.
DIMENSIONS X, Y

TREAT THIS PROBLEM EXACTLY LIKE THE CABLE PROBLEM.

STEP 1: FIND VERTICAL REACTIONS — MUST USE EQUILIBRIUM EQUATIONS.

$$\Sigma M_{@L} = 0 = 20^k(12') + 30^k(32') + 24^k(50') - V_R(60') \Rightarrow \boxed{V_R = 40^k}$$

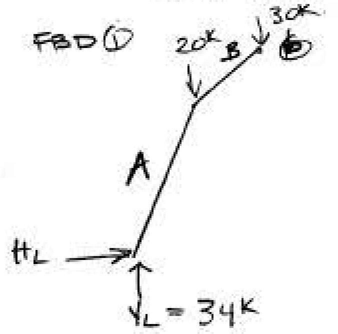
$$\Sigma F_y = 0 = V_L - 20^k - 30^k - 24^k + \underset{40^k}{\cancel{V_R}} \Rightarrow \boxed{V_L = 34^k}$$

STEP 2: FIND HORIZONTAL REACTIONS — USING METHOD 1.

METHOD 1: EQUILIBRIUM EQUATIONS — YOU KNOW THAT THERE IS NO MOMENT ANYWHERE IN THE ARCH — COMPRESSION FORCES ONLY!

JUST LIKE METHOD OF SECTIONS IN TRUSSES, CUT ARCH & DRAW FBD.

FBD①



$$\Sigma M_{@P} = 0 = -20^k(20') - H_L(32') + 34^k(32')$$

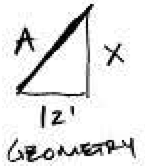
$$\boxed{H_L = 21.5^k}$$

$$\Sigma F_x = 0 = \underset{21.5^k}{\cancel{H_L}} - H_R \Rightarrow \boxed{H_R = 21.5^k}$$

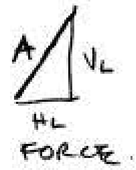
↑ BASED ON ENTIRE STRUCTURE

NOTE: THE FBD WAS TAKEN @ ⓑ BECAUSE IF TAKEN @ ⓐ, YOU WOULD BE UNABLE TO SOLVE IT SINCE THE DISTANCE "X" IS UNKNOWN.

STEP 3: FIND DIMENSIONS "X" + "Y" — USE SIMILAR TRIANGLES. (METHOD 2 FOR FINDING HORIZONTAL REACTIONS).

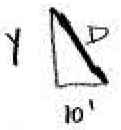


GEOMETRY



FORCE.

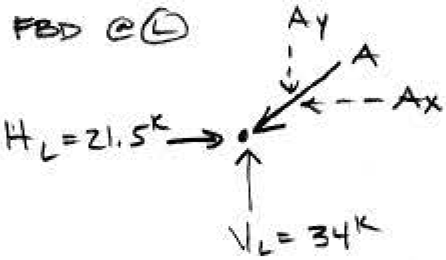
$$\frac{X}{34} = \frac{12'}{21.5^k} \Rightarrow \boxed{X = 18.98'}$$



$$\frac{Y}{40} = \frac{10'}{21.5^k} \Rightarrow \boxed{Y = 18.60'}$$

STEP 4: FIND MEMBER FORCES — USE FBDS SIMILAR TO METHOD OF JOINTS IN TRUSSES.

FBD @ Ⓛ

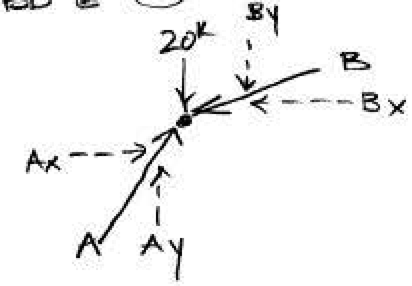


$A = \sqrt{34^2 + 21.5^2} = 40.22^k$

$A_x = 21.5^k$
$A_y = 34^k$

$\boxed{A = 40.22^k}$

FBD @ ⓐ



$\Sigma F_x = 0 = B_x - A_x$

$0 = B_x - 21.5^k \Rightarrow$ $B_x = 21.5^k$

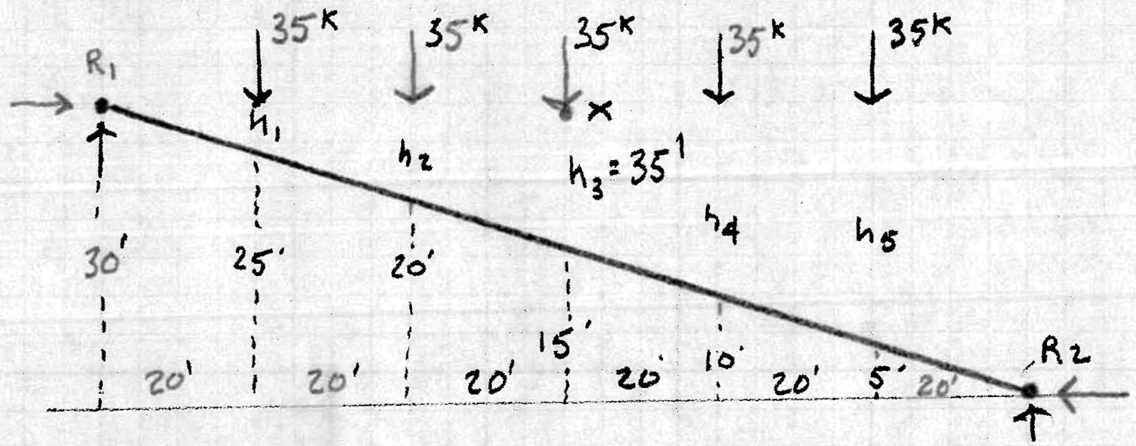
$\Sigma F_y = 0 = A_y - 20^k - B_y$

$0 = 34 - 20^k - B_y \Rightarrow$ $B_y = 14^k$

$B = \sqrt{14^2 + 21.5^2} \Longrightarrow$ $\boxed{B = 25.65^k}$

REPEAT SAME PROCEDURE TO FIND FORCES IN C ∳ D.

ARCH 331 | PROBLEM 9-9 b | 2 NOV 91 PvB



FIND REACTIONS:

$\Sigma M_{R_1} = 0 = 35(20) + 35(40) + 35(60) + 35(80) + 35(100) - V_R(120) + H(30)$

$V_R(120) = 10500 + H(30)$

$V_R = 87.5^K + 0.25H$

ΣM_x FOR FBD OF RIGHT HALF ONLY

$\Sigma M_x = 0 = 35(20) + 35(40) - V_R(60) + H(35+15)$

$V_R(60) = 2100 + H(50)$

$V_R = 35 + 0.8333H$

$35 + .8333H = 87.5 + 0.25H$

$0.5833H = 52.5$

$\underline{H = 90^K}$ $\Sigma F_H = 0 \therefore R_1H = R_2H = H = 90^K$

$V_R = 87.5 + 0.25H$

$= 87.5 + 22.5$

$\underline{V_R = 110^K}$

$\Sigma F_V = 0 = V_L - 35(5) + 110$

$V_L = 65^K$

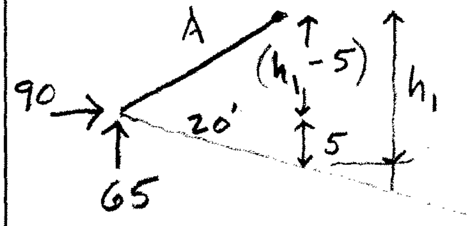
CHECK $\Sigma M_x = 0$ FOR WHOLE SYSTEM (NOTE SYMETRIC 35^K LOADS CANCEL)

$-90(20) + 65(60) + 90(50) - 110(60) = 0$ ✓

FIND $h_1 - h_5$

FBD ①

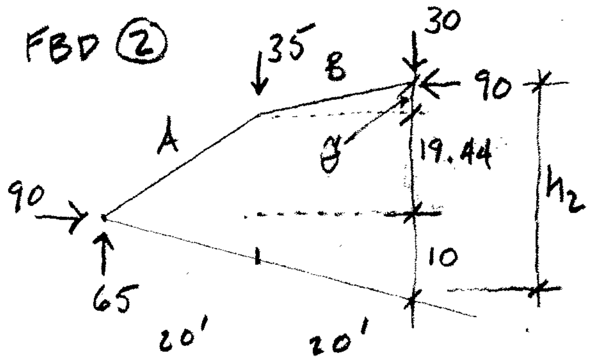
$\frac{90}{20} \quad \frac{65}{(h_1-5)} \quad (h_1-5) = 14.44$



$h_1 = 19.44'$

FORCE A $= \sqrt{90^2+65^2} = 111.02^K$

FBD ②



$\Sigma F_H = 90 - B_H$

$B_H = 90$

$\Sigma F_V = 65 - 35 - B_V$

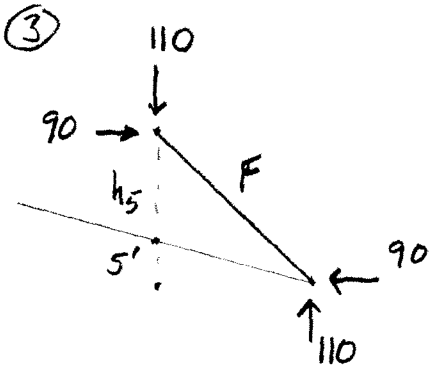
$B_V = 30$

$B = \sqrt{90^2+30^2} = 94.87^K$

$\frac{30}{y} \quad \frac{90}{20} \qquad y = 6.67'$

$h_2 = 6.67 + 19.44 + 10 = 36.12'$

FBD ③

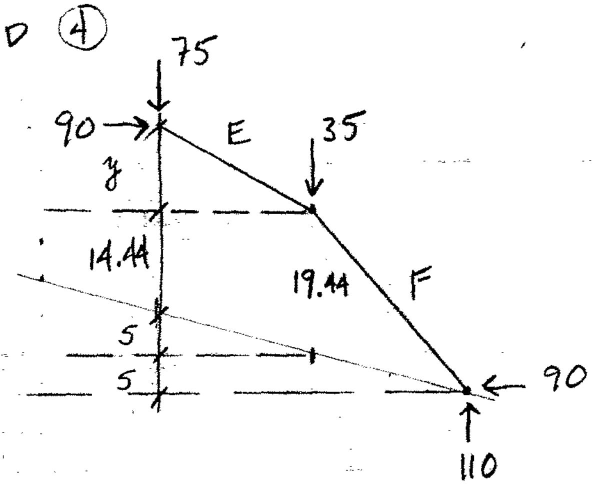


$\frac{90}{20} : \frac{110}{h_5+5} \qquad h_5 + 5 = 24.44$

$h_5 = 19.44$

$F = \sqrt{90^2+110^2} = 142.13^K$

FBD ④



$\Sigma F_H = -90 + E_H$

$E_H = 90$

$\Sigma F_V = 110 - 35 - E_V$

$E_V = 75$

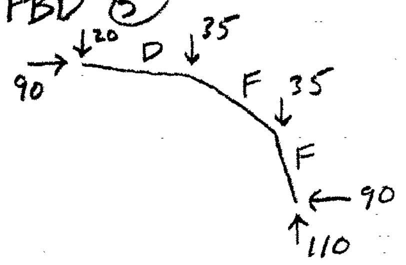
$E = \sqrt{90^2 + 75^2} = 117.15^K$

$\frac{75}{y}$ $\frac{90}{20}$ $y = 16.67'$

$h_4 = 14.44 + 16.47 = 31.12'$

FORCES IN REMAINING MEMBERS (BY CUTTING MEMBERS)

FBD ⑤



$\Sigma F_V = 0 = 110 - 35 - 35 - D_V$

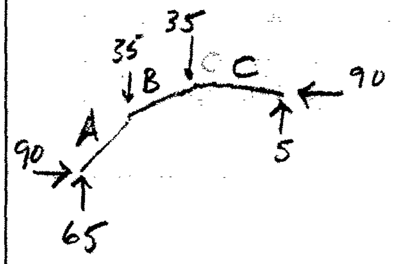
$D_V = 40$

$\Sigma F_H = 0 = -90 + D_H$

$D_H = 90$

$D = \sqrt{90^2 + 40^2} = 98.49^K$

FBD ⑥



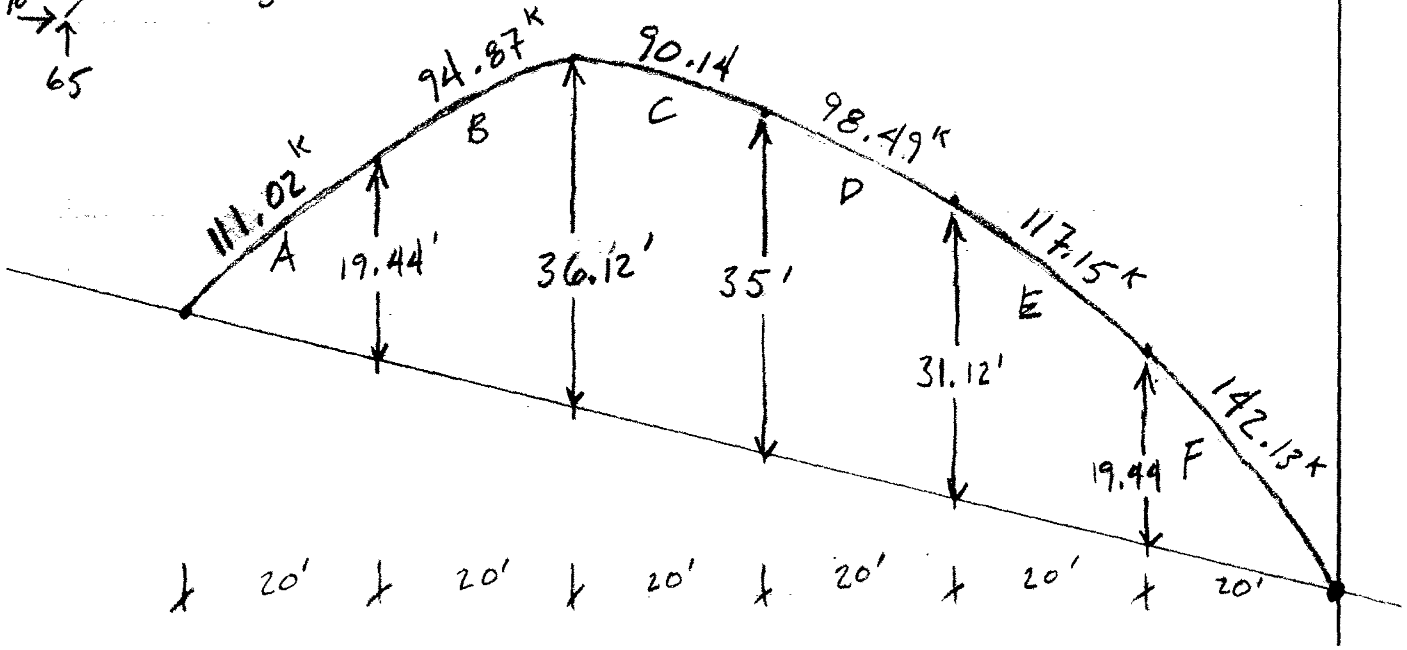
$\Sigma F_V = 0 = 65 - 35 - 35 + C_V$

$C_V = 5$

$\Sigma F_H = 0 = 90 - C_H$

$C_H = 90$

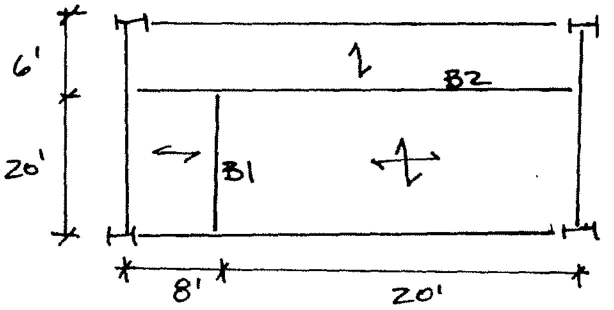
$C = \sqrt{90^2 + 5^2} = 90.14^K$



10.01.03
SR.

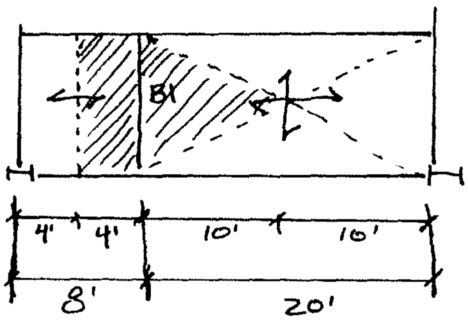
PROBLEM 5-2

(FROM TEXT BOOK)

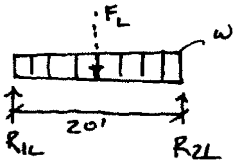


LIVE LOAD = 75 PSF
DEAD LOAD = 75 PSF
150 PSF

B1 SEES DISTRIBUTED LOADS FROM BOTH SIDES (i.e. LEFT SIDE & RIGHT SIDE)



$$\boxed{\begin{array}{l} R_1 = 13.5^K \\ R_2 = 13.5^K \end{array}}$$



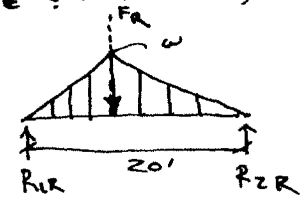
LEFT

$w_L = 150\text{PSF}\left(\frac{8}{2}\right) = 600\text{ PLF}$

$F_L = l \times w$ (AREA OF RECTANGLE)

$F_L = 20'(600\text{PLF}) = 12^K$

$R_{1L} = R_{2L} = \frac{12^K}{2} = \underline{\underline{6^K}}$



RIGHT

$w_R = 150\left(\frac{20}{2}\right) = 1500\text{ PLF}$

$F_R = \frac{1}{2} l \times w$ (AREA OF TRIANGLES)

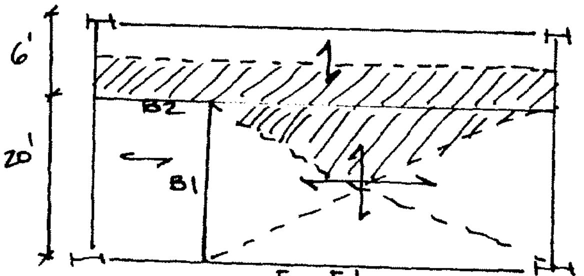
$F_R = \frac{1}{2}(20)(1500) = 15^K$

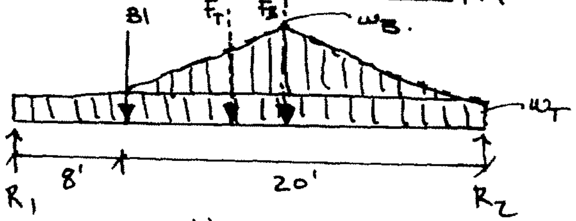
$R_{1R} = R_{2R} = \frac{15}{2} = \underline{\underline{7.5^K}}$

COMBINE THE REACTIONS FOR EACH LOADING.

$$R_1 = R_2 = 6^K + 7.5^K = \boxed{13.5^K}$$

B2 SEES DISTRIBUTED LOADS FROM BOTH SIDES (i.e. TOP & BOTTOM AS WELL AS THE POINT LOAD FROM BEAM B1.





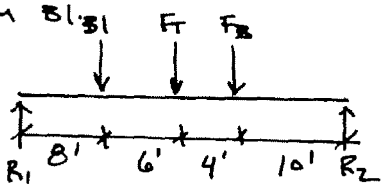
$w_T = 150\text{PSF}\left(\frac{6}{2}\right) = 450\text{ PLF}$

$w_B = 150\text{ PSF}\left(\frac{20}{2}\right) = 1500\text{ PLF}$

$B1 = 13.5^K$ (FROM ABOVE)

$F_T = l \times w = 28'(450\text{ PLF}) = 12.6^K$

$F_B = \frac{1}{2} l \times w = \frac{1}{2} 20'(1500\text{ PLF}) = 15^K$



$B1 = 13.5^K$

$F_T = 12.6^K$

$F_B = 15^K$

$\Sigma F_y = 0 = R_1 - 13.5 - 12.6 - 15 + R_2 = 0$

$R_1 = 41.1^K - R_2$

$R_1 = 41.1 - 19.8 = \underline{21.3^K}$

$\Sigma M_1 = 0 = 8'(13.5^K) + 14(12.6) + 18(15) - 28R_2 = 0$

$108 + 176.4 + 270 = 28R_2$

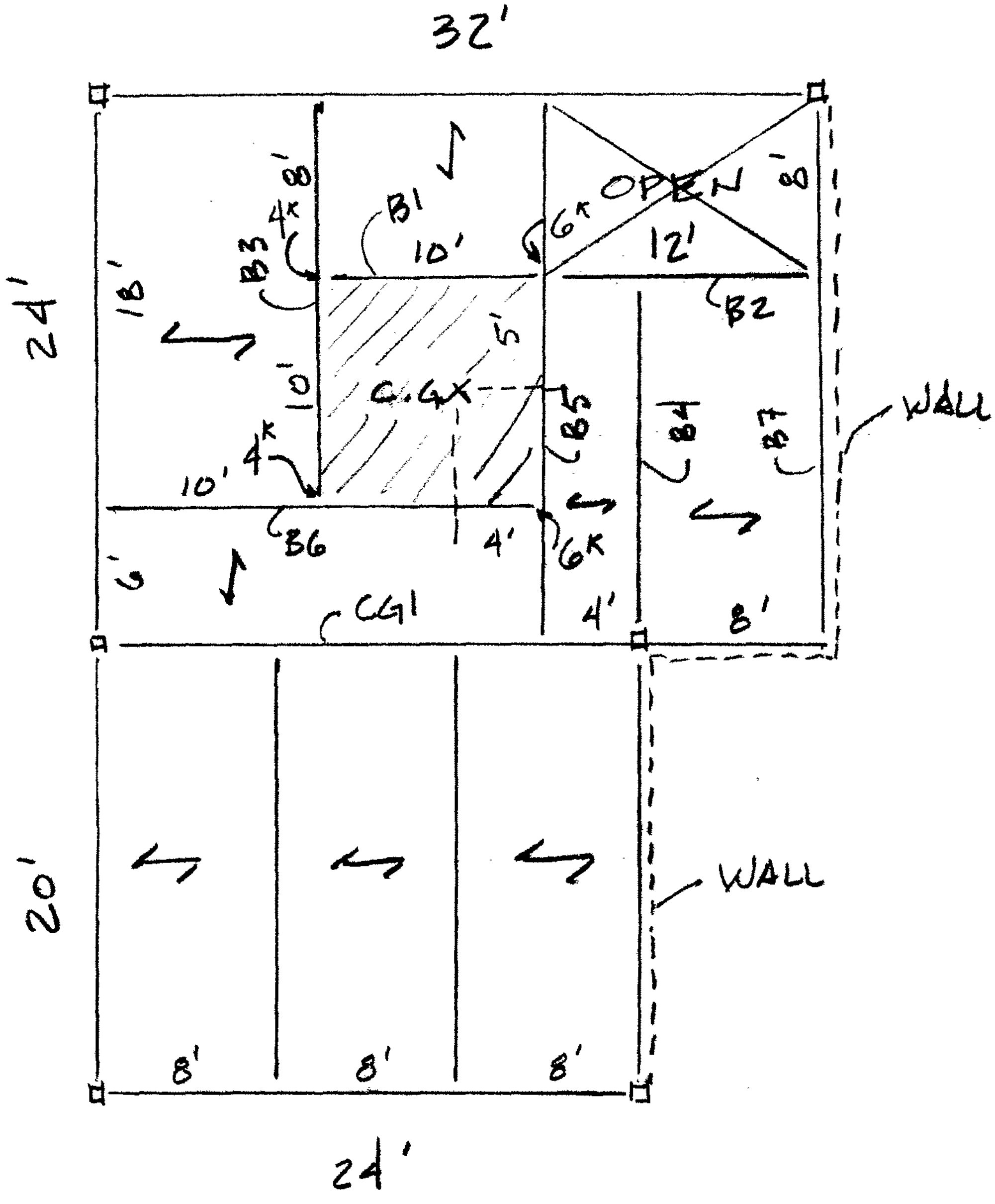
$554.4 = 28R_2$

$R_2 = \underline{\underline{19.8^K}}$

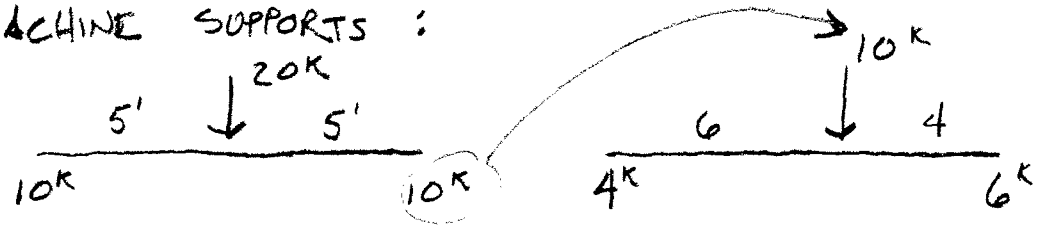
$$\boxed{\begin{array}{l} R_1 = 21.3^K \\ R_2 = 19.8^K \end{array}}$$

ARCH 331 | PROBLEM 5.3 | PVB

FRAMING PLAN



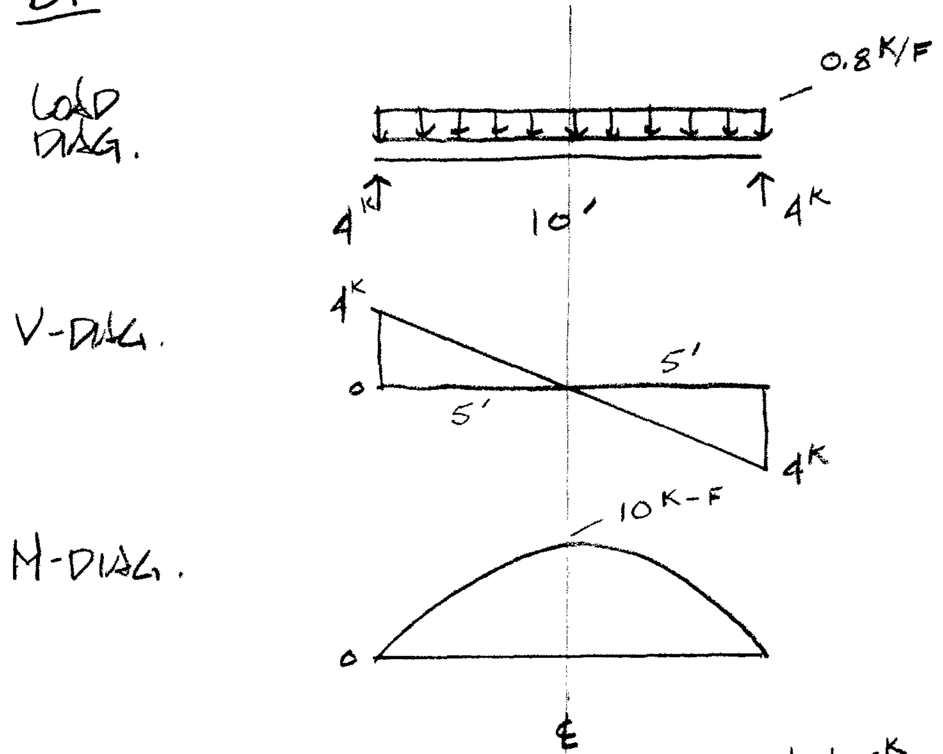
MACHINE SUPPORTS :



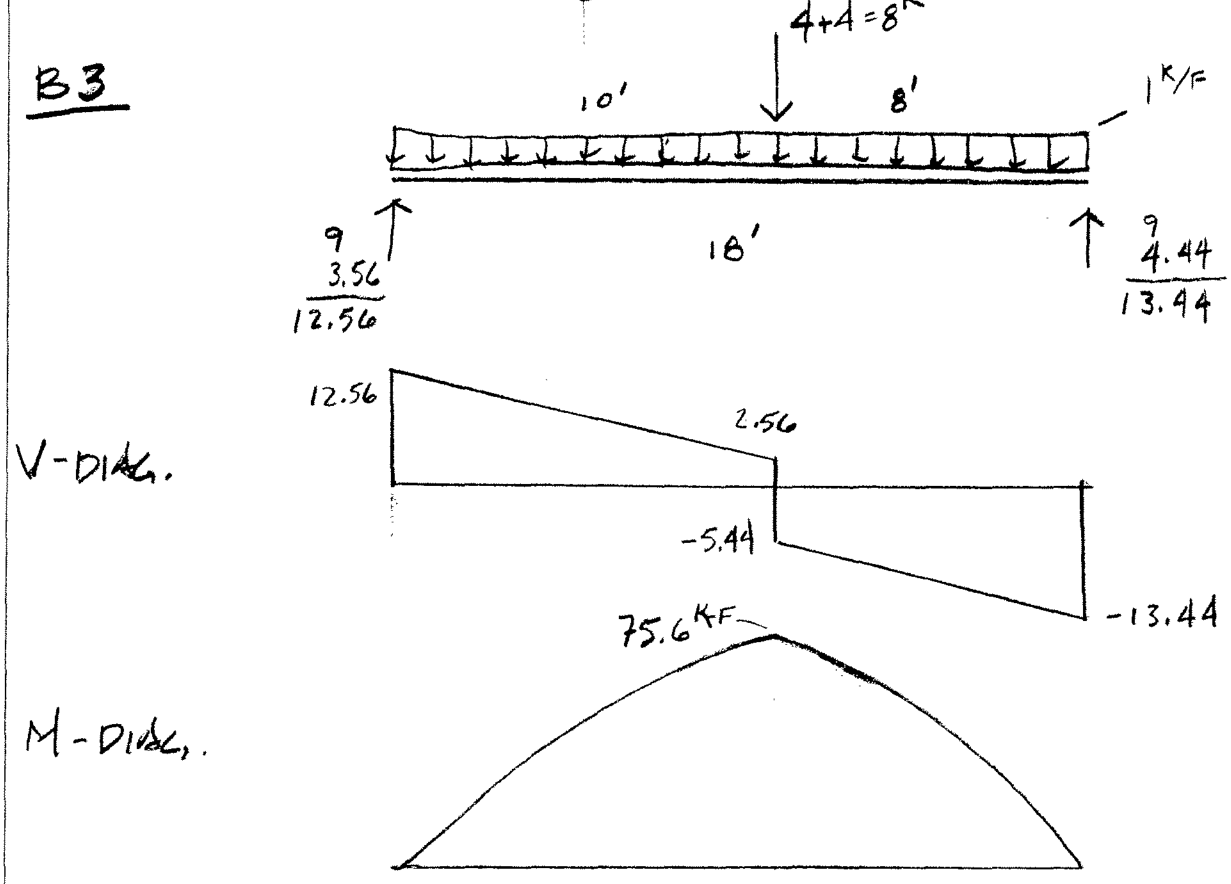
ON SIDES

ON CORNERS

B1


LOAD DIAG.
0.8 K/F
4K
10'
4K
V-DIAG.
4K
0
5'
5'
4K
M-DIAG.
10 K-F
0
℄

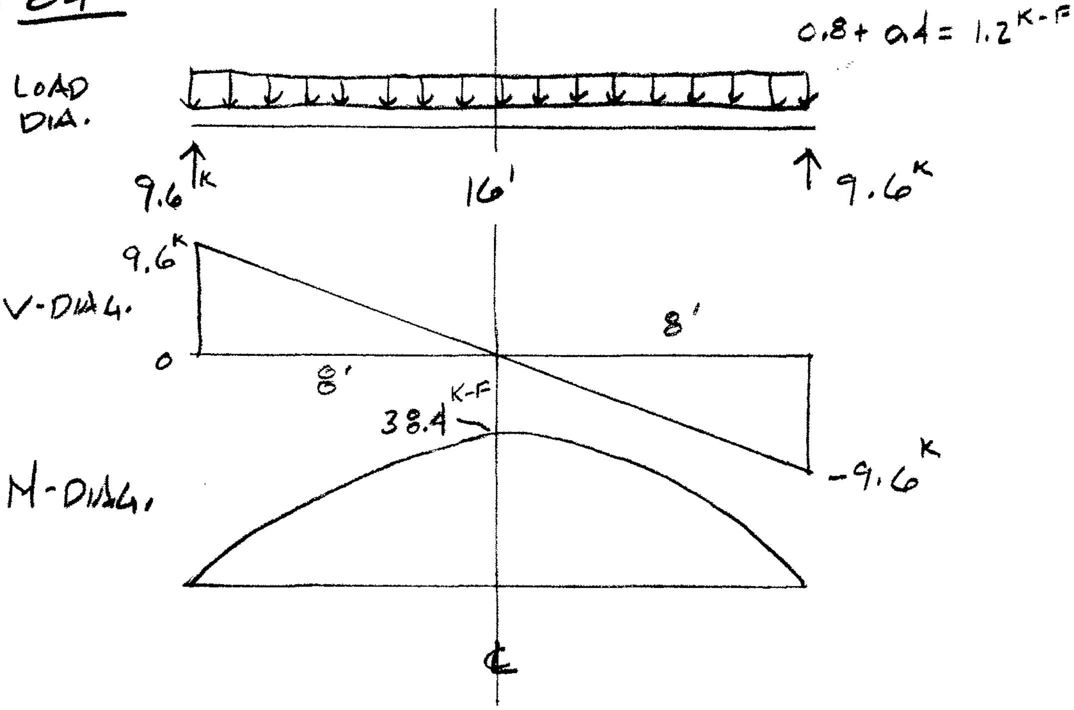
B3


4+4=8K
10'
8'
1 K/F
9
3.56
12.56
18'
9
4.44
13.44
V-DIAG.
12.56
2.56
-5.44
-13.44
M-DIAG.
75.6 KF

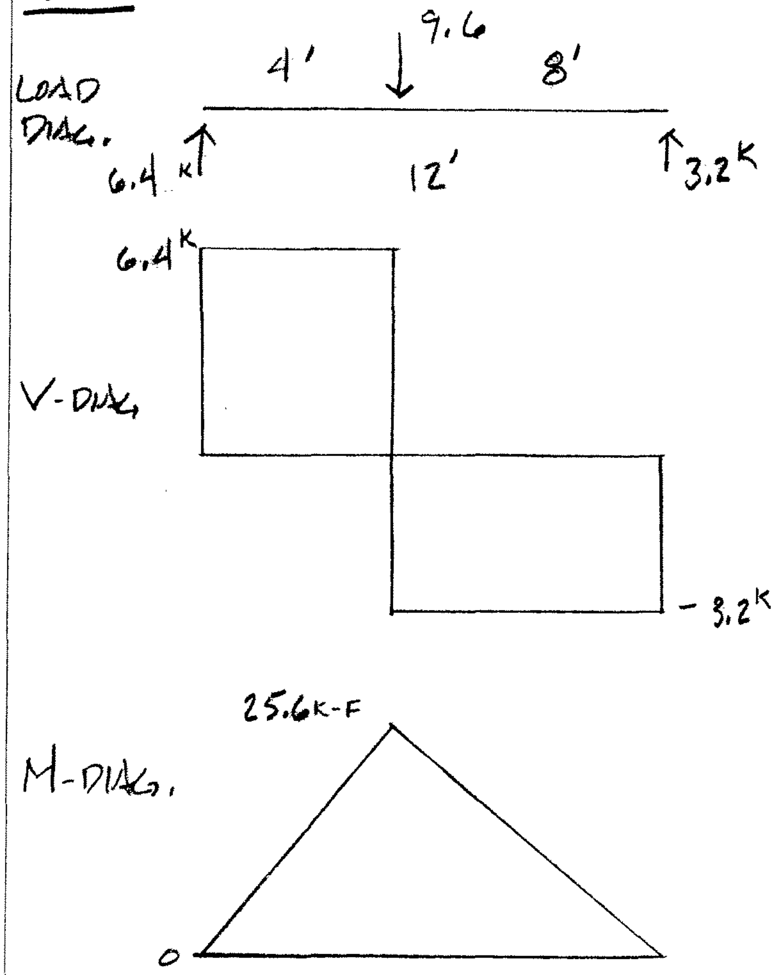
Hw 5.3 CONT

RJB

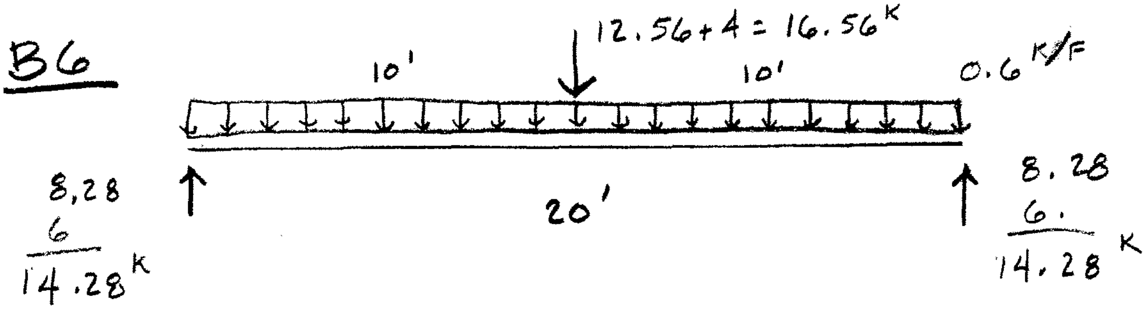
B4

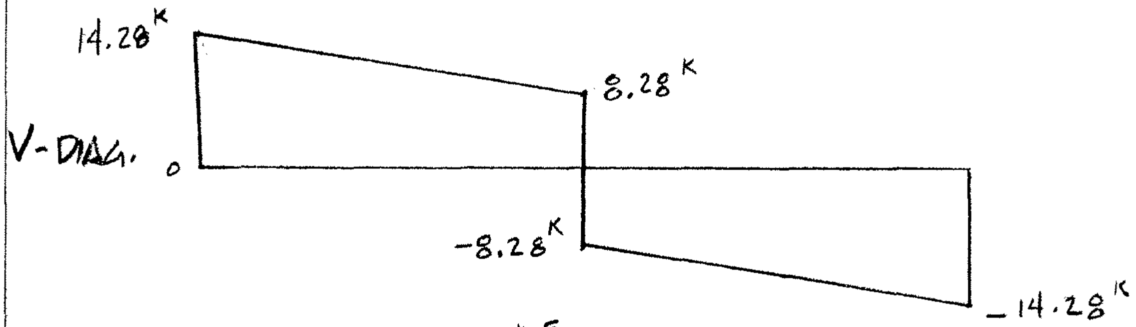


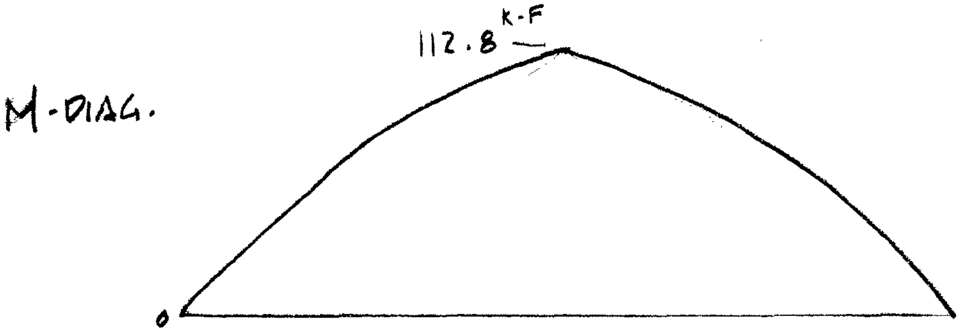
B2



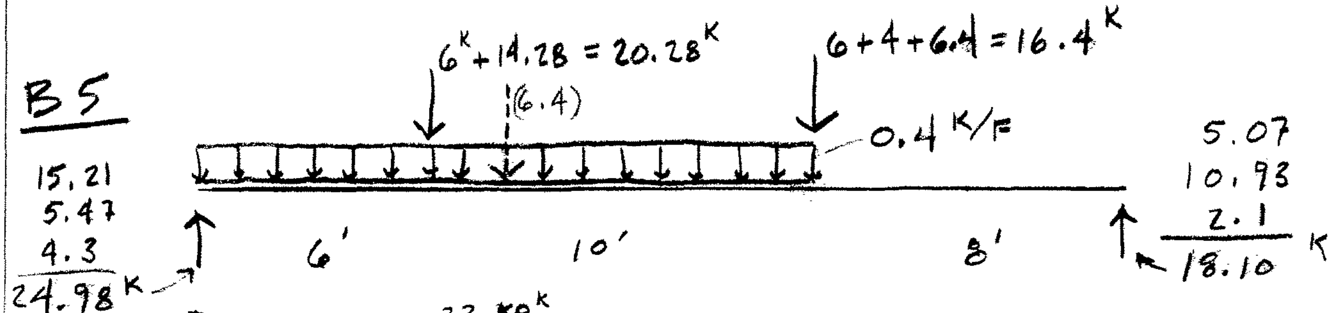
B6

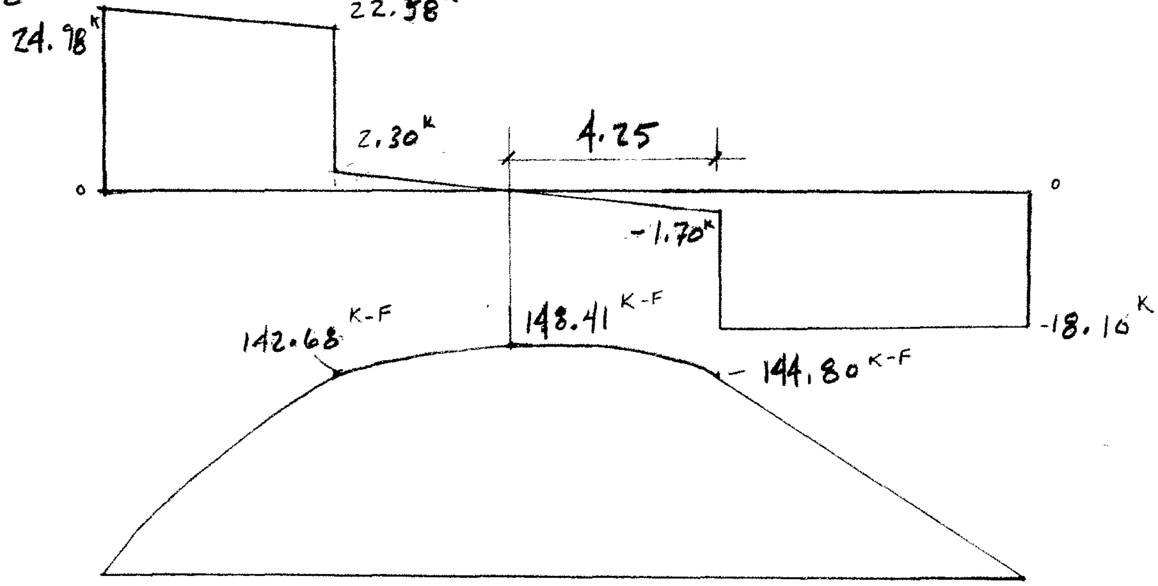

12.56+4 = 16.56^K
10'
10'
0.6 K/F
8,28
6
14.28^K
20'
8.28
6.
14.28 K


14.28^K
8.28^K
V-DIAG.
0
-8.28^K
-14.28^K

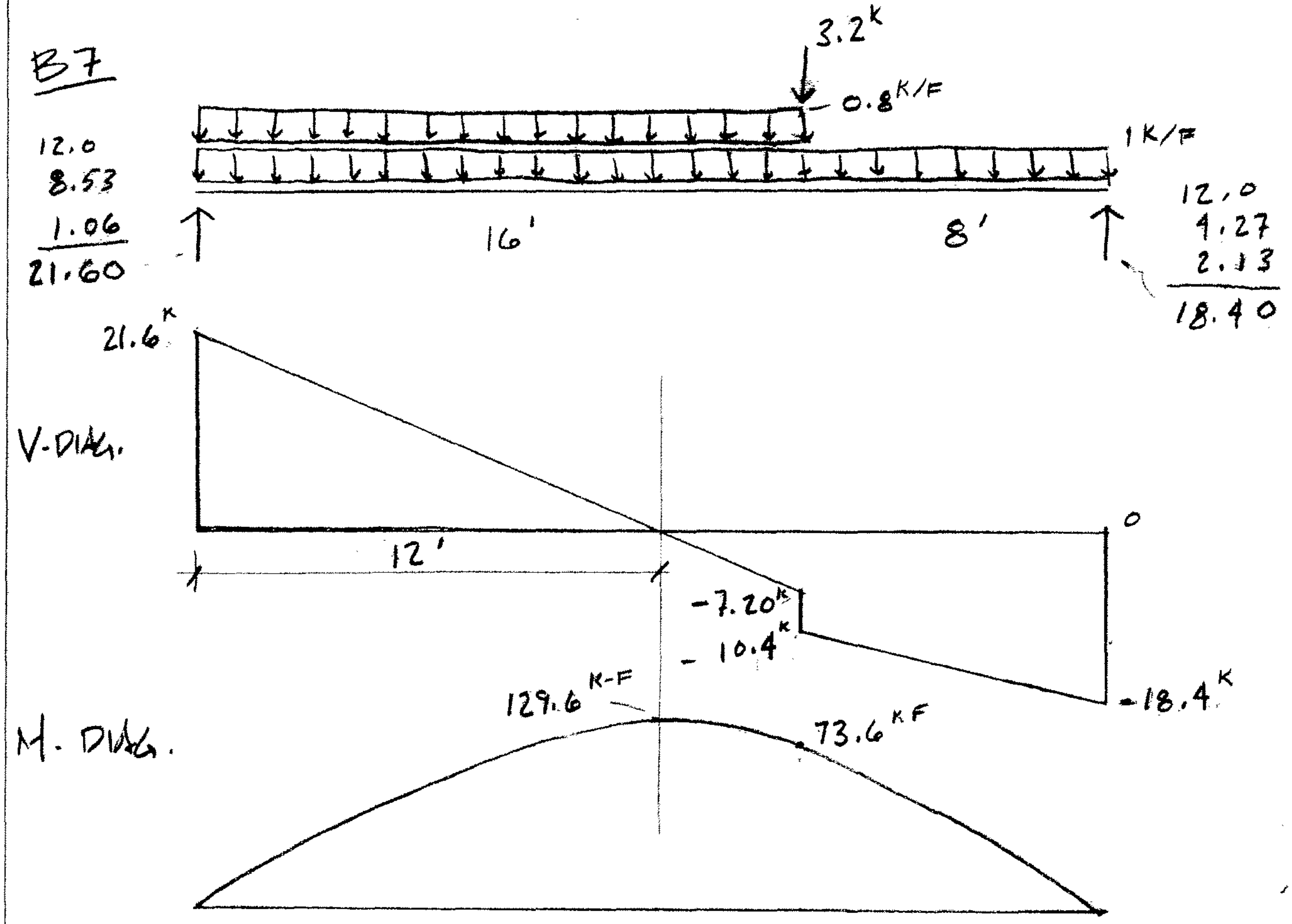

112.8^K-F
M-DIAG.
0

B5


6^K+14.28 = 20.28^K
(6.4)
6+4+6.4 = 16.4^K
0.4 K/F
15.21
5.47
4.3
24.98^K
6'
10'
8'
5.07
10.93
2.1
18.10 K


24.98^K
22.98^K
2.30^K
4.25
0
0
-1.70^K
-18.10^K
142.68^K-F
148.41^K-F
144.80^K-F

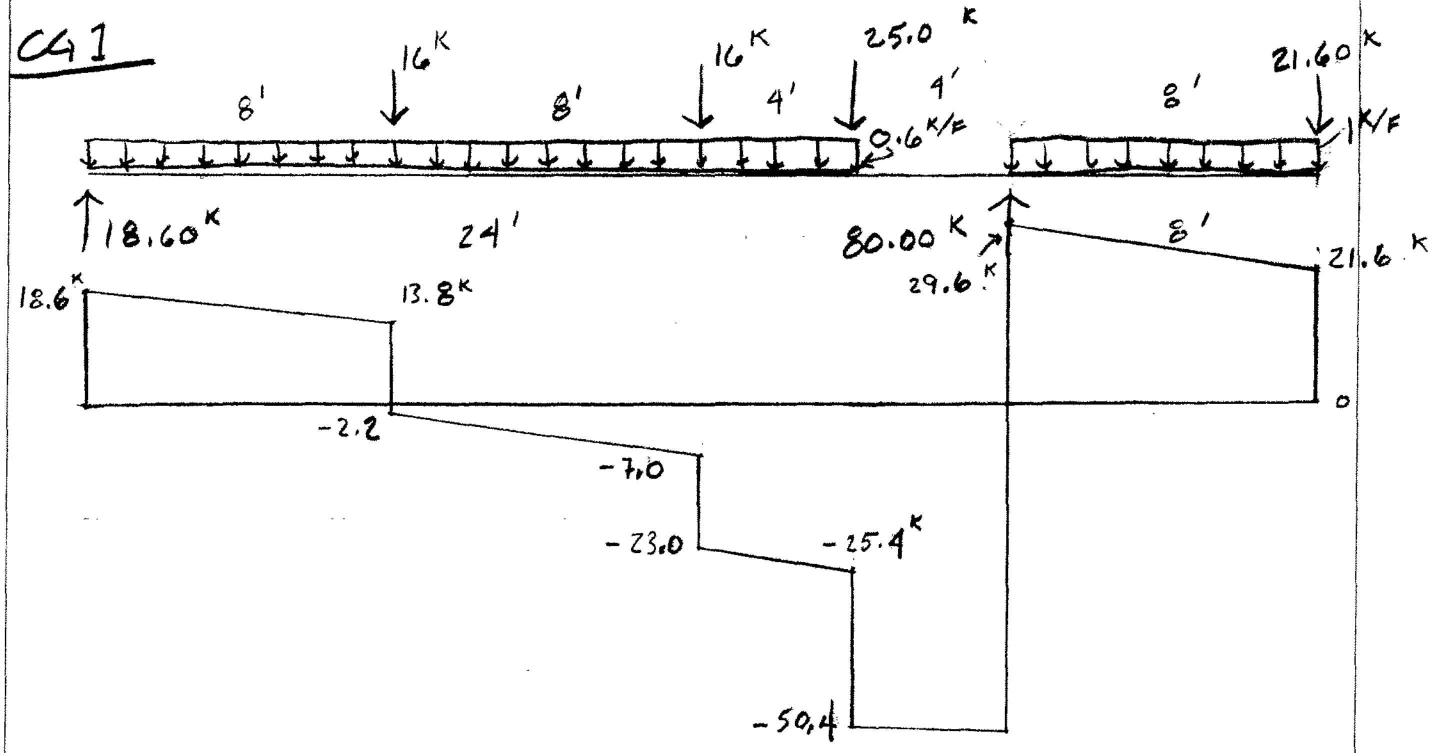
B7



12.0
8.53
1.06
21.60

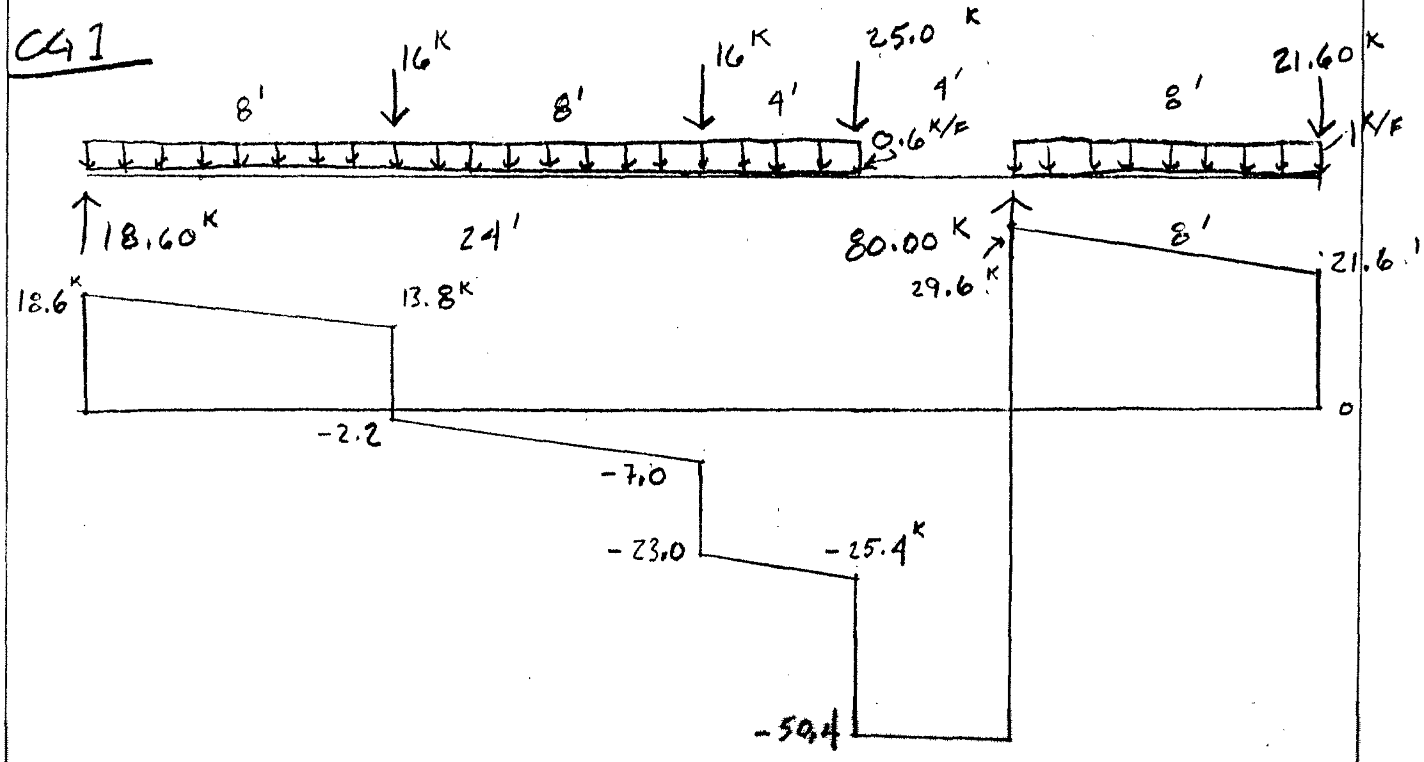
12.0
4.27
2.13
18.40

CG1

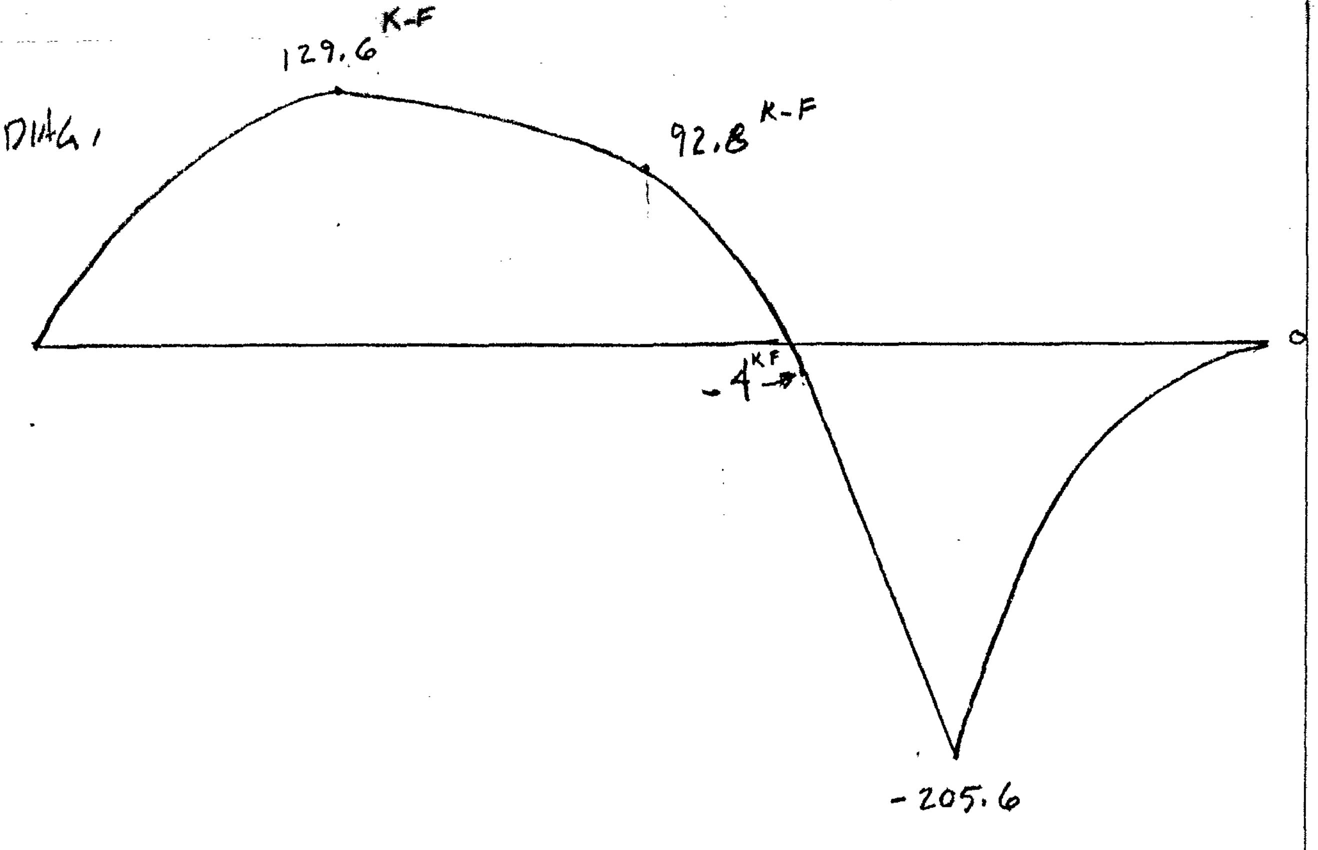


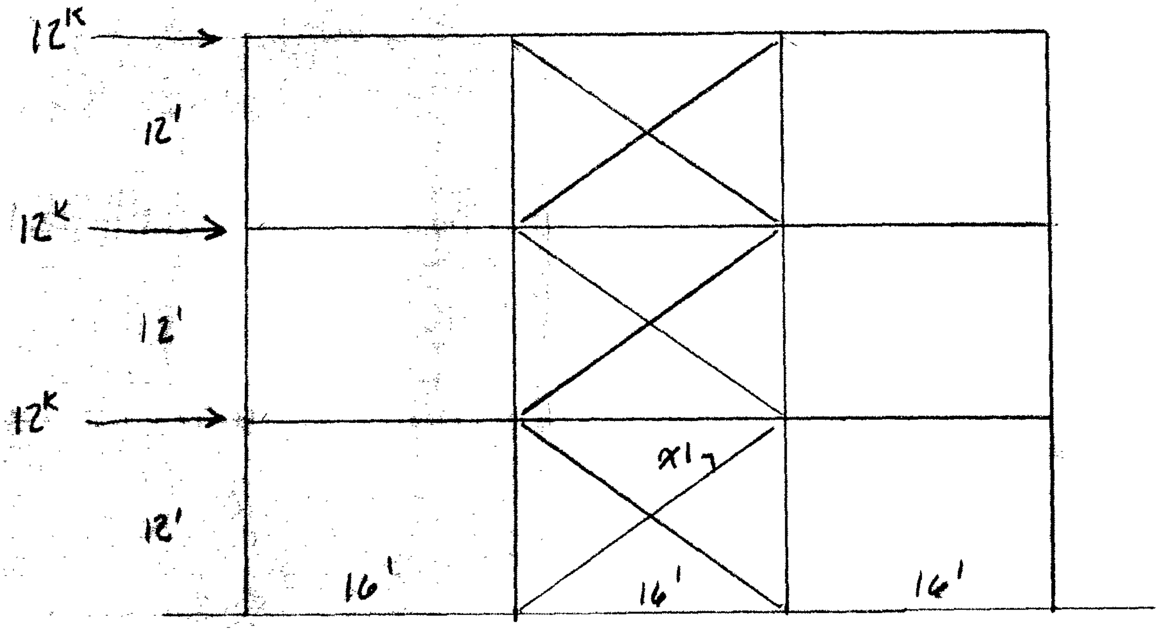
HW 5.3

CG1



M-DIAG.

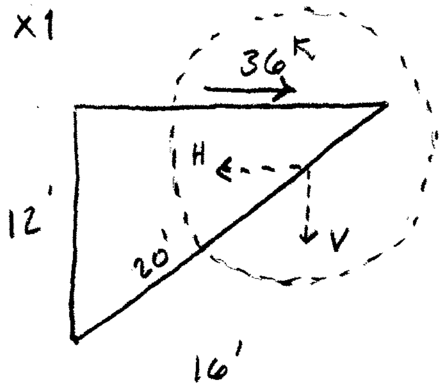




DETERMINE FORCE IN X1

FORCE ABOVE X1 :
$12 + 12 + 12 = 36^K$

FBD X1



$\Sigma F_H = 0 = 36 - H = 0$
$H = 36^K$

$\frac{H}{16} : \frac{X1}{20}$ $X1 = 45^K$

CHECK :

$\frac{V}{12} : \frac{45}{20}$ $V = 27^K$

$\sqrt{36^2 + 27^2} = 45$ ✓

STRESS vs. STRAIN

TENSION TEST:

TEST DATA

ε "/"	D"	P #	f KSI
.00035	.0021	1,960	10.00
.00070	.0041	3,920	20.00
.00105	.0063	5,880	30.00
.00140	.0084	7,840	40.00
.00175	.0105	9,800	50.00
.00210	.0126	11,760	60.00
.00250	.0150	12,050	61.48
.00300	.0180	12,030	61.38
.00500	.0300	12,040	61.43
.00800	.0480	12,060	61.53
.01000	.0600	12,050	61.48
.01500	.0900	13,100	66.84
.02000	.1200	14,100	71.94
.02500	.1500	15,050	76.79
.03000	.1800	16,000	81.63
.03500	.2100	16,900	86.22

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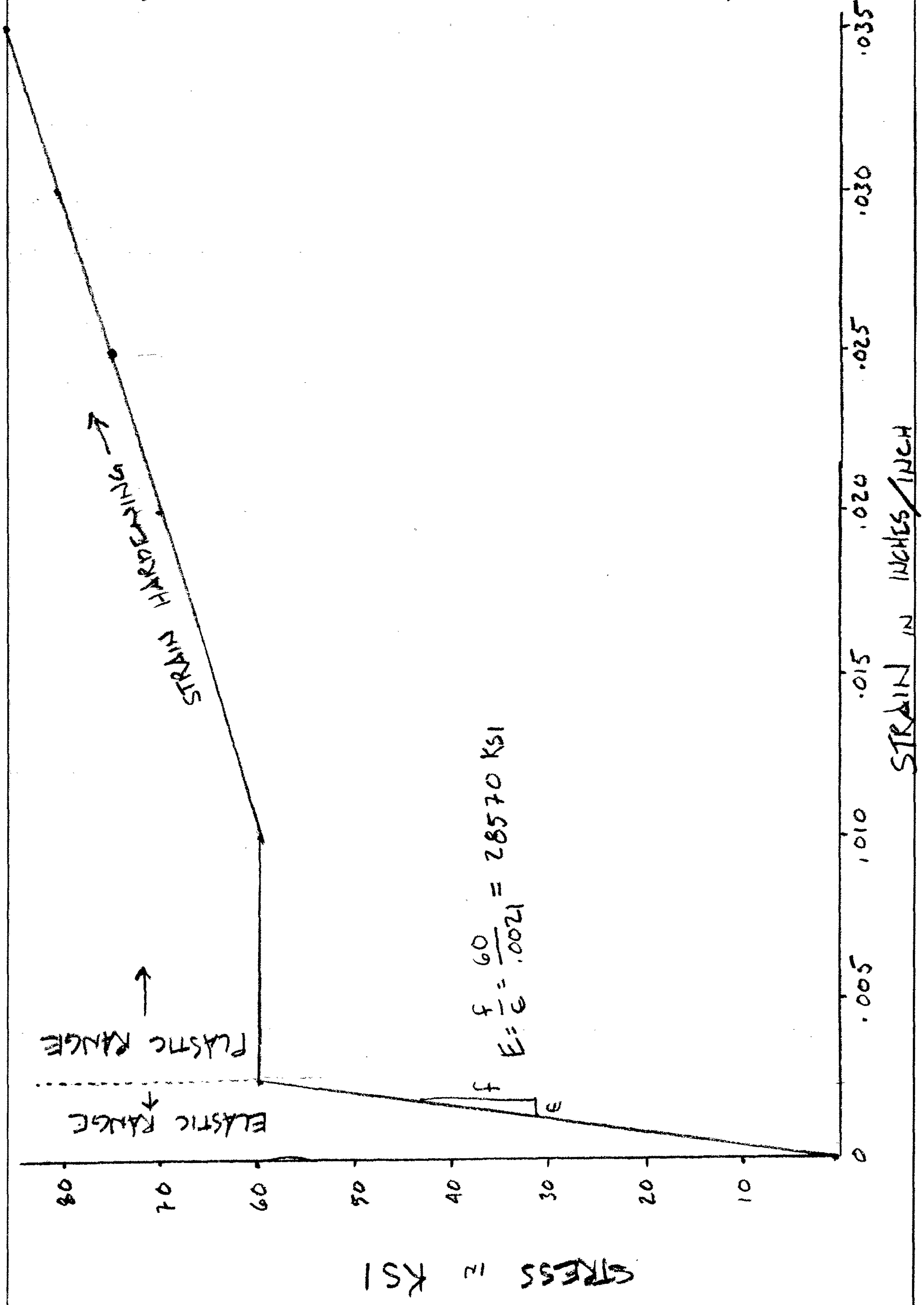
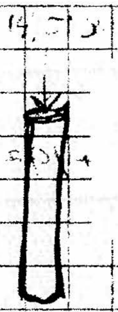


PROB 7-2

1. 75 stories
13'-4" per story



$f = 14{,}500$ psi

$E = \dfrac{f}{\epsilon}$

$29{,}000{,}000 \text{ psi} = \dfrac{14{,}500 \text{ psi}}{\epsilon}$

$\epsilon = 5 \times 10^{-4}$ "/"

1000' total - .5' deformation = $\boxed{999.5'}$

$\epsilon = \dfrac{D}{L}$ $\quad 5 \times 10^{-4}$ "/" $= \dfrac{D}{(1000')(12"/')}$

(deformation) $D = .5'$

2. 750' total

a. S1 grade: yield strength = 36 k.s.i.

stress, $f = \frac{1}{2}$ 36 k.s.i = 18 k.s.i

$\epsilon = \dfrac{f}{E} = \dfrac{18 \text{ k.s.i.}}{29{,}000 \text{ k.s.i}}$

$= 6.21 \times 10^{-4}$ "/"

total loss = $D = L * \epsilon$

$= (750')(6.21 \times 10^{-4}$ "/"$)$

$= .46575' (12"/')$

$= 5.6"$

b. S2 grade: yield strength = 50 k.s.i.

stress, $f = \frac{1}{2}$ 50 ksi = 25 k.s

$\epsilon = \dfrac{f}{E} = \dfrac{25 \text{ k.s.i.}}{29{,}000 \text{ ks.i}}$

$= 8.62 \times 10^{-4}$

total loss = $D = L * \epsilon$

$= (750')(8.62 \times 10^{-4})$

$= .6465' (12"/')$

$= 7.76"$

c. S3 grade: yield strength = 100 k.s.i

stress, $f = \frac{1}{2}(100 \text{ k.s.i}) = 50$ ksi

$\epsilon = \dfrac{f}{E} = \dfrac{50 \text{ k.s.i.}}{29{,}000 \text{ k.s.i.}}$

$= .00173$ "/"

total loss = $D = L * \epsilon$

$= (750')(.00173$ "/"$)$

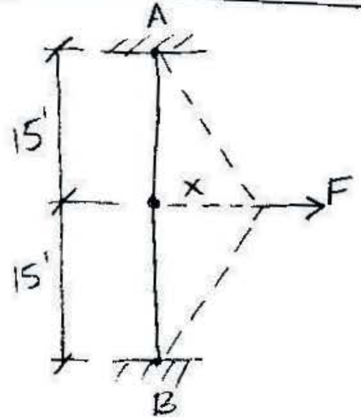
$= 1.29' (12"/')$

$= 15.52"$

PROBLEM # 7-2 (3)

KLP 11/9/04

ARCH 314



7-WIRE STRAND CABLE

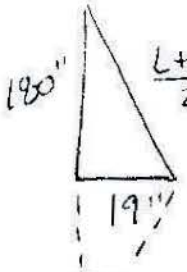
CROSS-SECTIONAL AREA = 0.0799 in²

DETERMINE TENSION, P

AND FORCE, F

a) $X = 19''$

$L = 30' (12''/') = 360''$



$\frac{L+D}{2} = \sqrt{180^2 + 19^2} = 181''$ $\Rightarrow$ CHANGE IN LENGTH, $D = 2''$

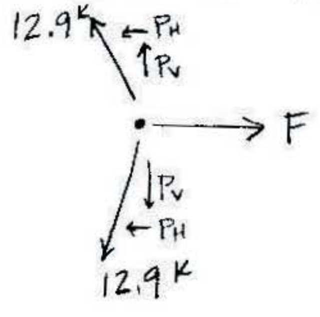
STRAIN: $\epsilon = \frac{D}{L} = \frac{2''}{360''} = .00556$

STRESS: $f = \epsilon * E = (.00556) * (29{,}000 \text{ ksi}) = \underline{161.1 \text{ ksi}}$

TENSILE FORCE: $P = f * A = (161.1 \text{ ksi}) * (0.0799 \text{ in}^2) = \underline{\underline{12.9^K}}$

APPLIED FORCE:

USE FBD OF POINT 'C':



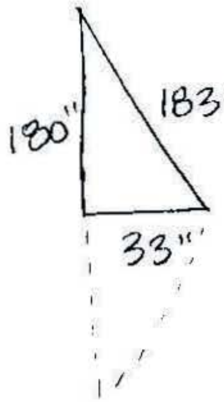
$P_H = \frac{19}{181}(12.9^k) = 1.35^K$

$\Sigma F_x = 0 = F - 2P_H$

$\Rightarrow \underline{\underline{F = 2.70^K}}$

PROBLEM # 7-2 (3) KLP 11/9/04 ARCH 314 2/2

b) $\underline{X = 33''}$



$183'' = \sqrt{180^2 + 33^2}$ $\Rightarrow$ CHANGE IN LENGTH, D = 6"

STRAIN: $\epsilon = \frac{D}{L} = \frac{6''}{360''} = 0.0167$

STRESS: $f = \epsilon * E = (0.0167) * (29{,}000 \text{ ksi}) = 483 \text{ ksi}$

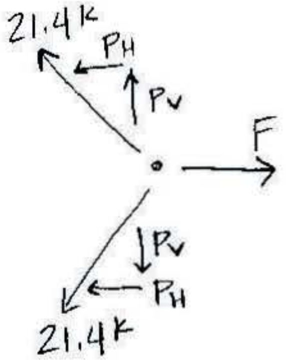
* BUT: FROM DATA SHEET D-7, WE KNOW THE ELASTIC LIMIT OF 7-WIRE STRAND IS 190 ksi.

FROM THE GRAPH ON D-7,
A STRAIN OF $\epsilon = 0.0167$
YIELDS A STRESS OF $f = 268$ ksi

TENSILE FORCE: $P = f * A = (268 \text{ ksi}) * (0.0799 \text{ in}^2) = \underline{21.4^K}$

APPLIED FORCE:

FBD OF 'C':



$P_H = \frac{33}{183}(21.4^K) = 3.86^K$

$\Sigma F_x = 0 = F - 2P_H$

$F = 2 * 3.86^K$

$\underline{F = 7.72^K}$

PROBLEM 7-3 (2)

KLP 11/9/04

ARCH 314

STEEL WIRE, 0.1" DIAMETER
STRETCHED @ 70°F BETWEEN FIXED SUPPORTS
TENSILE FORCE OF 40#

USE $E = 30 \times 10^6$ PSI
$C = .00000667 \text{ "/"/}{}^{\circ}\text{F}$

$A = \pi \frac{D^2}{4} = .00785 \text{ in}^2$

a) WHAT IS THE TENSION IN THE WIRE @ 30°F?

CHANGE IN STRESS: $f = E * C * \Delta t$

MATERIAL SHRINKS AS TEMP. DECREASES

$= (30 * 10^6 \text{ PSI})(.00000667 \text{ "/"/}{}^{\circ}\text{F})(70^{\circ} - 30^{\circ}\text{F})$

$= 8004 \text{ PSI}$

TENSION FORCE $= 40\# + (8004 \text{ PSI})(.00785 \text{ in}^2)$

$= \underline{\underline{102.9 \#}}$

b) AT WHAT TEMPERATURE WILL THE STRESS BE ZERO?

MATERIAL EXPANDS AS TEMP. INCREASES

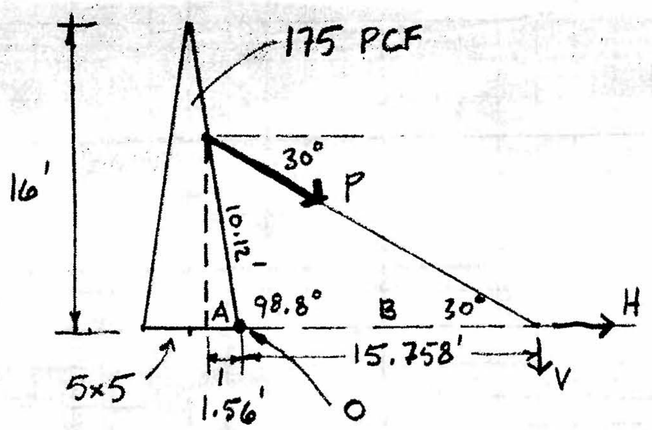
CHANGE IN STRESS: $f = E * C * \Delta t = 40\#/A$

$(30 * 10^6 \text{ PSI})(.00000667 \text{ "/"/}{}^{\circ}\text{F})(\Delta t) = 40\#/.00785 \text{ in}^2$

$\Delta t = 25.5^{\circ}\text{F}$

$T = 70^{\circ}\text{F} + 25.5^{\circ}\text{F}$

$= \underline{\underline{95.5^{\circ}\text{F}}}$



$$\text{VOLUME} = \frac{5 \times 5 \times 16}{3} = 133.33 \text{ CF}$$

$$\text{WEIGHT} = 133.33(175) = 23333.33 \text{ LBS}$$

C.G. 1/3 ABOVE BASE (TRIANGLE)

OVERTURNING MOMENT:
DEAD WEIGHT MOMENT

$$M_O = 23333.33\ (2.5) = 58333.33 \text{ FT-LBS}$$

SLOPE OF PYRAMID:

$$\frac{16}{2.5} : \frac{10}{A} \qquad A = 1.5625$$

$$\sqrt{1.5625^2 + 10^2} = 10.1213$$

$$\text{ANGLE AT BASE} = \text{ARCTAN } \frac{16}{2.5} = 81.12^\circ$$

SUPPLEMENT = 98.88°
COMPLIMENT = 8.88°

$$\angle b = 90^\circ - 30^\circ - 8.88^\circ = 51.12^\circ$$

$$\frac{\sin b}{B} = \frac{\sin 30^\circ}{10.1213} \qquad B = 15.758'$$

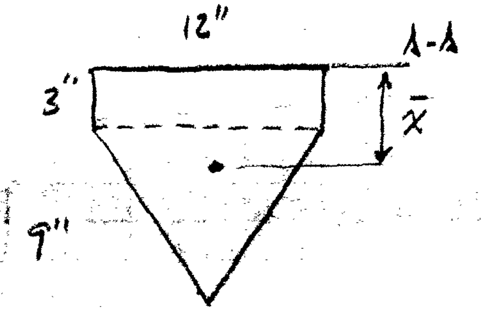
$$M_o = 58333.33 = V(15.758); \quad V = 3701.8 \text{ lbs} \downarrow$$

$$P = V/\sin 30^\circ = 3822.6(2) = \underline{7404 \text{ LBS}} \searrow 30^\circ$$

ARCH 331 | PROB 4.3(b, d, f) 4 SEP 1991 PVB

FOR EACH OF THE FOLLOWING DETERMINE THE LOCATION OF THE CENTROID $\bar{x}$, MEASURED FROM REF AXIS A-A

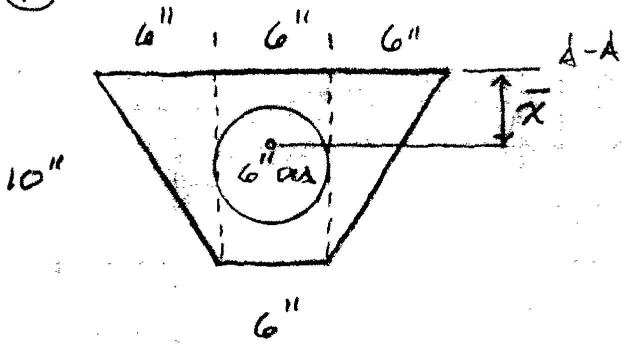
(B)



$$\frac{\Sigma Ad}{\Sigma A} = \frac{36(1.5)+54(6)}{36+54} = \bar{x}$$

$$\bar{x} = \frac{378}{90} = 4.2''$$

(D)

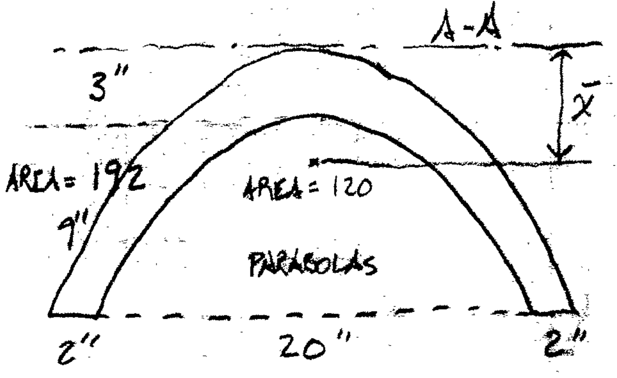


200 300 -141

$$\bar{x} = \frac{\Sigma Ad}{\Sigma A} = \frac{2(30)(\frac{10}{3}) + 60(5) - 28.27(5)}{30+30+60-28.27}$$

$$\bar{x} = \frac{358.63}{91.73} = 3.91''$$

(F)

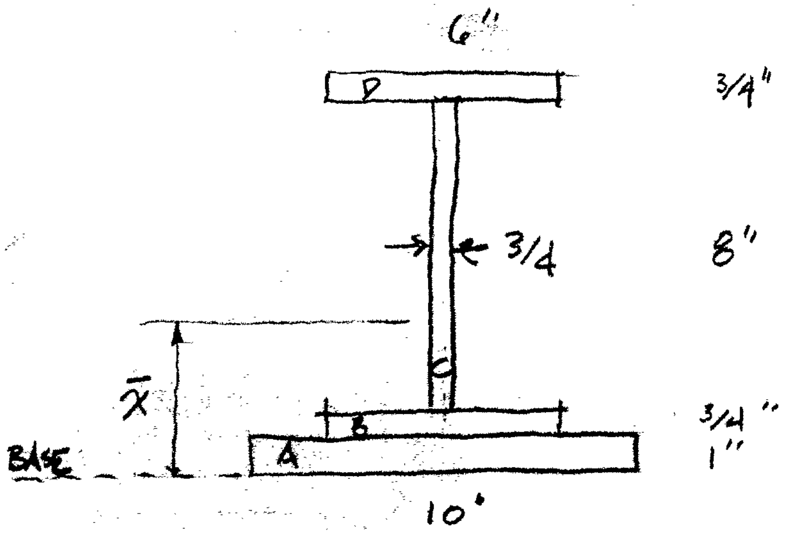


$$\bar{x} = \frac{\Sigma Ad}{\Sigma A} = \frac{192(\frac{3}{5}12) - 120(\frac{3}{5}9+3)}{192-120}$$

$$\bar{x} = \frac{1382.4-1008}{72} = 5.2''$$

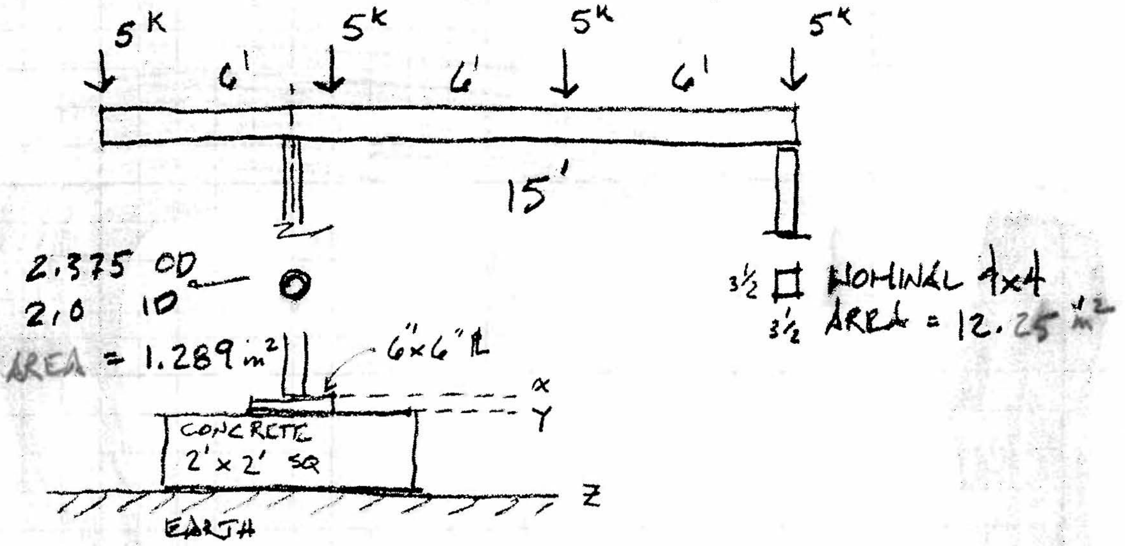
ARCH 331 | PROB 4.4 (e) | 4 SEP '91 PUB

DETERMINE THE LOCATION OF THE CENTROID $\bar{x}$ FROM BASE



SHAPE	AREA	d	Ad
A	10	.5	5
B	4.5	1.375	6.1875
C	6	5.75	34.5
D	4.5	10.125	45.5625
	$\Sigma A = 25$		$\Sigma Ad = 91.25$

$$\bar{x} = \frac{\Sigma Ad}{\Sigma A} = \frac{91.25}{25} = \underline{3.65''}$$



FOR THE SYSTEM SHOWN DETERMINE:

1. BEAM REACTIONS
2. UNIT STRESS IN 2" STEEL PIPE
3. UNIT STRESS IN 4x4 POST
4. UNIT STRESSES AT CONTACT AT X, Y AND Z

REACTIONS:

$\Sigma M_A = 0 = -5(3) + 5(3) + 5(9) + 5(15) - B(15)$

$\underline{B = 8^k \uparrow}$

$\Sigma M_B = 0 = -5(18) + A(15) - 5(12) - 5(6)$

$\underline{A = 12^k \uparrow}$

$\Sigma F_V = 0 = 8 + 12 - 20$ ✓

COMP. STRESS IN PIPE:

$S_c = P/A = 12/1.289 = \underline{9.31 \text{ KSI}}$

COMP. STRESS IN WOOD POST:

$S_c = P/A = 8/12.25 = \underline{0.653 \text{ KSI}}$

BEARING STRESS @ X

$S_b = P/A = 12/1.289 = \underline{9.31 \text{ KSI}}$

BEARING STRESS @ Y

$S_b = P/A = 12/36 = \underline{0.333 \text{ KSI}}$

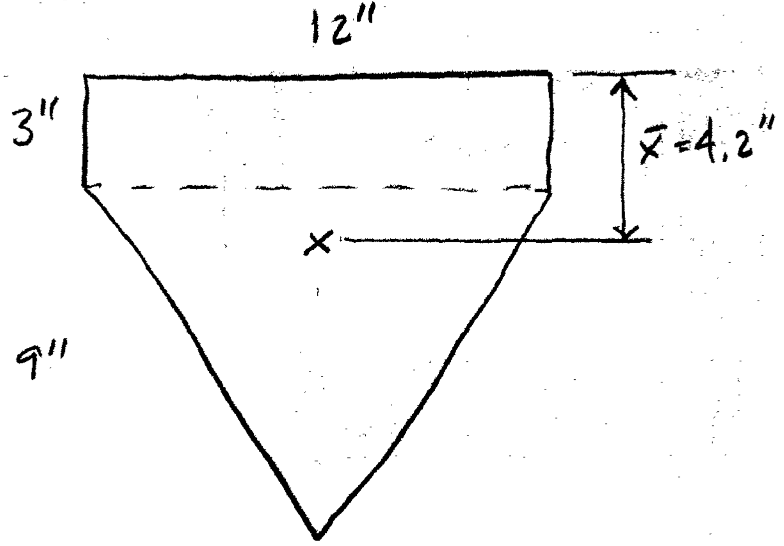
BEARING STRESS @ Z

$S_b = P/A = 12/576 = \underline{0.0208 \text{ KSI}}$ OR 20 PSI

ARCH 331 | PROB 7-5(b) | 7 OCT 91 RVB

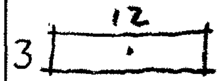
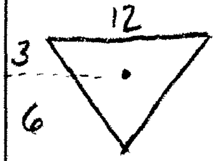
DETERMINE THE MOMENT OF INERTIA WITH RESPECT TO THE NEUTRAL AXIS

(SEC 4.3)



$\bar{x} = 4.2$

(SEE PROB 4-3)

SHAPE	A	d	Ad^2	I	
	$36 in^2$	2.7"	$262.44 in^4$	$27 in^4$	$\frac{bd^3}{12}$
	$54 in^2$	1.8"	$174.96 in^4$	$243 in^4$	$\frac{bd^3}{36}$
			$437.4 in^4$	$270 in^4$	

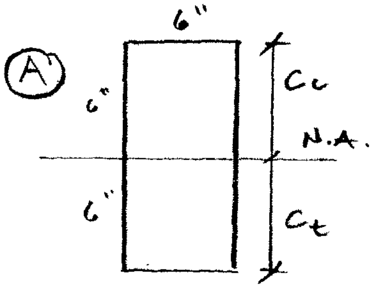
$$I_{TOTAL} = \Sigma I + \Sigma Ad^2$$
$$= 270 + 437.4$$
$$= 707.4 \ in^4$$

GIVEN: E (COMP.) = E (TENS.)

$F_{ALLOWABLE}$ = 12 KSI (FOR COMPRESSION OR TENSION).

FIND: RESISTING MOMENT CAPACITY.

(A)



STEP 1: FIND MOMENT OF INERTIA.

$$I = \frac{bd^3}{12} = \frac{6(12)^3}{12} = 864 \text{ IN}^4$$

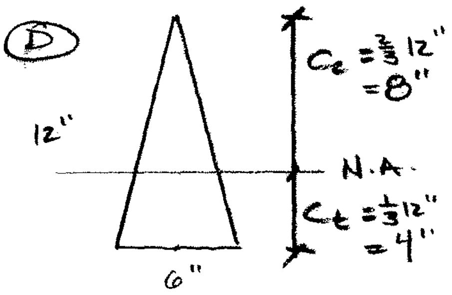
STEP 2: FIND MAX. RESISTING MOMENT.

$$M = \frac{fI}{c} = \frac{12 \text{ KSI } (864 \text{ IN}^4)}{6 \text{ IN}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

CONVERSION.

$$\underline{\underline{M = 144 \text{ K}\cdot\text{ft}}}$$

(D)



STEP 1: FIND MOMENT OF INERTIA

$$I = \frac{bd^3}{36} = \frac{6(12)^3}{36} = 288 \text{ IN}^4.$$

↑ FROM D-13 (pg. 331) ENGEL'S

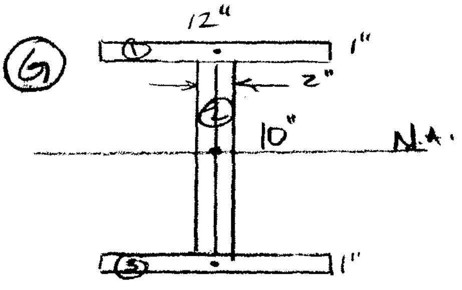
STEP 2: FIND MAX. RESISTING MOMENT

$$M = \frac{fI}{c} = \frac{12 \text{ KSI } (288 \text{ IN}^4)}{8 \text{ IN}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

C_c MAX C, SO USE TO FIND MAX M.

$$\underline{\underline{M = 36 \text{ K}\cdot\text{ft}}}$$

(G)



STEP 1: FIND MOMENT OF INERTIA

	I (IN⁴)	A (IN²)	$\bar{x}$ (IN)	$I + A\bar{x}^2$ (IN⁴)
①	$12(1)^3/12 = 1$	$12(1) = 12$	5.5	364
②	$2(10)^3/12 = 166.7$	$2(10) = 20$	0	166.7
③	$12(1)^3/12 = 1$	$12(1) = 12$	5.5	364

$$I = 894.7 \text{ IN}^4$$

STEP 2: FIND MAX. RESISTING MOMENT

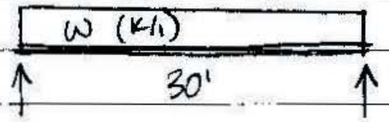
$$M = \frac{fI}{c} = \frac{12 \text{ KSI } (894.7 \text{ IN}^4)}{6 \text{ IN}} \left(\frac{1 \text{ ft}}{12 \text{ IN}}\right)$$

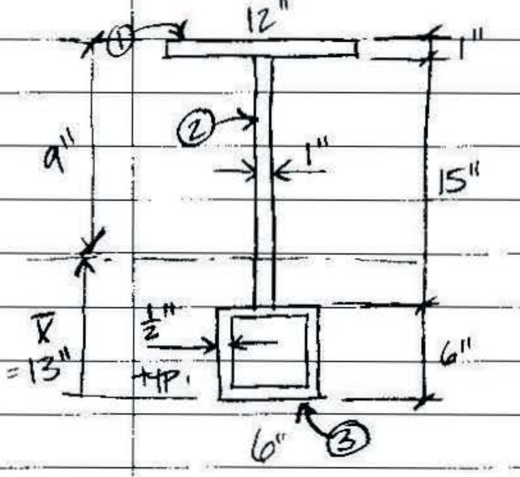
$$\underline{M = 149 \text{ K}\cdot\text{ft}}$$

PROBLEM 7-7:

Max. safe load-carrying capacity?

Allow. bending stress = 18 ksi tension
16 ksi compression





CENTROID:

$$\bar{x} = \frac{\Sigma(A*d)}{\Sigma A} = \frac{(12*1)(21.5)+(15*1)(13.5)+(6*6-5*5)(3)}{(12*1+15*1+6*6-5*5)}$$

$$= 13.0''$$

MOMENT OF INERTIA:

Element	I	A	$\bar{x}$	$A\bar{x}^2$	$I = I_g + A\bar{x}^2$
①	$\frac{bh^3}{12} = \frac{(12)(1)^3}{12} = 1\text{ in}^4$	12 in^2	8.5"	867 in^4	868 in^4
②	$\frac{bh^3}{12} = \frac{(1)(15)^3}{12} = 281.3\text{ in}^4$	15 in^2	0.5"	3.75 in^4	285.1 in^4
③	$\frac{bh^3}{12} = \frac{(6)(6)^3}{12} - \frac{5(5)^3}{12} = 55.9\text{ in}^4$	11 in^2	10"	1100 in^4	1155.9 in^4

$$I_{TOT} = 2309\text{ in}^4$$

Find max. moment: $f = \frac{Mc}{I}$

$$f_t = \frac{Mc_t}{I} \longrightarrow M = \frac{f_t \cdot I}{c_t} = \frac{(18\text{ ksi})(2309\text{ in}^4)}{13''} = \underline{\underline{3197\ ''\cdot k}} = 266'\cdot k$$

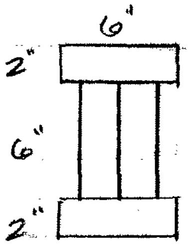
$$f_c = \frac{Mc_c}{I} \longrightarrow M = \frac{f_c \cdot I}{c_c} = \frac{(16\text{ ksi})(2309\text{ in}^4)}{9''} = 4105\ ''\cdot k$$

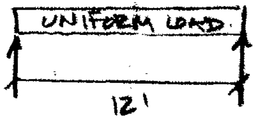
for distributed loads, $M = \frac{wl^2}{8} \longrightarrow 266'k = \frac{w(30')^2}{8} \rightarrow w = 2.36\ k/'$

↖ Data sheet D-25

Total load $W = w*l = 2.36\ k/' * 30' = \underline{\underline{70.9\ k}}$

Ⓐ GIVEN: 4 - 2x6's (FULL DIMENSION)
ALLOWABLE STRESS = 1500 PSI.
SIMPLY SUPPORTED BEAM



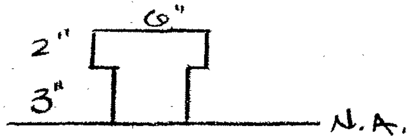


FIND: (A) MOMENT OF INERTIA
(B) LOAD CARRYING CAPACITY

(A) MOMENT OF INERTIA

$$\boxed{I = \frac{bd^3}{3}}$$

METHOD 1:



$$I_{TOP} = \frac{6(5)^3}{3} - \frac{2(3)^3}{3} = 232 \text{ IN}^4$$

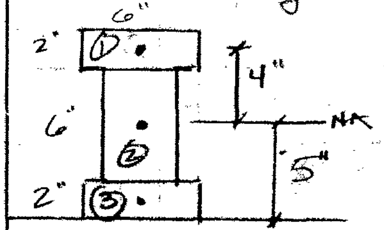
I_{BOTTOM} = SAME DUE TO SYMMETRY ABOUT N.A. = 232 IN^4

$$I_{TOTAL} = I_{TOP} + I_{BOTTOM} = 232 + 232$$

$$\underline{\underline{I = 464 \text{ IN}^4}}$$

METHOD 2: $\boxed{I = \frac{bd^3}{12}}$

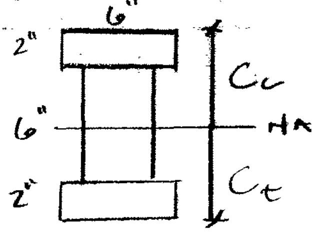
$I_a = I_g + A\bar{x}^2$ N.A. = 5" DUE TO SYMMETRY



	I (IN^4)	A (IN^2)	$\bar{X}$	$I + A\bar{X}^2$ (IN^4)
①	$6(2)^3/12 = 4$	$2(6) = 12$	4	$4 + 12(4)^2 = 196$
②	$4(6)^3/12 = 72$	$4(6) = 24$	0	$72 + 24(0)^2 = 72$
③	$6(2)^3/12 = 4$	$2(6) = 12$	4	$4 + 12(4)^2 = 196$
				464

$$\Sigma(I + A\bar{x}^2) = \underline{464 \text{ IN}^4} = I$$

(B) LOAD CARRYING CAPACITY



$$\boxed{M = \frac{fI}{C}}$$

f = 1500 PSI (GIVEN)
$c = 5" = C_c = C_t$
I = 464 IN^4

$$M = \frac{1500 \text{ PSI } (464 \text{ IN}^4)}{5 \text{ IN}}$$

$$= 139200 \text{ lb}\cdot\text{in} = 11600 \text{ lb}\cdot\text{ft}.$$

→

Ⓐ CONT.

(B) $M = 11600$ lb.ft.

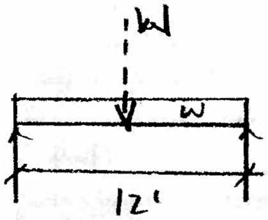
MAX. MOMENT OCCURS @ MIDSPAN.

$$M_{MAX} = \frac{wl^2}{8} = 11600 \text{ lb.ft}$$

$$\frac{w(12\text{ft})^2}{8} = 11600 \text{ lb.ft}$$

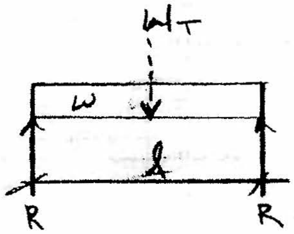
$$w = \frac{11600 \text{ lb ft}\,(8)}{(12\text{ ft})^2}$$

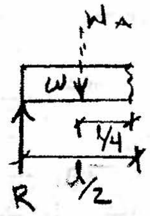
$$\underline{\underline{w = 644 \text{ PLF}}}$$



$$W = 644 \text{ PLF } (12 \text{ ft})$$

$$\underline{\underline{W = 7.73^{k}}}$$





WHERE:

$W_T = w \times l$

$R = w \times l/2$

$W_A = W_T/2 = w\frac{l}{2}$

SO:

$$M = R\left(\frac{l}{2}\right) - W_A\left(\frac{l}{4}\right)$$

$$= \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{wl}{2}\left(\frac{l}{4}\right)$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

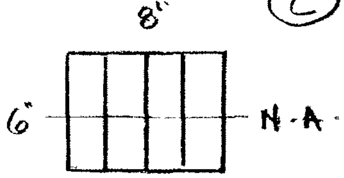
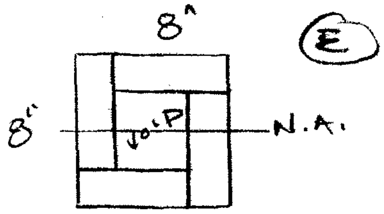
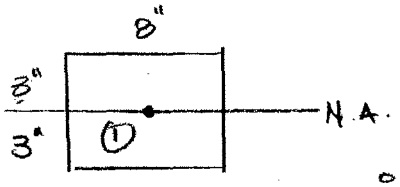
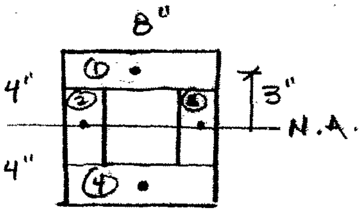
$$\boxed{M_{MAX} = \frac{wl^2}{8}}$$

FOR UNIFORMLY DISTRIBUTED LOAD

OR MORE SIMPLY USE FORMULA TABLES FROM:
D-25 (p.343) ENGEL
ASD (p. 2-295 - 312)

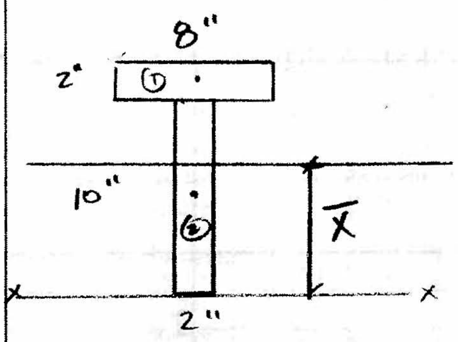
STRUCTURES I - A314 | PROB. 7-8 (I) | S. ROBINSON 11/6/03

(A) MOMENT OF INERTIA

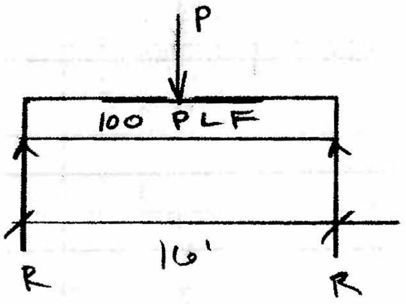
	Ⓒ	Ⓔ
		
METHOD 1: $I = \frac{bd^3}{3}$		
I_{TOP}	$(8)(3)^3/3 = 72$ IN^4	$8(4)^3/3 - 4(2)^3/3 = 160$ IN^4
I_{BTTM}	$(8)(3)^3/3 = 72$ IN^4	$8(4)^3/3 - 4(2)^3/3 = 160$ IN^4
$I_{TOP} + I_{BTTM} = I_{TOTAL}$	$72 + 72 = \underline{\underline{144 \text{ IN}^4}}$	$160 + 160 = \underline{\underline{320 \text{ IN}^4}}$
METHOD 2: $I = \frac{bd^3}{12}$ $I_a = I_g + A\bar{x}^2$		
SHAPE ①	$I_1 = 8(6)^3/12 + \cancelto{0}{(8)(6)0^2}$ $I_1 = 144$ IN^4	$I_1 = (8)(2)^3/12 + 8(2)(3)^2$ $I_1 = 149.3$ IN^4
SHAPE ②	—	$I_2 = (2)(4)^3/12 + \cancelto{0}{(2)(4)0^2}$ $I_2 = 10.7$ IN^4
SHAPE ③	—	DUE TO SYMMETRY $I_3 = I_2$
SHAPE ④	—	DUE TO SYMMETRY $I_4 = I_1$
$\sum I_a = I_{TOTAL}$	$\underline{\underline{144 \text{ IN}^4}}$ ✓	$149.3 \times 2 + 10.7 \times 2 = \underline{\underline{320 \text{ IN}^4}}$ ✓

(B) $M = fI/c$ WHERE $f = 1500$ PSI		
c	3" $\quad c_c = c_t$	4" $\quad c_c = c_t$
M	= 1500 PSI (144 IN^4) / 3 IN = 72,000 lb.in	= 1500 PSI (320 IN^4) / 4 IN = 120,000 lb in
M_{MAX}	= 6000 lb.ft	= 10,000 lb.ft
$M_{MAX} = \frac{wl^2}{8}$ $\quad w$	6000 lb.ft (8)/(12 ft)^2 = $\underline{\underline{333.3 \text{ PLF}}}$	10,000 lb.ft (8)/(12 ft)^2 = $\underline{\underline{555.6 \text{ PLF}}}$
$W = wl$ $\quad W$	333.3 PLF (12 ft) = $\underline{\underline{4^K}}$	555.6 PLF (12 ft) = $\underline{\underline{6.7^K}}$

GIVEN: $F_{ALLOW} = 1800$ PSI. IN BENDING.



SECTION



LOADING DIAG.

FIND: MAXIMUM POINT LOAD "P"

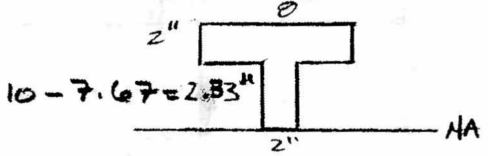
STEP 1: FIND LOCATION OF N.A. OF SECTION.

	A (IN²)	$\bar{X}$ (IN)	A$\bar{X}$ (IN³)
①	(8)(2) = 16	11	176
②	(2)(10) = 20	5	100
	$\Sigma A = 36$		$\Sigma A\bar{X} = 276$

$$\bar{X} = \frac{\Sigma A\bar{X}}{\Sigma A} = \frac{276}{36} = 7.67 \text{ IN FROM BASE.}$$

STEP 2: FIND MOMENT OF INERTIA.

METHOD 1



TOP.

$$I_{TOP} = I_{SOLID} - I_{VOID} \qquad I = \frac{bd^3}{3}$$

$$= \frac{8(2+2.33)^3}{3} - \frac{6(2.33)^3}{3}$$

$$= 191.2 \text{ IN}^4$$

$$I_{BOTTOM} = \frac{2(7.67)^3}{3} = 300.8 \text{ IN}^4$$

$$I_{TOTAL} = I_{TOP} + I_{BOTTOM}$$

$$= 191.2 + 300.8$$

$$I = 492 \text{ IN}^4$$

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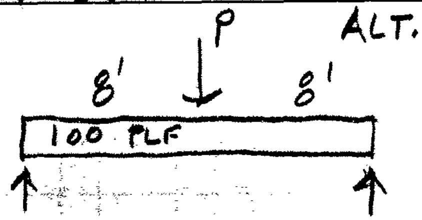
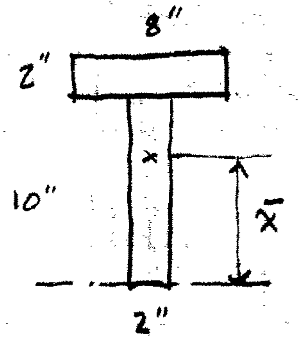
ALT. METHOD w/ FBD



CENTROID:

$$\frac{\Sigma Ad}{\Sigma A} = \frac{16(11)+20(5)}{16+20}$$

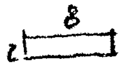
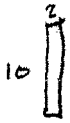
$$\bar{x} = 7.67''$$



$F_b = 1800 \text{psi}$

MOMENT OF INERTIA:

$$I_o = \Sigma I_g + \Sigma Ad^2$$

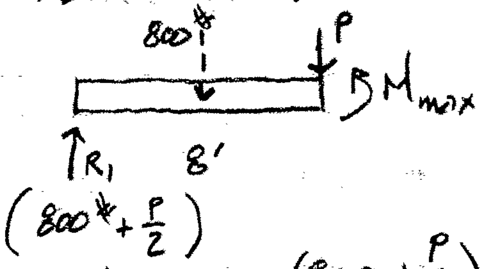
SHAPE	Ad^2	I_g
	$16(3.33)^2 =$ 177.78	$\frac{8(2)^3}{12} =$ 5.333
	$20(2.67)^2 =$ 142.22	$\frac{2(10)^3}{12} =$ 166.67

$\Sigma\ 320 + \Sigma\ 172$

$I_a = 492 \text{ in}^4$

FOR SYMETRIC LOAD M_{max} @ ℄

FBD OF BEAM CUT @ ℄



$$\Sigma M_{@℄} = 0 = \left(800 + \frac{P}{2}\right)8 - 800(4) - M_{max}$$

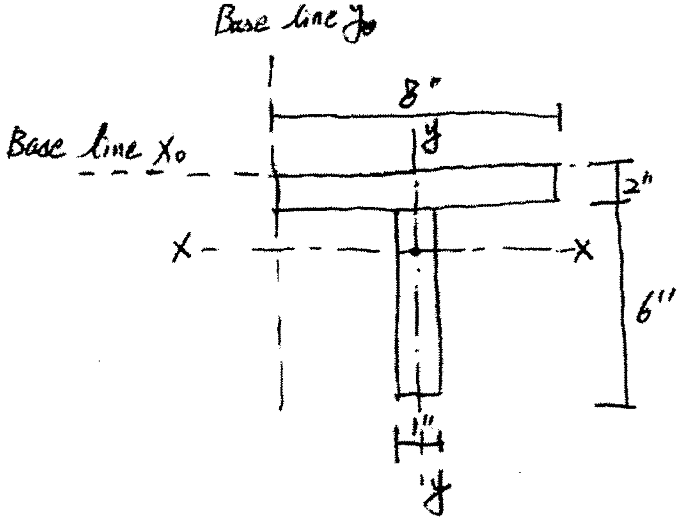
$$M_{max} = 4P + 3200 \quad \text{FT-LBS}$$

$$F_{allow} = 1800 = \frac{Mc}{I} = \frac{(4P+3200)\times 12\,(7.67)}{492} = .7480P + 598.4$$

$$0.7480P = 1800 - 598.4 = 1204.6$$

$$P = 1610.4 \text{ LBS}$$

$$\approx 1.6^{K}$$

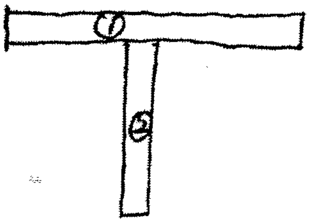


Find the moment of inertia of the T-section, that is, find I_x, I_y.

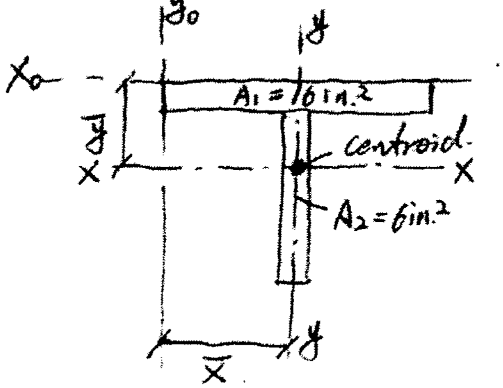
(Note: x-x and y-y are the two axes which pass the centroid of the T-section)

Solution 1:

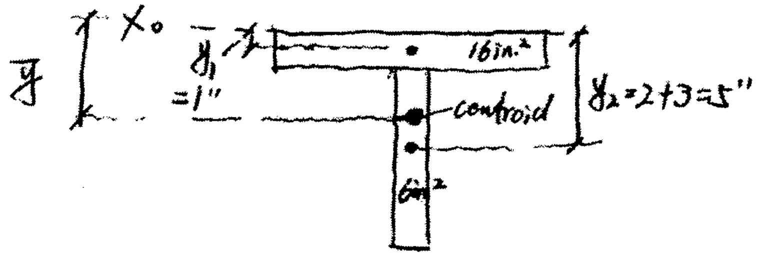
Step 1: Divide the T-section into two rectangular shapes (Shape ① and Shape ②)



Step 2: Find centroid of the T-section. That is, the ~~distance~~ location of the centroid relative to the Base Line X_o and y_o



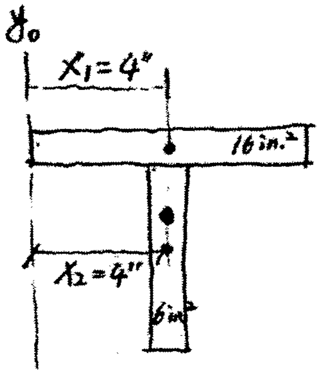
① Find $\bar{y}$.



$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(16in.^2)(1") + (6in.^2)(5")}{16in.^2 + 6in.^2}$$

$$= 2.09 \text{ in.}$$

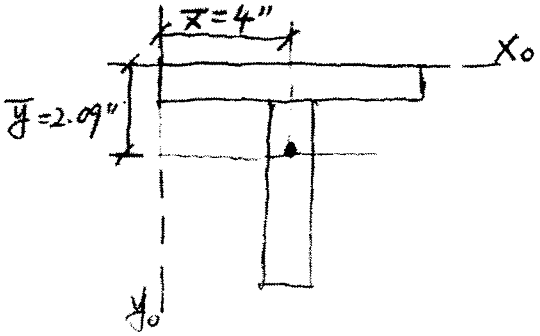
② Find $\bar{x}$



$$\bar{x} = \frac{\Sigma AX}{\Sigma A} = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2} = \frac{(16in.^2)(4") + (6in.^2)(4")}{16in.^2 + 6in.^2}$$

$$= 4 \text{ in.}$$

From ① and ②,
the centroid of the T-section is :



tep 3: Find I_x, I_y

$$I_x = \Sigma \bar{I}_x + \Sigma A d_y^2 \quad , \quad I_y = \Sigma \bar{I}_y + \Sigma A d_x^2$$

- Find I_x first.

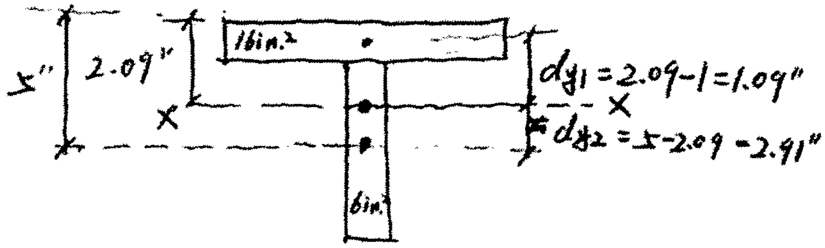
① $\Sigma \bar{I}_x = I_{x_1} + I_{x_2}$ ← Find $\Sigma \bar{I}_x$.

$$I_{x_1} = \frac{8''(2'')^3}{12} = 5.33 \text{ in.}^4$$

$$I_{x_2} = \frac{1''(6'')^3}{12} = 18 \text{ in.}^4$$

$$\therefore \Sigma \bar{I}_x = 5.33 \text{ in.}^4 + 18 \text{ in.}^4 = 23.33 \text{ in.}^4$$

② $\Sigma A d_y^2 = A_1 d_{y1}^2 + A_2 d_{y2}^2$ ← Find $\Sigma A d_y^2$



$$A_1 d_{y1}^2 = (16 \text{in.}^2)(1.09 \text{in.})^2 = 19.01 \text{ in.}4$$

$$A_2 d_{y2}^2 = (6 \text{in.}^2)(2.91 \text{in.})^2 = 50.81 \text{ in.}4$$

$$\therefore \Sigma A d_y^2 = 19.01 \text{in.}^4 + 50.81 \text{ in.}^4 = 69.82 \text{ in.}4$$

③ $I_x = \Sigma \bar{I}_x + \Sigma A d_y^2$ ← Find I_x

$$I_x = 23.33 \text{ in.}^4 + 69.82 \text{ in.}^4 = \boxed{93.15 \text{ in.}^4}$$

- Find I_y.

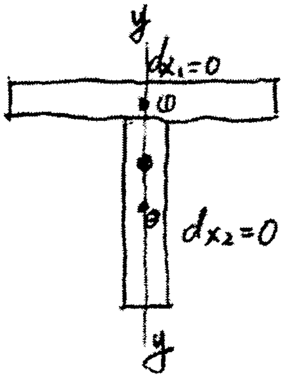
① $\Sigma I_y = I_{y_1} + I_{y_2}$ ← Find ΣI_y.

$$I_{y_1} = \frac{2''(8'')^3}{12} = 85.33 \text{ in.}^4$$

$$I_{y_2} = \frac{6''(1'')^3}{12} = 0.5 \text{ in.}^4$$

$$\therefore \Sigma I_y = 85.33 + 0.5 = 85.83 \text{ in.}^4$$

② $\Sigma A d_{\cancel{y}}^2 = A_1 d_{x_1}^2 + A_2 d_{x_2}^2$ ← Find $\Sigma A d_x^2$



d_{x_1}, $d_{x_2} = 0$ because the centroids of Shape ① and Shape ② are right on the y-y axis, which pass the centroid of the T-section.

$$\therefore \Sigma A d_x^2 = 0$$

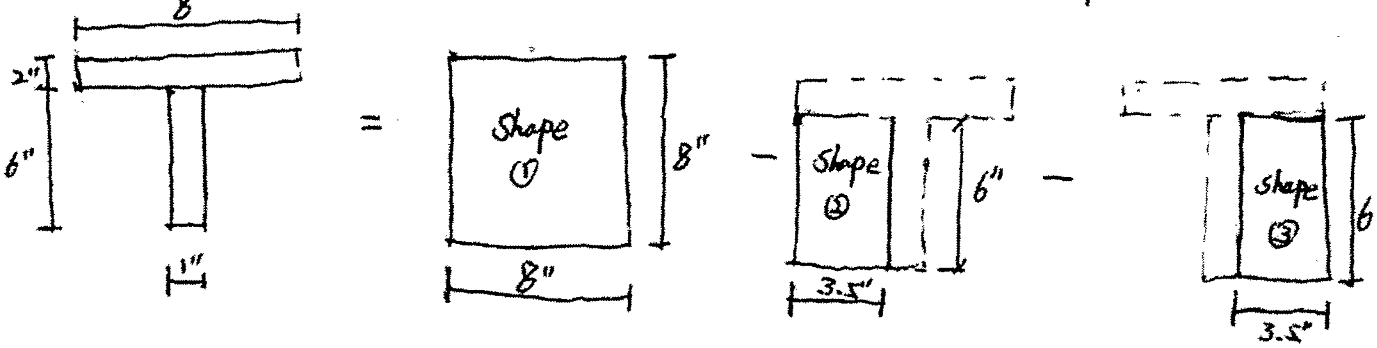
③ $I_y = \Sigma I_y + \Sigma A d_x^2$ ← Find I_y

$$I_y = 85.83 \text{ in.}^4 + 0 = \boxed{85.83 \text{ in.}^4}$$

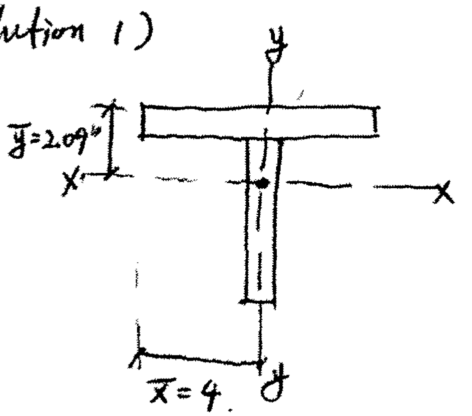
Thus, $\boxed{I_x = 93.15 \text{ in.}^4 \quad , \quad I_y = 85.83 \text{ in.}^4}$

Solution 2 :

Step 1 : Divide the T-section into three regular shapes.



Step 2 : Find centroid of the T-section. (Same as the step 2 in solution 1)



Step 3 : Find I_x, I_y.

$$I_x = \Sigma \bar{I}_x + \Sigma A d_y^2 , \quad I_y = \Sigma \bar{I}_y + \Sigma A d_x^2$$

- Find I_x

① $\Sigma \bar{I}_x = I_{x_1} \overset{-}{\cancel{+}} I_{x_2} \overset{-}{\cancel{+}} I_{x_3}$ ← Find $\Sigma \bar{I}_x$

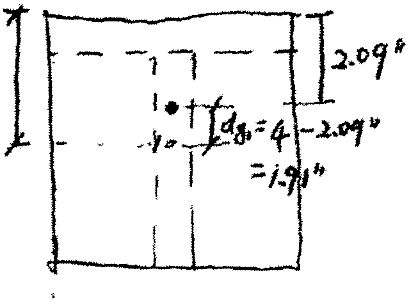
$$I_{x_1} = \frac{8''(8'')^3}{12} = 341.33 \text{ in.}^4$$

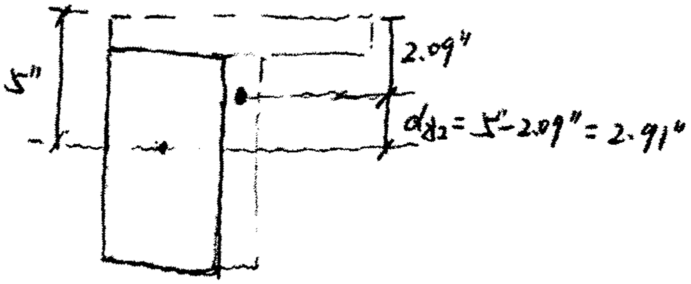
$$I_{x_2} = \frac{3.5''(6'')^3}{12} = 63 \text{ in.}^4$$

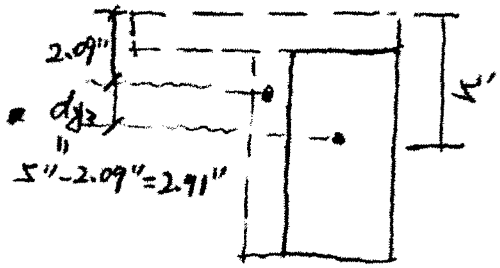
$$I_{x_3} = \frac{3.5''(6'')^3}{12} = 63 \text{ in}^4$$

$$\therefore \Sigma \bar{I}_x = 341.33 - 63 - 63 = 215.33 \text{ in}^4$$

② $\Sigma A d_y^2 = A_1 d_{y_1}^2 - A_2 d_{y_2}^2 - A_3 d_{y_3}^2 \cdot$







$A_1 d_{y_1}^2 = (8" \times 8")(1.91")^2 = 233.48 \text{ in.}^4$

$A_2 d_{y_2}^2 = (3.5" \times 6")(2.91")^2 = 177.83 \text{ in.}^4$

$A_3 d_{y_3}^2 = (3.5" \times 6")(2.91")^2 = 177.83 \text{ in.}^4$

$\therefore \Sigma A d_y^2 = 233.48 \text{ in.}^4 - 177.83 \text{ in.}^4 - 177.83 \text{ in.}^4 = -122.18 \text{ in.}^4$

③ $I_x = \Sigma \bar{I}_x + \Sigma A d_y^2 \longleftarrow$ Find I_x

$I_x = 215.33 \text{ in.}^4 - 122.18 \text{ in.}^4 = \boxed{93.15 \text{ in.}^4}$

- Find I_y.

① $\Sigma \bar{I}_y = I_{y1} - I_{y2} - I_{y3} \longleftarrow$ Find $\Sigma \bar{I}_y$.

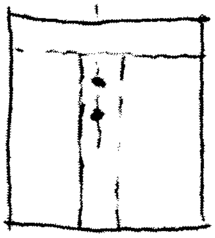
$$I_{y1} = \frac{8''(8'')^3}{12} = 341.33\,in.^4$$

$$I_{y2} = \frac{6''(3.5'')^3}{12} = 21.44\,in^4$$

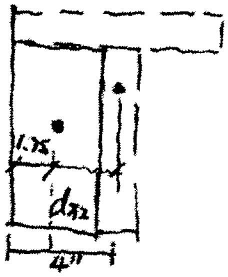
$$I_{y3} = \frac{6''(3.5'')^3}{12} = 21.44\,in.^4$$

$\therefore\ \Sigma \bar{I}_y = 341.33 - 21.44 - 21.44 = 298.45\ in.^4$

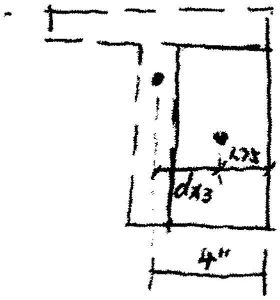
② $\Sigma A d_x^2 = A_1 d_{x1}^2 - A_2 d_{y2}^2 - A_3 d_{x3}^2$



$d_{x1} = 0$



$d_{x2} = 4'' - 1.75'' = 2.25''$



$d_{x3} = 4'' - 1.75'' = 2.25''$

$A_1 d_{x1}^2 = 0$

$A_2 d_{x2}^2 = (6'' \times 3.5'')(2.25'')^2 = 106.31\ in.^4$

$A_3 d_{x3}^2 = (6'' \times 3.5'')(2.25'')^2 = 106.31\ in.^4$

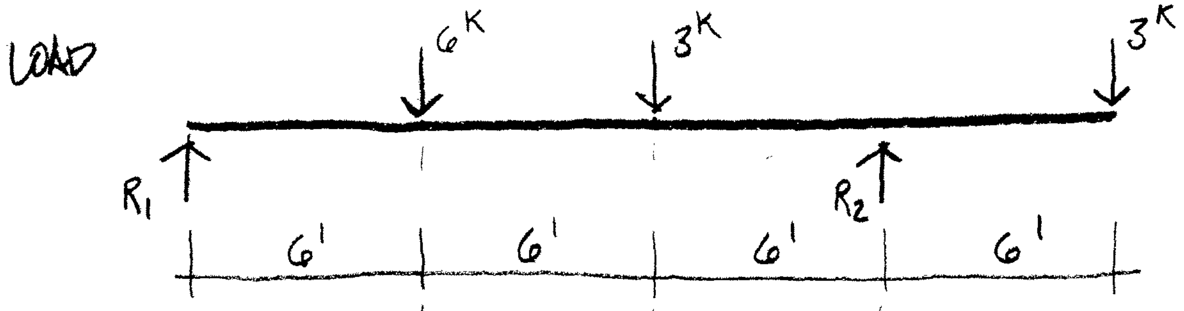
$\therefore\ \Sigma A d_x^2 = 0 - 106.31\,in.^4 - 106.31\,in.^4 = -212.63\ in.^4$

③ $I_y = \Sigma \bar{I}_y + \Sigma A d_x^2 \longleftarrow$ Find I_y

$I_y = 298.45\,in.^4 - 212.63\ in.^4 = \boxed{85.82\ in.^4}$

Thus, $\boxed{I_x = 93.15\,in.^4\ ,\quad I_y = 85.82\,in^4}$

ARCH 331 | PROBLEM 8-1 e | 23 OCT 91 RJB



REACTIONS:

$\Sigma M_{R_1} = 0 = 6(6) + 3(12) - R_2(18) + 3(24)$

$R_2 18 = 144$

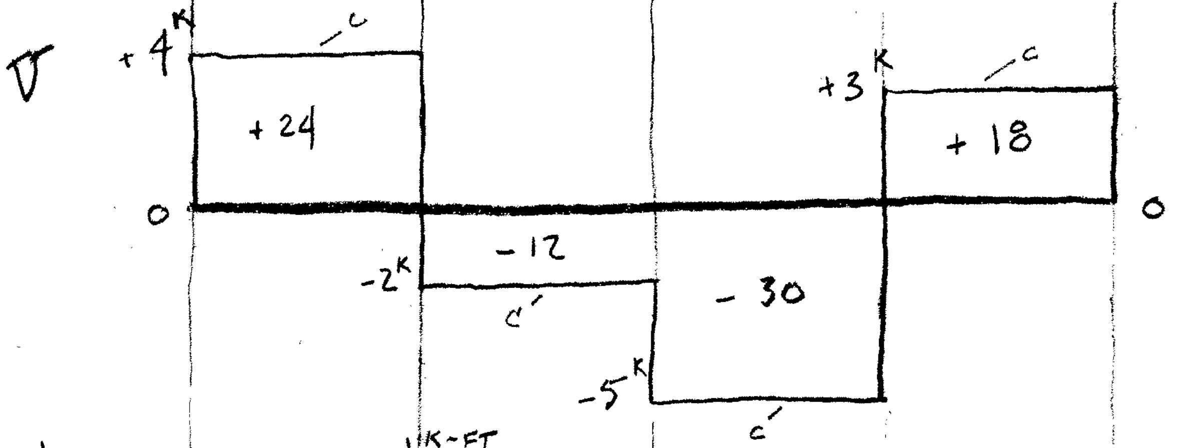
$R_2 = 8^K$

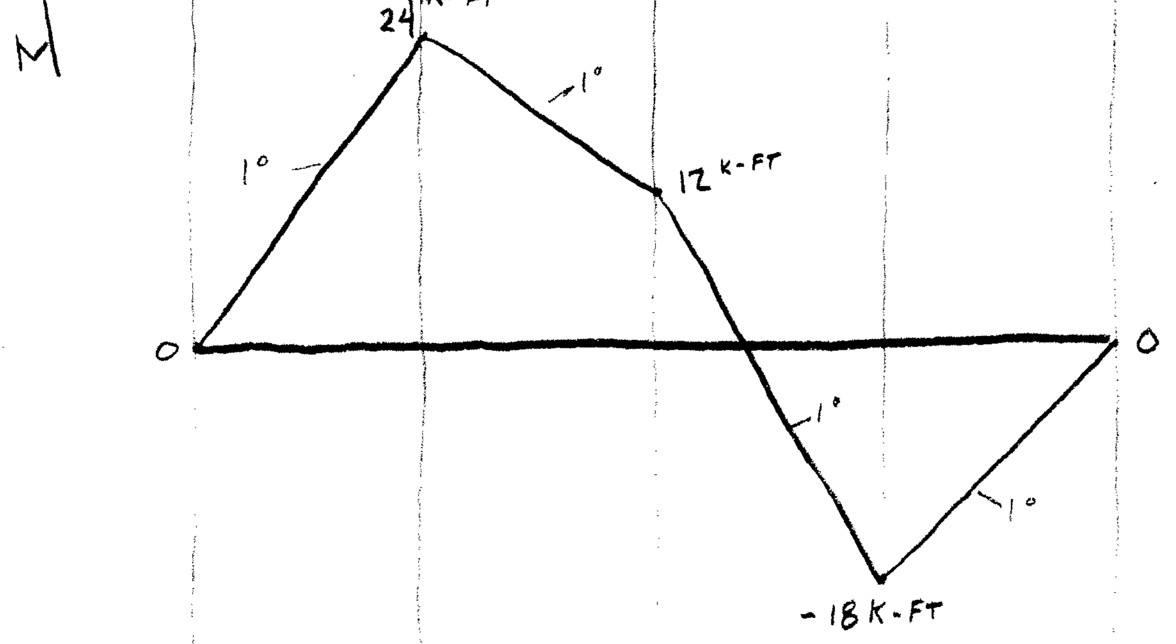
$\Sigma M_{R_2} = 0 = R_1(18) - 6(12) - 3(6) + 3(6)$

$R_1 18 = 72$

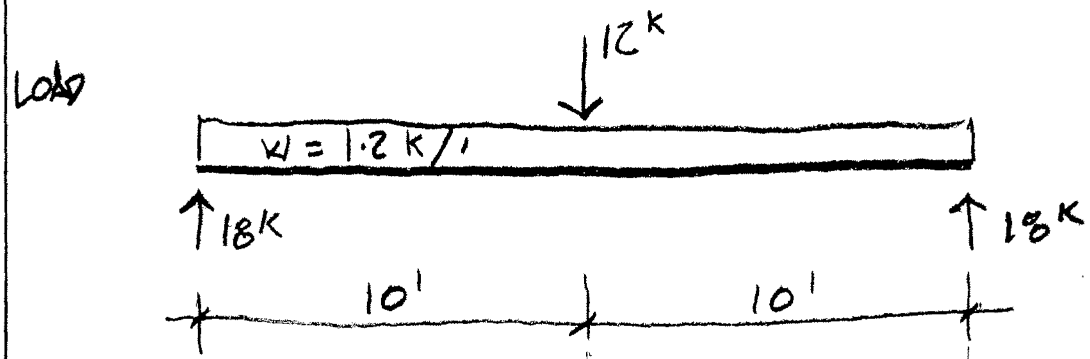
$R_1 = 4$

CHECK $\Sigma F_V = 0 = 8 + 4 - 6 - 3 - 3 = 0$ ✓



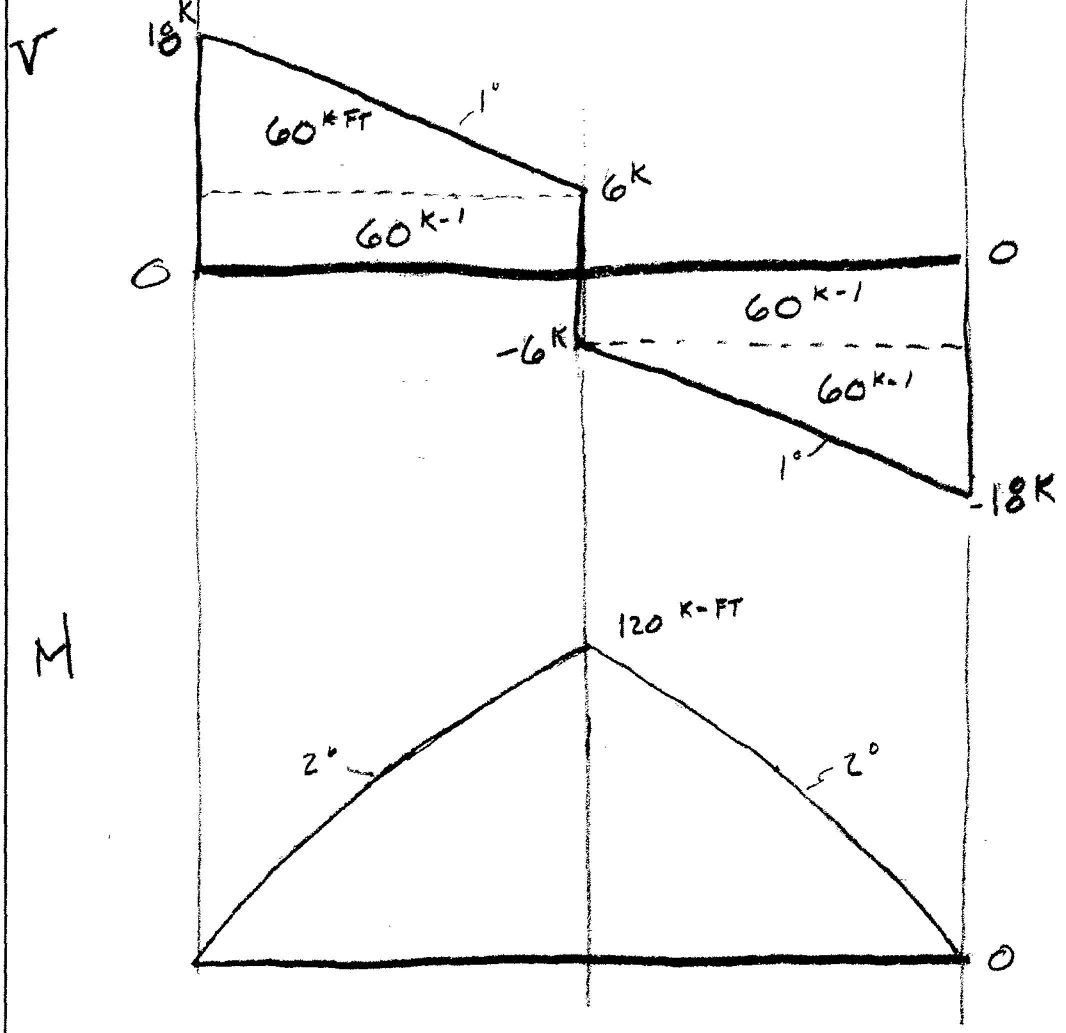


ARCH 331 | PROBLEM 8.1K | 22 OCT 91 PVB

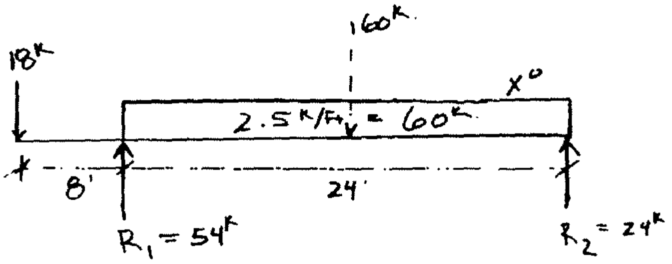


REACTIONS BY SYMMETRY:

12 + 1.2(20) = 36   36/2 = 18K



PROBLEM 8-2E:



FIRST FIND REACTIONS:

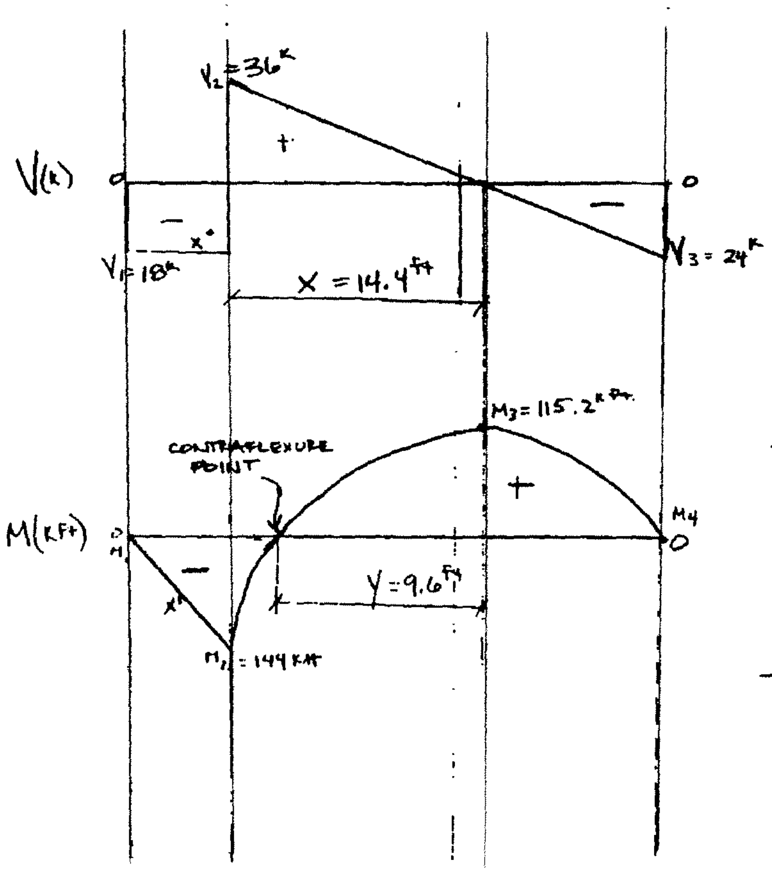
$\Sigma M_{eR_1} = 0 = -18^k(8') + 60^k(12') - R_2(24')$

$R_2 = 24^K$

$\Sigma F_y = 0 = R_1 - 18^k - 60^k + R_2$

$0 = R_1 - 18^k - 60^k + 24^K$

$R_1 = 54^K$



FIND V FORCES:

V_1 = POINT LOAD = -18^K

$V_2 = V_1 + R_1 = -18 + 54^k = 36^k$

V_3 : V_2 + AREA OF LOAD

$= 36 + (-2.5\ k/ft)(24\ ft) = 24^K$

$V_3 = R_2$ ✓

FIND WHERE V-DIAG. CROSSES ZERO, X

ΔV = AREA OF LOADING DIAG.

$V_2 - 0 = 2.5^{k/ft}(x)$

$36^k = 2.5x \Longrightarrow x = 14.4^{ft}$

FIND M FORCES:

$M_1 = 0$

$M_2 = M_1$ + AREA OF V-DIAG.

$= 0 + -18^k(8') = -144^{kft}$

$M_3 = M_2$ + AREA OF V-DIAG.

$= -144 + \frac{1}{2}(14.4^{ft})(36^k) = 115.2^{kft}$

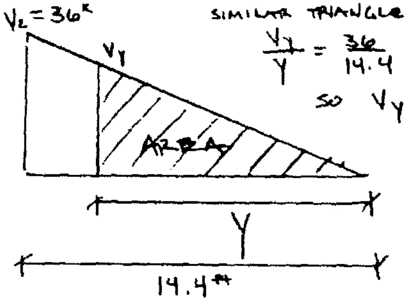
$M_4 = M_3$ + AREA OF V-DIAG.

$= 115.2 + \frac{1}{2}(24-14.4)(-24) = 0$ ✓

MMAX OCCURS WHERE V-DIAG. CROSSES ZERO $\Longrightarrow$ $115.2^{kft} = M_{MAX}$

$36^k = V_{MAX}$

FROM V-DIAG



SIMILAR TRIANGLES:

$\frac{V_Y}{Y} = \frac{36}{14.4}$

SO $V_Y = 2.5Y$

FIND POINT OF CONTRAFLEXURE → WHERE M-DIAG CROSSES ZERO:

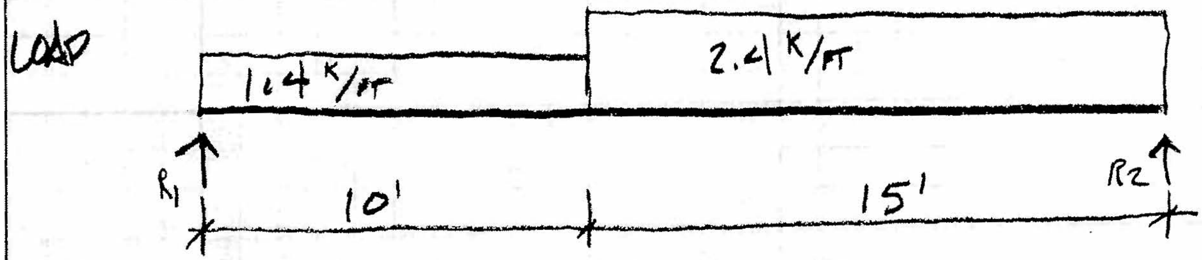
ΔM = AREA OF V-DIAG.

$M_3 - 0 = \frac{1}{2}Y(V_Y)$

$115.2^{kft} = \frac{1}{2}Y(2.5Y)$

$115.2 = 1.25Y^2 \Longrightarrow Y = 9.6^{ft}$

ARCH 331 | PROBLEM 8.21d 23 OCT 91 PVB



REACTIONS:

$\Sigma M_{R_1} = 0 = 14(5) + 36(17.5) - R_2(25)$

$R_2(25) = 700$

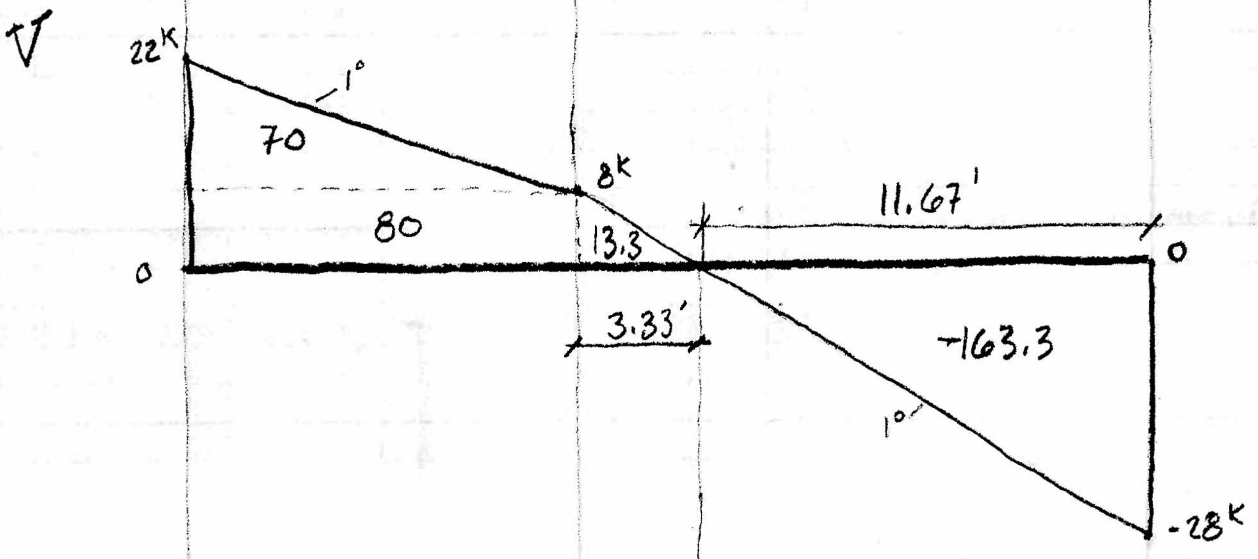
$R_2 = 28^K$

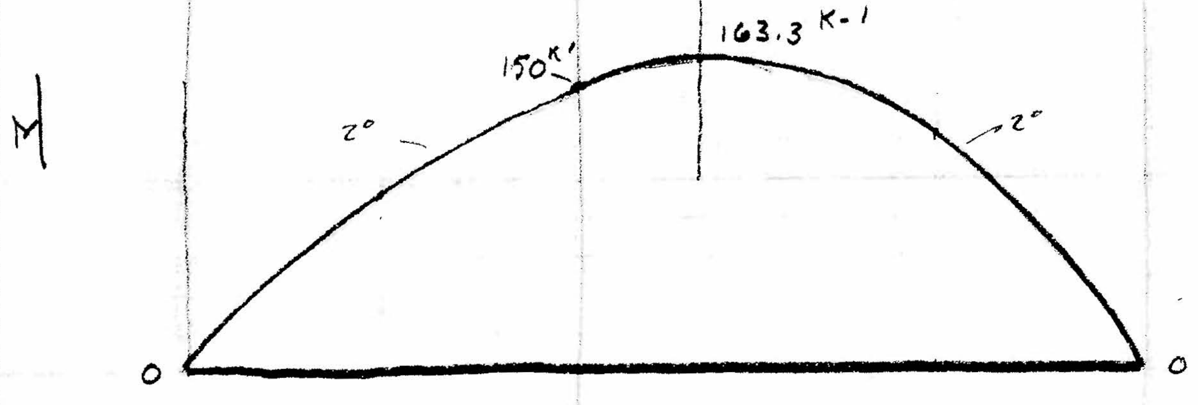
$\Sigma M_{R_2} = 0 = -14(20) - 36(7.5) + R_1(25)$

$R_1(25) = 550$

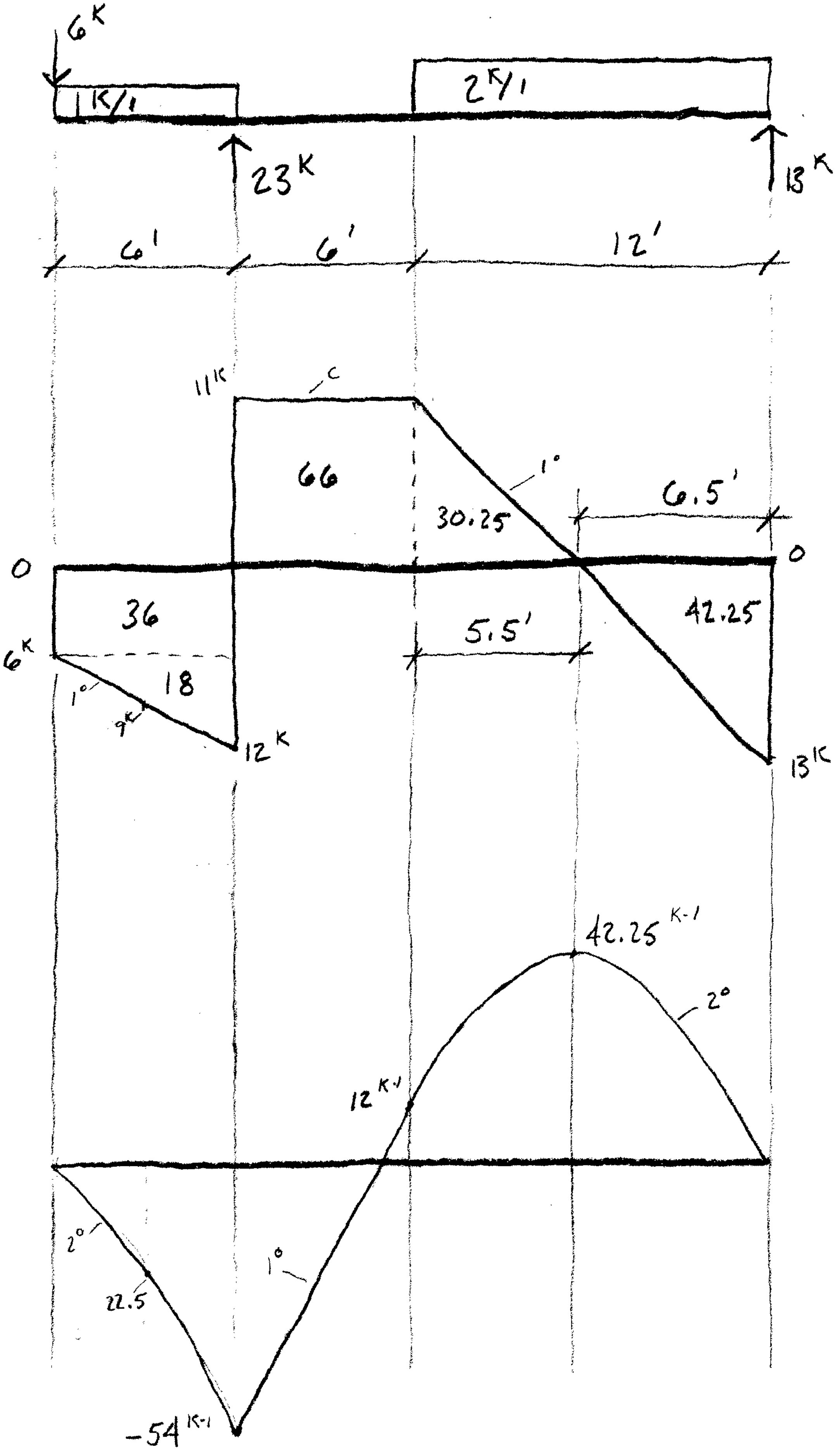
$R_1 = 22^K$

CHECK $F_V = 28 + 22 - 14 - 36 = 0$ ✓

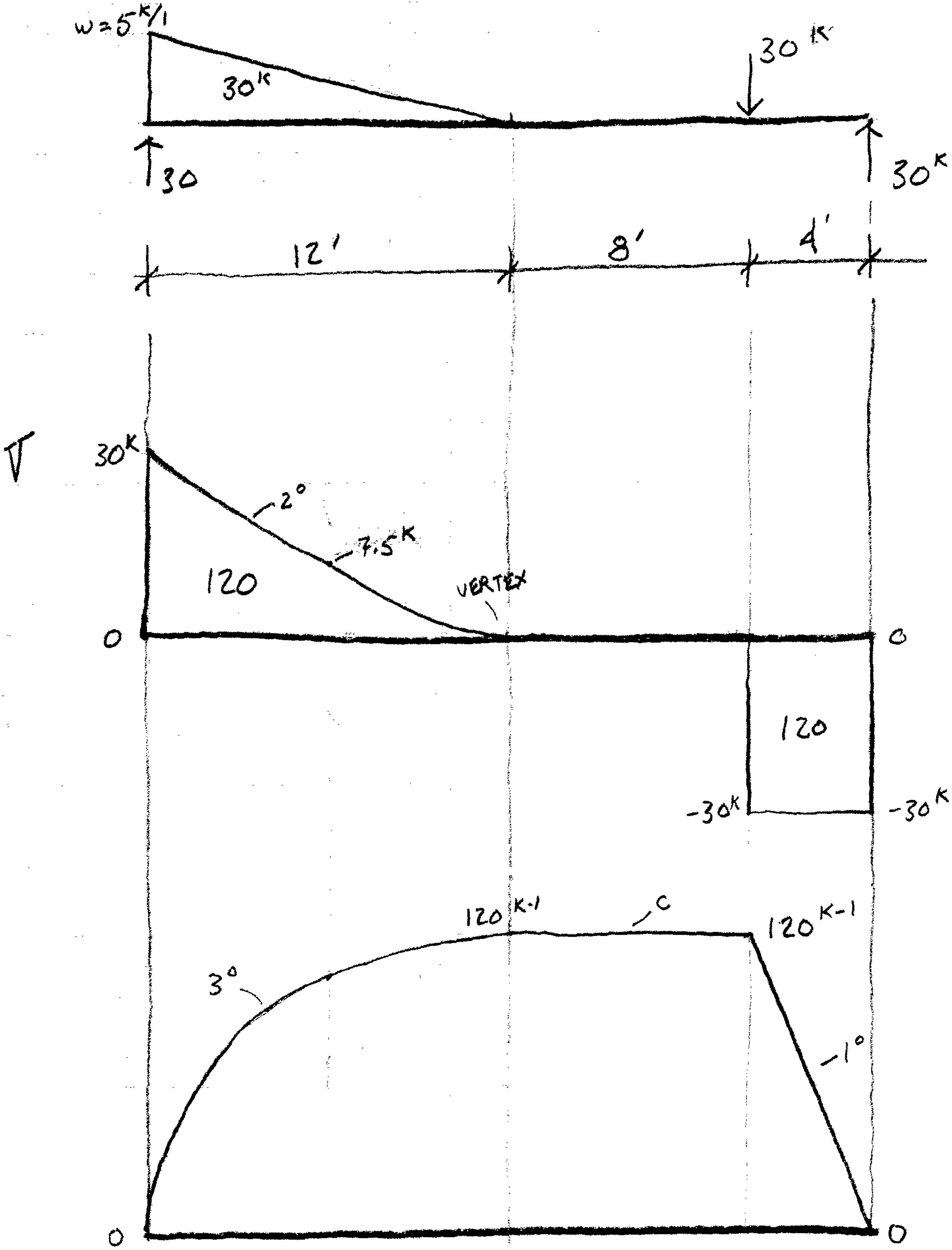




ARCH 331 | PROBLEM 8.21c | 23 OCT 91 PVB

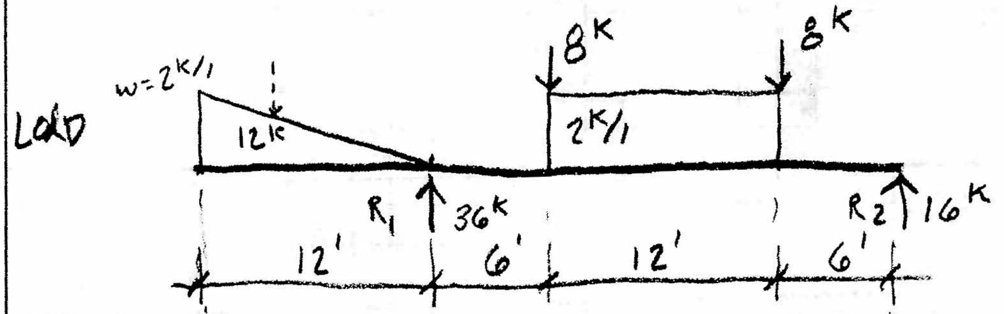


ARCH 331 | PROBLEM 8-3f | 23 OCT 91 PvB



ARCH 331 | PROBLEM 8-6F | 30 OCT 91 RB

REV. 06



REACTIONS:

$\Sigma M_{R_1} = 0 = -12(8) + 8(6) + 24(12) + 8(18) - R_2(24)$

$R_2(24) = 384$

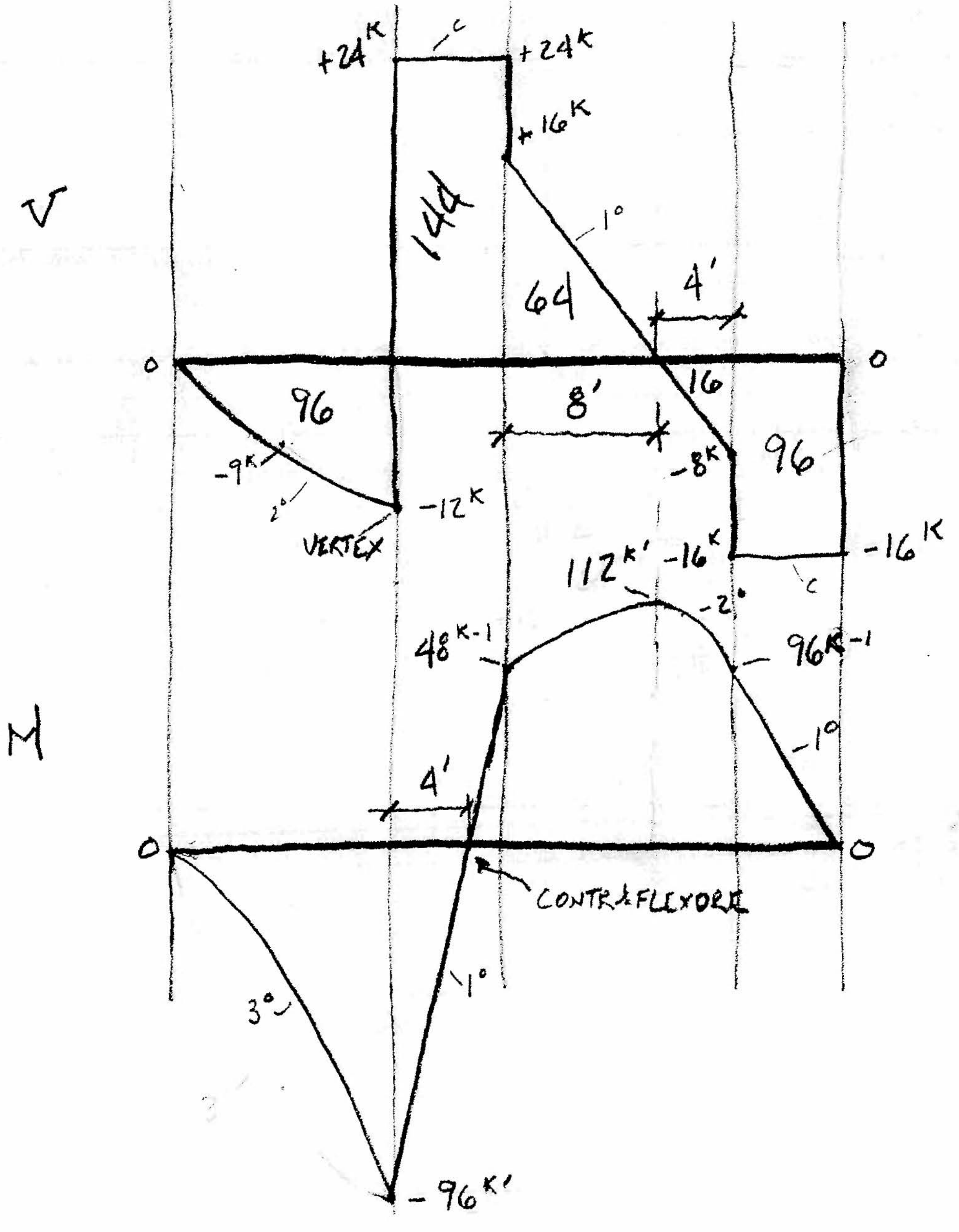
$R_2 = 16^K$

$\Sigma M_{R_2} = 0 = -12(32) + R_1(24) - 8(18) - 24(12) - 8(6)$

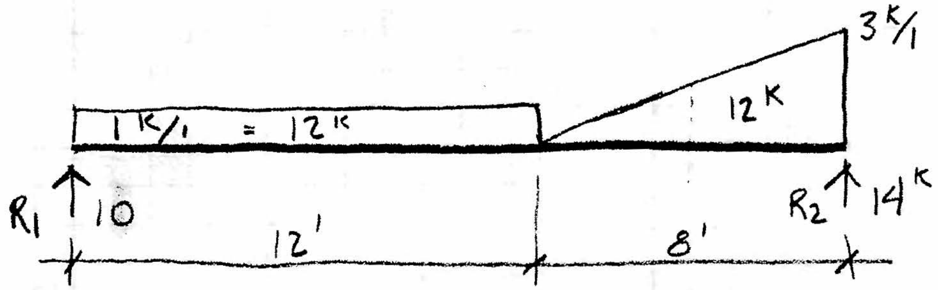
$R_1(24) = 864$

$R_1 = 36^K$

CHECK $\Sigma F_V = 16 + 36 - 12 - 8 - 8 - 24 = 0$ ✓



ARCH 331 | PROBLEM 8.7 B | 23 OCT 71 RVB



REACTIONS:

$\Sigma M_{R_1} = 12(6) + 12(17.33) - R_2(20) = 0$

$R_2 20 = 280$

$R_2 = 14^K$

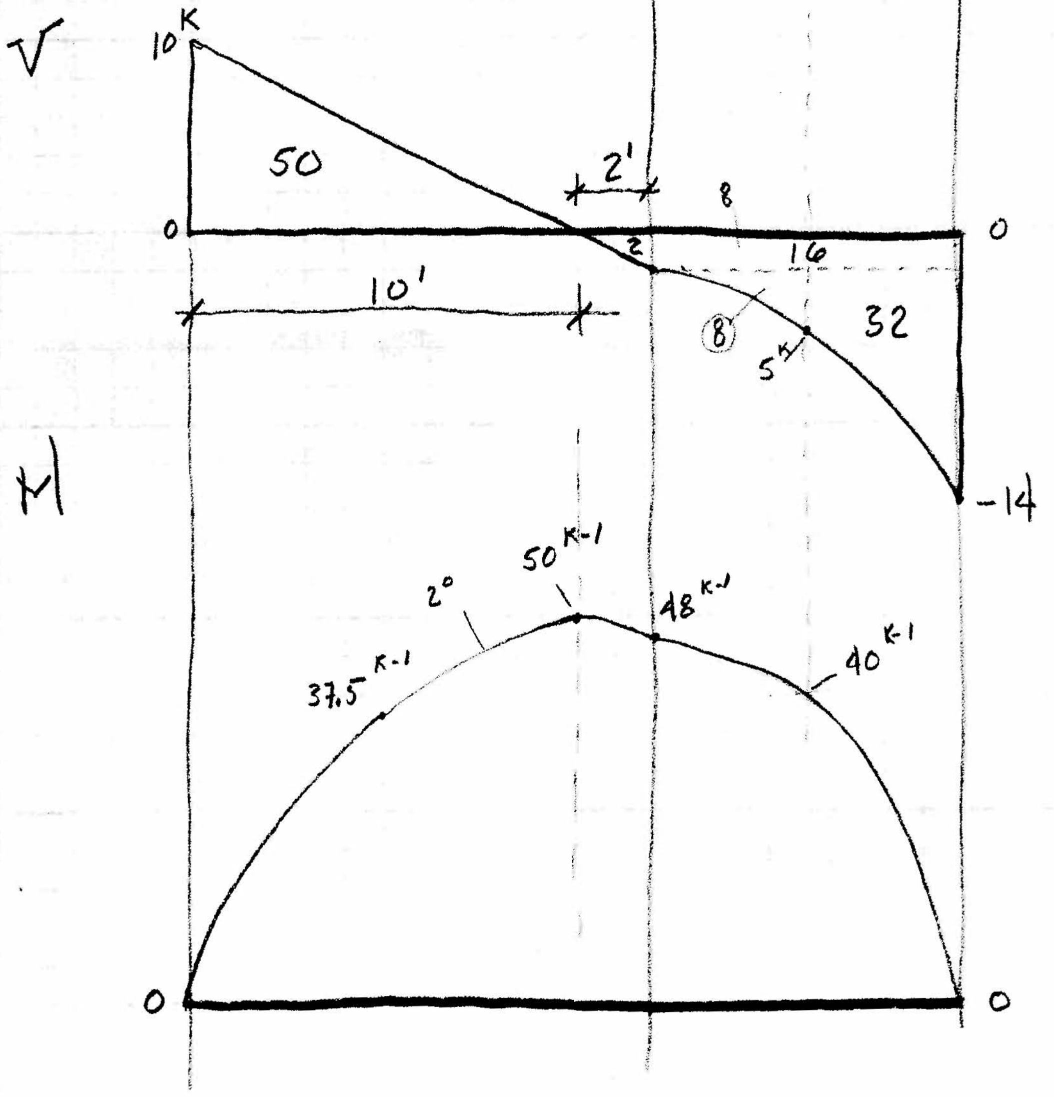
$\Sigma M_{R_2} = 0 = -12(14) - 12(2.67) + R1(20)$

$R_1 20 = 200$

$R_1 = 10$

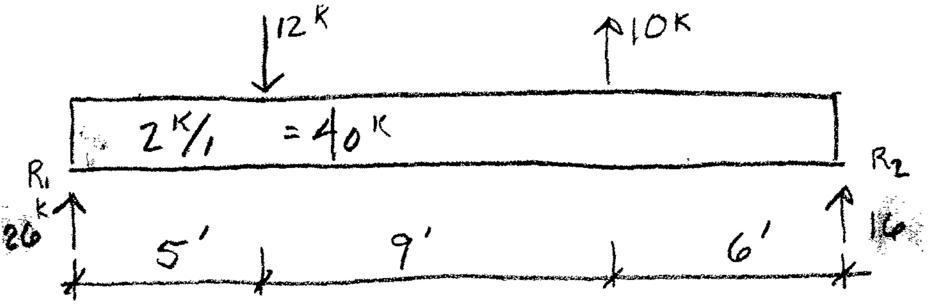
CHECK $F_V = 14 + 10 - 12 - 12 =$ ✓

×2



÷2

ARCH 331 | PROBLEM 8-7d | PvB Nov 2, 1991



REACTIONS

$\Sigma M_{R_1} = 0 = 12(5) + 40(10) - 10(14) - R_2(20)$

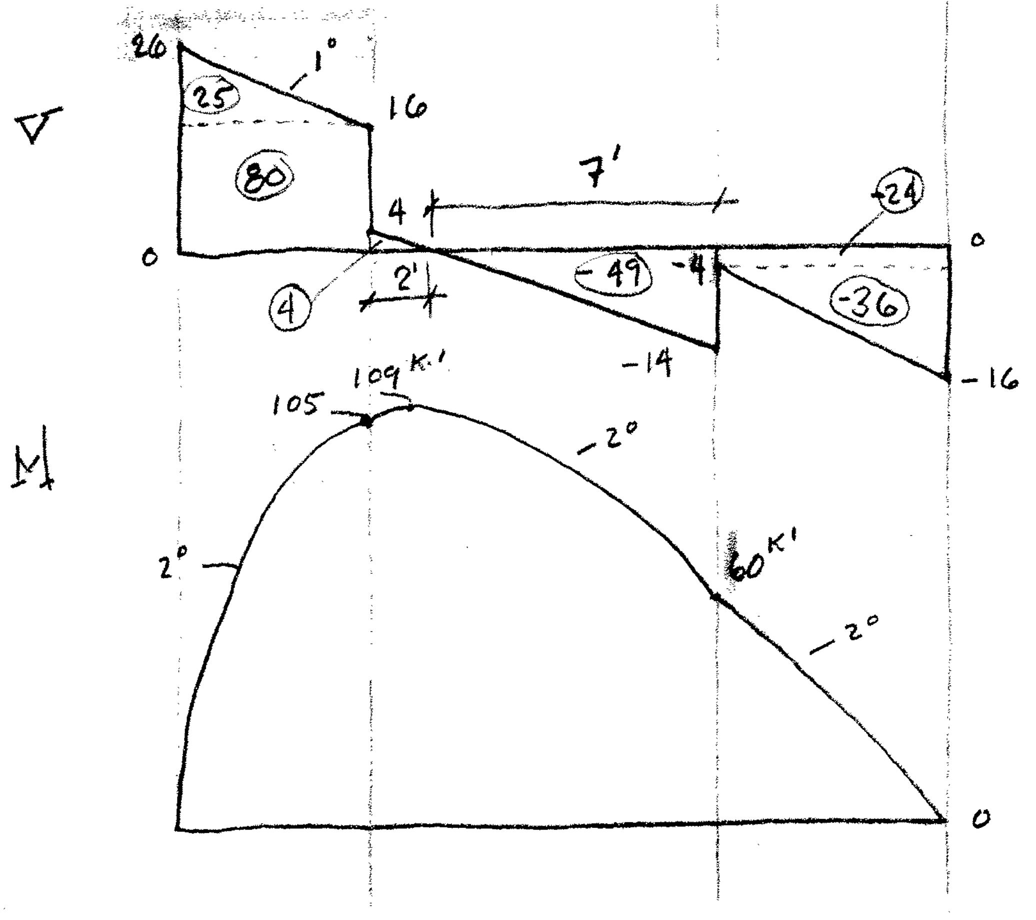
$R_2(20) = 320$

$R_2 = 16$

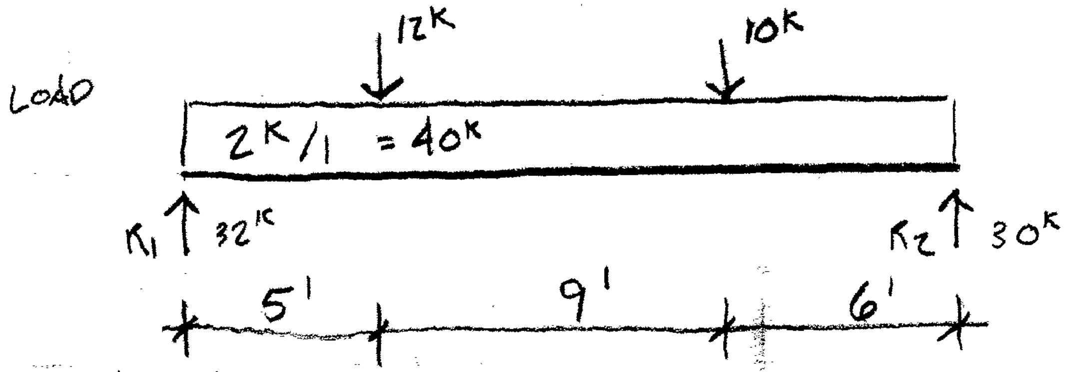
$\Sigma M_{R_2} = 0 = -12(15) - 40(10) + 10(6) + R_1(20)$

$R_1(20) = 520$

$R_1 = 26$



ARCH 331 | PROBLEM 8-7 dx | 23 OCT 91 RVB



REACTIONS:

$\Sigma M_{R_1} = 0 = 12(5) + 40(10) + 10(14) - R_2(20)$

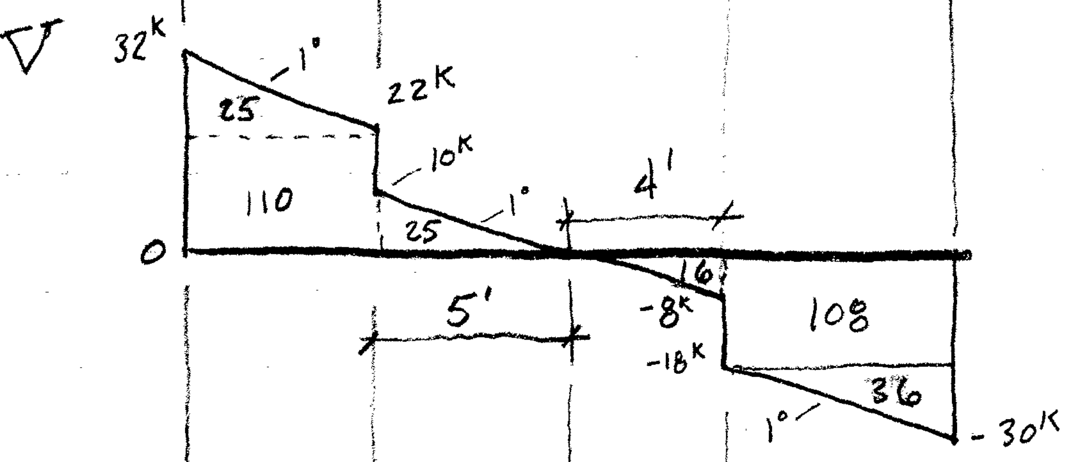
$R_2 20 = 600$

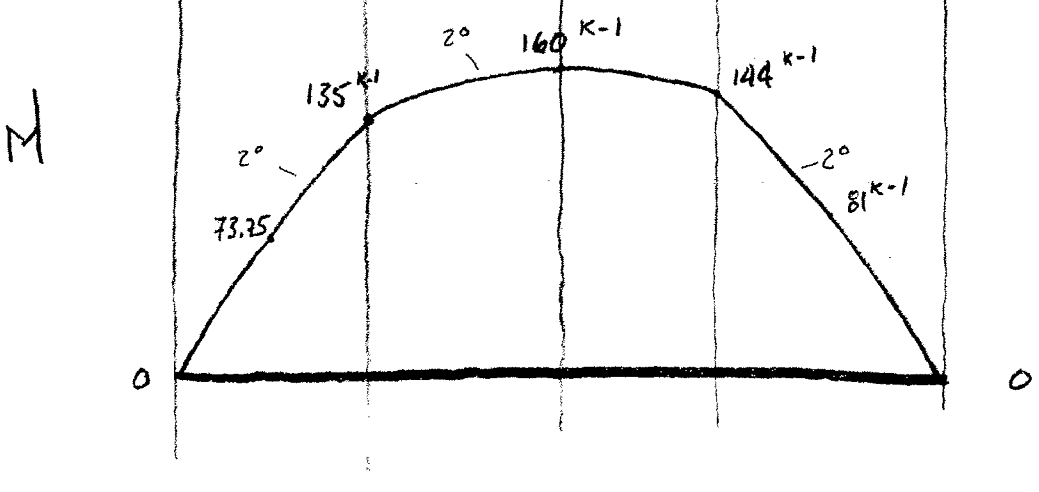
$R_2 = 30^k$

$\Sigma M_{R_2} = 0 = R_1(20) - 12(15) - 40(10) - 10(6)$

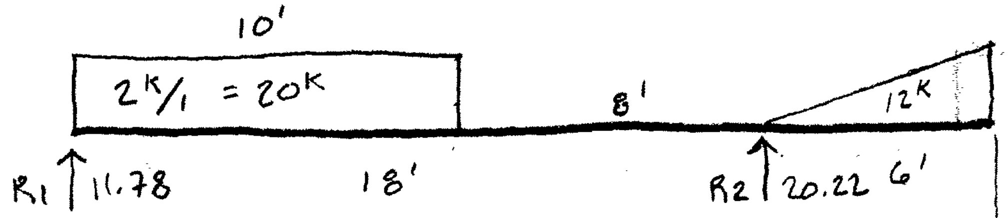
$R_1(20) = 640$

$R_1 = 32^k$ CHECK $\Sigma F_V = 30 + 32 - 12 - 40 - 10 = 0$ ✓





ARCH 331 | PROBLEM 8-7F | 23 OCT 91 PvB



REACTIONS:

$\Sigma M_{R_1} = 0 = 20(5) + 12(22) - R_2(18)$

$R_2 18 = 364$

$R_2 = 20.22^K$

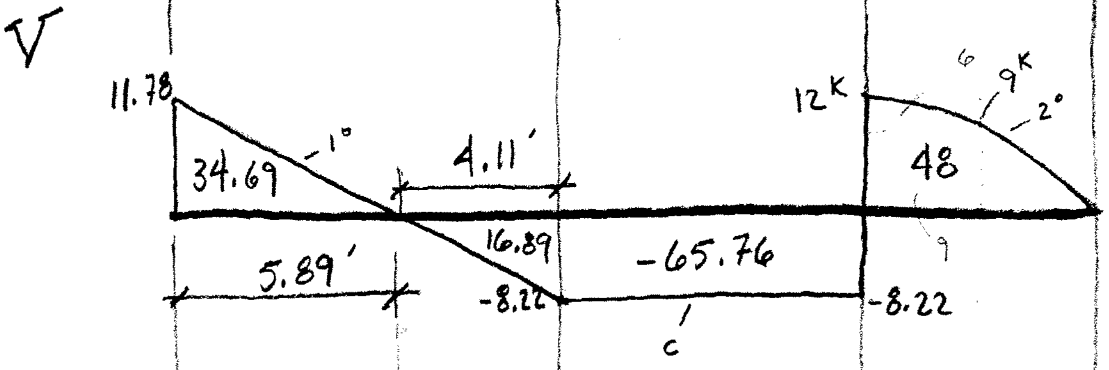
$\Sigma M_{R_2} = 0 = R_1(18) - 20(13) + 12(4)$

$R_1 18 = 212$

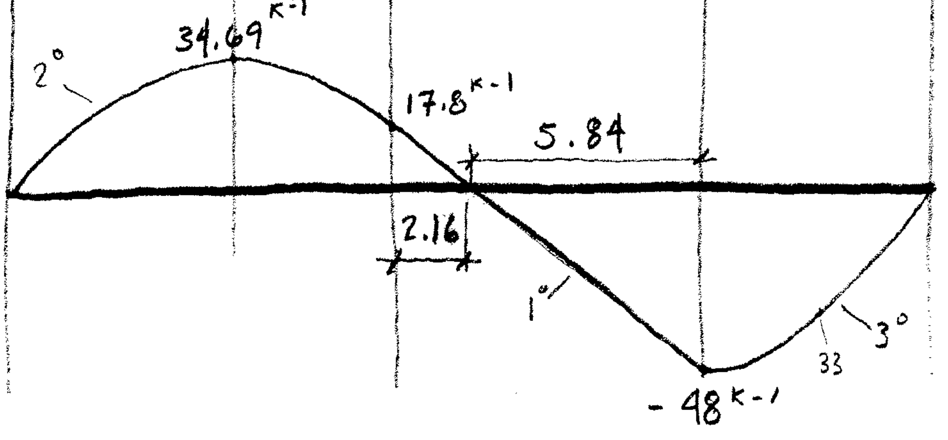
$R_1 = 11.78^K$

CHECK $\Sigma F_V = 0 = -20 - 12 + 20.22 + 11.78 = 0$ ✓

x2



÷2



STEP 2: (CONT.) FIND MOMENT OF INERTIA.

METHOD 2

NOTE, DIFFERENT $\bar{X}$ THAN $\bar{X}$ FOUND IN STEP 1.

$$I_x = I_g + A\bar{x}^2$$

WHERE

$$I = \frac{bd^3}{12}$$

	I (IN^4)	A (IN^2)	$\bar{X}$	I+A$\bar{X}^2$ (IN^4)
①	$8(2)^3/12 = 5.3$	$8(2) = 16$	3.3	183.1
②	$2(10)^3/12 = 166.7$	$2(10) = 20$	2.7	308.9

$I = 492$ IN^4 ✓

STEP 3: FIND MAXIUM RESISTING MOMENT

$$M = \frac{FI}{C} = \frac{1800 \text{ PSI } (492 \text{ IN}^4)}{7.67 \text{ IN}}$$

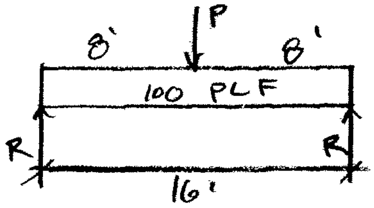
$C_c = 4.33$ IN. (ABOVE N.A.)

$C_t = 7.67$ IN (BELOW N.A.)

$$= 115,463 \text{ lb} \cdot \text{in}$$

$$= 9622 \text{ lb} \cdot \text{ft}$$

STEP 4: FIND MAXIMUM MOMENT DUE TO EXTERNAL LOADING.



M_{MAX} @ MIDSPAN DUE TO LOADING CONDITIONS.

$$M_{MAX} = (M_{MAX})_{\substack{\text{DIST.} \\ \text{LOAD}}} + (M_{MAX})_{\substack{\text{POINT} \\ \text{LOAD}}}$$

$$= \frac{w l^2}{8} + \frac{PL}{4}$$

$$M_{MAX} = \frac{100(16)^2}{8} + \frac{P(16)}{4}$$

$$M_{MAX} = 3200 + 4P \quad \text{lb} \cdot \text{ft}$$

STEP 5: EQUATE MMAX RESISTING TO MMAX LOADING.

$$\underset{\text{(RESISTING)}}{M_{MAX}} = \underset{\text{(LOADING)}}{M_{MAX}}$$

$$9622 = 3200 + 4P$$

$$6422 = 4P$$

$$P = 1606 \text{ lb.} = \underline{1.6^K}$$

$\boxed{\text{SHEAR + MOMENT DIAGRAMS}}$

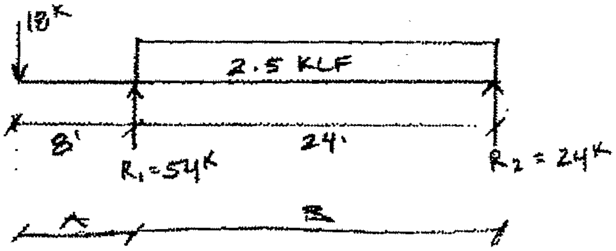
11.24.04

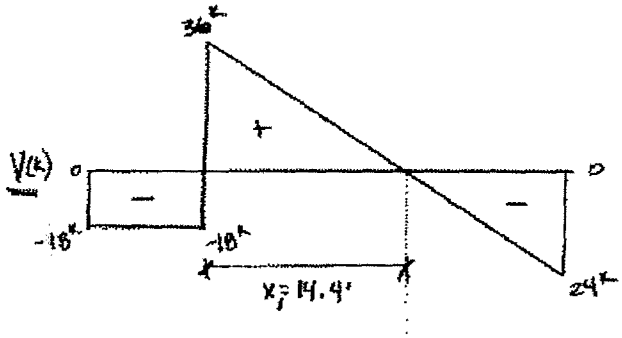
ALTERNATIVE METHOD USING CALCULUS:

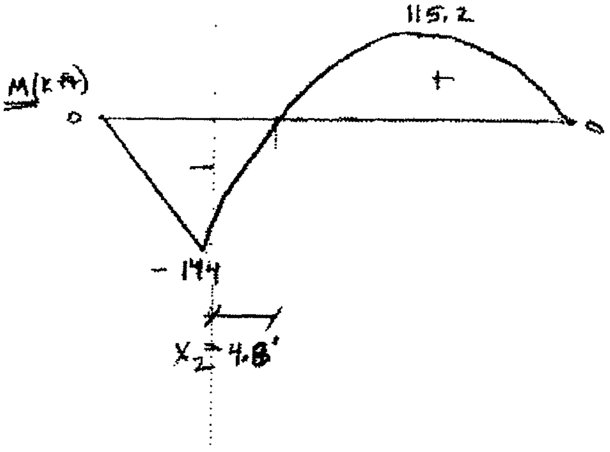
$$V=\int p\,dx + C_1 \qquad (EQ-1)$$

$$M=\int V\,dx + C_2 \qquad (EQ-2)$$

PROBLEM 8-2E







DIVIDE BEAM INTO TWO SECTIONS: A & B — WRITE INDIVIDUAL EQUATIONS:

SECTION $\boxed{A}$:
NO EQUATION

SECTION $\boxed{B}$:
$p_B = -2.5$

FOR V (SHEAR) DIAGRAM:

SECTION $\boxed{A}$:
DRAW V DIAG. BY VISUAL INSPEC. RESULTING IN

$$\boxed{V_A = -18}$$

SECTION $\boxed{B}$:
USE (EQ-1) ABOVE TO FIND V_B;

$$V_B = \int p_B\,dx + C_1 \qquad (EQ-1)$$

$$= \int -2.5\,dx + C_1$$

$$V_B = -2.5x + C_1$$

$$\underset{\text{AT LOCATION } R_1}{V_B(x=0)} = -2.5(0) + C_1 = 36$$

$$C_1 = 36 \quad \text{SO,}$$

$$\boxed{V_B = -2.5x + 36}$$

USING THE V_B EQUATION WE CAN DETERMINE THE SHEAR AT CRITICAL POINTS, MAINLY:

- @ FAR RIGHT => $X = 24'$

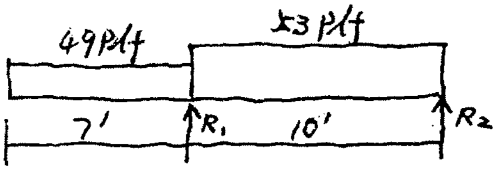
$$V_B(x=24) = -2.5(24) + 36 = \boxed{24}$$

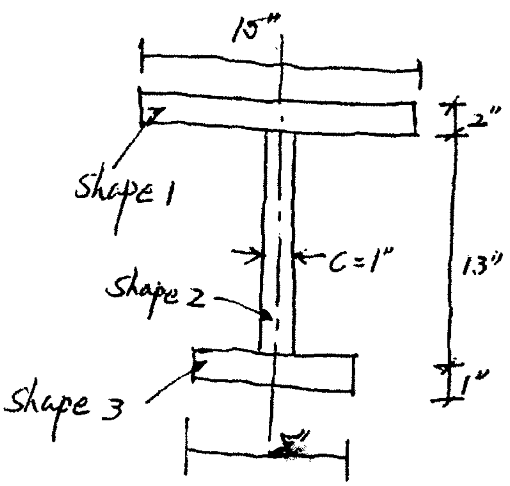
TO FIND X_1, WHERE V DIAG. CROSSES ZERO AXIS, $V=0$:

$$V_B = -2.5x + 36 = 0$$

$$\boxed{X_1 = 14.4'}$$

Problem 13.

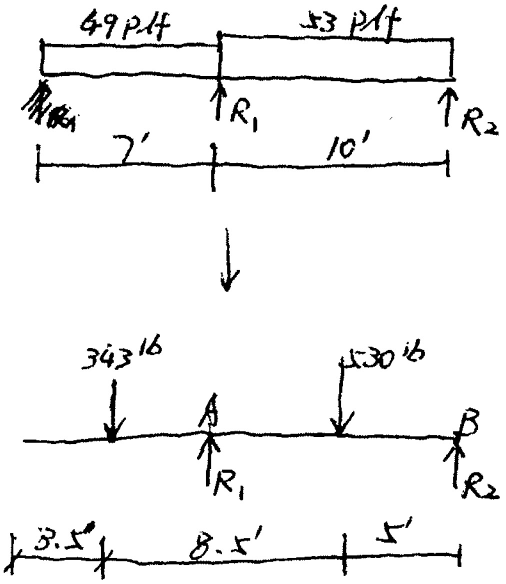




Find horizontal shear stress at the top of shape 2, at the base of Shape 2, and at the location of maximum horizontal shear (the neutural axis — N.A.) ?

Solution:

Step 1: Find reactions R_1, R_2.



$\Sigma M_B = 0$, assume ↻+

$-343^{lb}(13.5') + 10' R_1 - 530^{lb}(15') = 0$

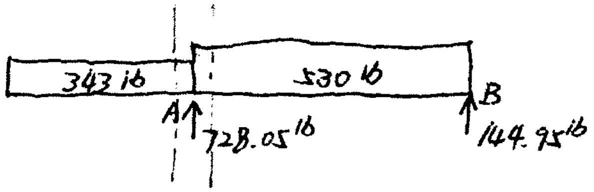
$R_1 = 728.05^{lb}$ ↑

$\Sigma F_X = 0$, assume ↑+

$R_1 + R_2 - 343^{lb} - 530^{lb} = 0$

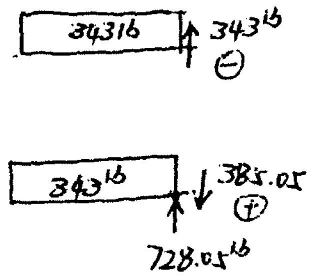
$R_2 = 144.95^{lb}$ ↑

Step 2. Draw shear diagram

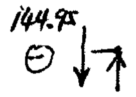


- Find shears at the ~~three~~ two special points A and B

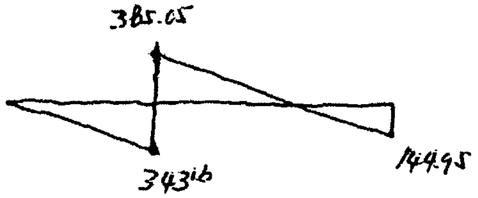
A.



B.



- Use the relationship between loading pattern and shear diagram to connect the shears at the ~~two~~ special points



Step 3. Decide the maximum shear force.

From the shear diagram, $V_{max.} = 385.05^{lb}$, at point B.

Step 4. Find neutral axis (NA)

$$\bar{y} = \frac{\Sigma A y}{\Sigma A}$$

$$= \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

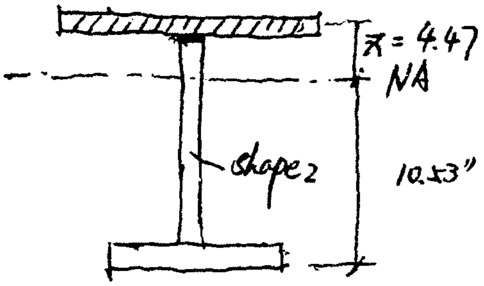
$$= \frac{(10'' \times 2'')(15'') + (13'' \times 1'')(7.5'') + (5'' \times 1'')(0.5'')}{(10'' \times 2'') + (13'' \times 1'') + (5'' \times 1'')}$$

$$= \boxed{10.53''}$$

Step 5. Find static moment (Q)

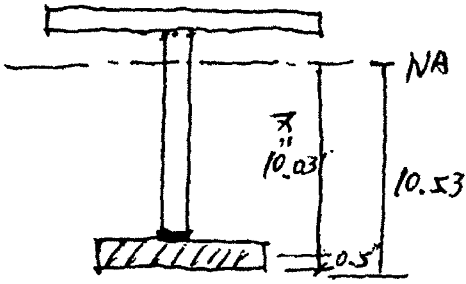
- Q at top of shape 2.

$$Q_T = A\bar{x} = (10''\times 2'')(4.47'')$$
$$= \boxed{89.4\ in^3}$$



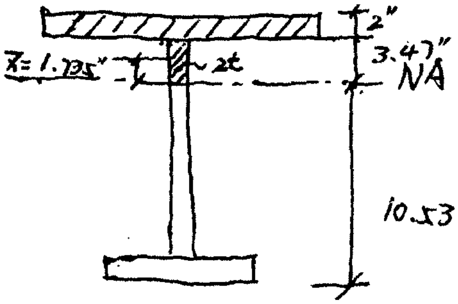
- Q at base of shape 2.

$$Q_b = A\bar{x} = (5''\times 1'')(10.03'')$$
$$= \boxed{50.15\ in^3}$$



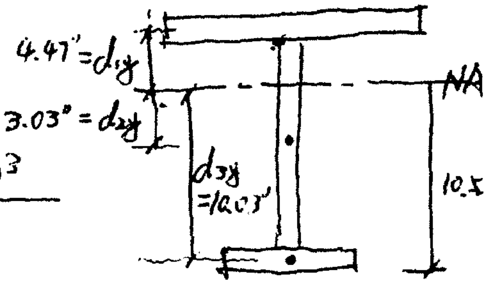
- Q_N at the NA.

$$Q_N = \Sigma A\bar{x} = Q_T + Q_{2t}$$
$$= 89.4 in.^3 + (3.47''\times 1'')(1.735'')$$
$$= \boxed{95.42\ in.^3}$$



Step 6. Find I about the NA.

$$I = I_x + \Sigma A d_y^2$$
$$I_x = I_1 + I_2 + I_3$$
$$= \frac{10(2)^3}{12} + \frac{1(13)^3}{12} + \frac{5(1)^3}{12}$$
$$= 190.16\ in.^4$$



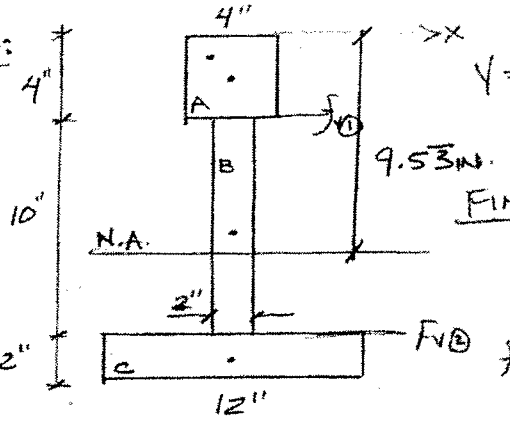
$$\Sigma A d_y^2 = A_1 d_{1y}^2 + A_2 d_{2y}^2 + A_3 d_{3y}^2$$
$$= (10''\times 2'')(4.47)^2 + (13''\times 1'')(3.03)^2 + (5''\times 1'')(10.03)^2$$
$$= 1021.97\ in.^4$$

$$\therefore I = 190.16 + 1021.97 = \boxed{1212.13\ in^4}$$

12/3/03
SR.

PROBLEM 10-1 Ⓕ

GIVEN:



$V = 1827^{\text{lb}}$

FIND:

$f_{v①}$ @ TOP OF SHAPE B

$f_{v②}$ @ BOTOM OF SHAPE B

$f_{v\text{MAX}}$ @ N.A.

STEP ①: FIND N.A.

$$\boxed{N.A. = \frac{\sum A\bar{x}}{\sum A}}$$

REF. TOP.

	A (IN²)	$\bar{x}$ (IN)	$A\bar{x}$ (IN³)
A	(4)(4) = 16	2	32
B	(2)(10) = 20	9	180
C	12 (2) = 24	15	360
	$\sum A = 60$		$572 = \sum A\bar{x}$

$$N.A. = \frac{572}{60} = \underline{9.5\bar{3}\text{ IN.}}$$

REFERENCE FROM TOP.

STEP ②: FIND MOMENT OF INERTIA

$$\boxed{I = \frac{bh^3}{12} + Ad^2}$$

$$I_A = \frac{4(4)^3}{12} + (4)(4)(9.53-2)^2 = 928.5 \text{ IN}^4$$

$$I_B = \frac{2(10)^3}{12} + (2)(10)(9.53-9)^2 = 172.3$$

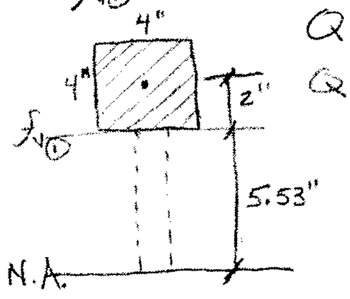
$$I_c = \frac{12(2)^3}{12} + (12)(2)(9.53-15)^2 = 726.1$$

$$\sum I = \underline{1826.9 \text{ IN}^4}$$

STEP ③: FIND STATIC MOMENT

$$\boxed{Q = A\bar{x}}$$

FOR $f_{v①}$



$Q = (4)(4)(2+5.53)$

$Q = \underline{120.5 \text{ IN}^3}$

STEP ④: FIND f_v

$$\boxed{f_v = \frac{VQ}{IB}}$$

FOR $f_{v①}$ THERE ARE TWO SURFACES:

ABOVE — SHAPE A ⇒ B = 4"

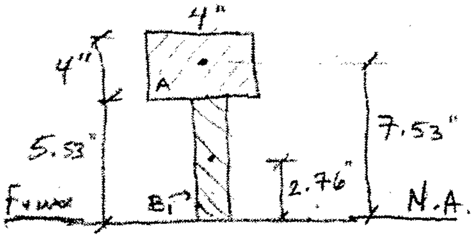
$$f_{v①} = \frac{1827^{\text{lb}}(120.5 \text{ IN}^3)}{1826.9 \text{ IN}^4 (4")} = \underline{30.1 \text{ PSI}}$$

BELOW — SHAPE B ⇒ B = 2"

$$f_{v①} = \frac{1827 \text{ lb.}(120.5 \text{IN}^3)}{1826.9 \text{ IN}^4 (2")} = \underline{60.2 \text{ PSI}}$$

STEP ③. FIND STATIC MOMENT

FOR $f_{v\,MAX}$ – AT THE N.A.



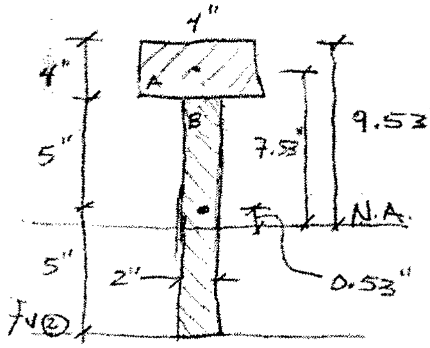
$$Q_{TOTAL} = Q_A + Q_{B_1}$$
$$= (4)(4)(7.53) + (2)(5.53)(2.76)$$
$$= 120.5 + 30.6$$
$$Q = \underline{151.1 \text{ IN}^3}$$

STEP ④: FIND f_v

$$f_{v\,MAX} = \frac{1827(151.1)}{1826.9(2)}$$
$$f_{v\,MAX} = \underline{75.6 \text{ PSI}}$$

STEP ③: FIND STATIC MOMENT

FOR $f_{v②}$



$$Q_{TOTAL} = Q_A + Q_B$$
$$= (4)(4)(7.53) + (2)(10)(0.53)$$
$$Q = \underline{131.1 \text{ IN}^3}$$

STEP ④: FIND f_v

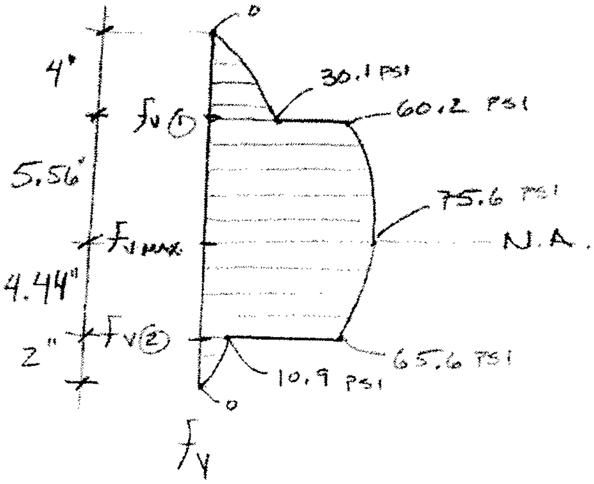
FOR $f_{v②}$ THERE ARE TWO SURFACES:

ABOVE – SHAPE B ⟹ B = 2"

$$f_{v②} = \frac{1827(131.1)}{1826.9(2)} = \underline{65.6 \text{ PSI}}$$

BELOW – SHAPE C ⟹ B = 12"

$$f_{v②} = \frac{1827(131.1)}{1826.9(12)} = \underline{10.9 \text{ PSI}}$$



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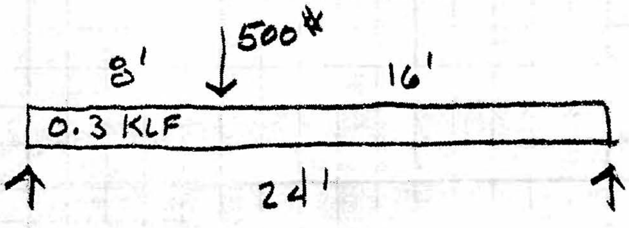
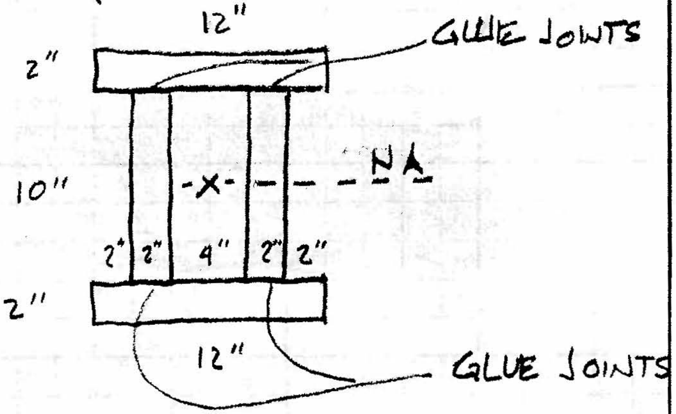


GLUE $F_v = 50$ psi
WOOD $F_v = 120$ psi



BEAM REACTIONS:

$$R_1 = 7.2/2 + 16/24(.500) = 3.93^K$$
$$R_2 = 7.2/2 + 8/24(.50) = 3.77^K$$
$$7.70^K = \Sigma F_V$$ ✓

$$V_{max} = R_1 = 3.93^K$$

FIND I:

$$\frac{12(14)^3}{12} - \frac{8(10)^3}{12} = 2744 - 666.7 = 2077.3 \text{ in}^4$$

FIND Q's

Q AT GLUE LINE:
$$Q = A\bar{x} = 2(12)(6) = 144 \text{ in}^3$$

Q AT N.A.:
$$Q = \Sigma A\bar{x} = 144 + 2(2(5))(2.5) = 194 \text{ in}^3$$

COMPARE STRESS LEVELS:

AT GLUE LINE:
$$f_v = \frac{VQ}{Ib} = \frac{3.93(144)}{2077.3(4)} = 0.068 \text{ KSI} = 68 \text{ psi} > 50 \therefore \underline{\underline{NG!}}$$

AT N.A.:
$$f_v = \frac{VQ}{Ib} = \frac{3.93(194)}{2077.3(4)} = .092 \text{ ksi} = 92 \text{ psi} < 120 \therefore \text{OK HERE}$$

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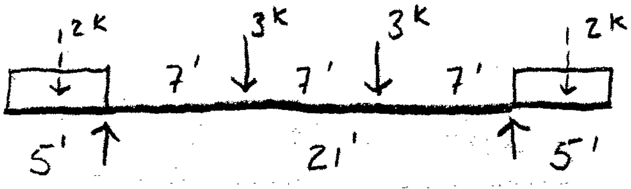
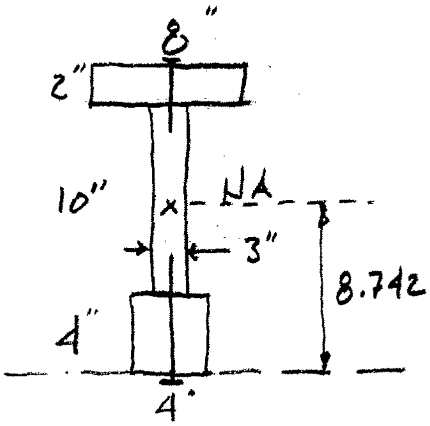




CONNECTORS $F_v = 200^{\#}$ EACH

DETERMINE NUMBER & SPACING FOR CONNECTORS REQUIRED TO FASTEN THE FLANGES TO THE WEB

REACTIONS:

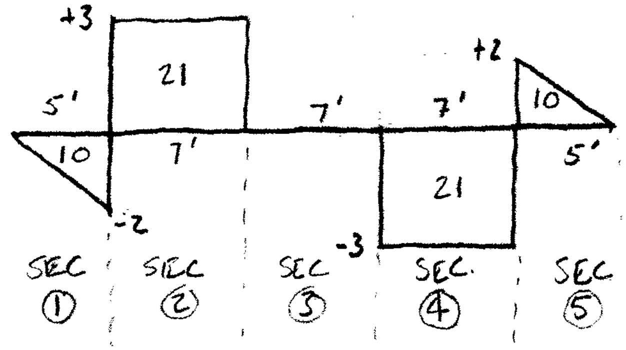
$\Sigma M_{R_1} = 0 = -2(2.5) + 3(7) + 3(14) - R_2(21) + 2(23.5)$

$R_2(21) = 105$

$R_2 = 5^k$

$R_1 = 5^k$ BY SYMMETRY

V DIA.



FIND I:

SHAPE	A	$\bar{x}$ (TO BOTTOM)	$A\bar{x}$	d	Ad^2	I_g
	16	15	240	6.258	626.6	$\frac{8(2)^3}{12} = 5.33$
	30	9	270	0.258	2.0	$\frac{3(10)^3}{12} = 250$
	16	2	32	6.742	727.3	$\frac{4(4)^3}{12} = 21.33$
	$\Sigma A = 62$		$\Sigma A\bar{x} = 542$		1355.9	276.7

$\frac{\Sigma A\bar{x}}{\Sigma A} = 8.742''$ FROM BOTTOM

$I_0 = \Sigma I_g + \Sigma Ad^2$

$= 276.7 + 1355.9 = \underline{1632.6 \text{ in}^4}$

FIND Q'S :

AT TOP –

$Q = A\bar{x}_{(NA)} = 16\,(6.258) = 100.128$

AT BOTTOM –

$Q = A\bar{x}_{(NA)} = 16\,(6.742) = 107.872$

FIND f_v :

SECTION ① + ⑤

$$f_{V_{TOP}} = \frac{VQ}{Ib} = \frac{2000\,(100.128)}{1632.6\,(3)} = 40.89 \text{ psi or } 122.67\ \#/\text{in}$$

$$f_{V_{BOTTOM}} = 40.89\,\frac{107.872}{100.128} = 44.05 \text{ psi or } 132.15\ \#/\text{in}$$

MAX SPACING FOR MAX SHEAR :

AT REACTION END –

TOP → $200/122.67 = 1.63''$

BOTTOM → $200/132.15 = 1.51''$

NOTE ⟶ (FROM HERE ON I WILL JUST USE THE BOTTOM JOINT TO FIND SPACINGS SINCE THEY ARE ABOUT THE SAME WITH BOTTOM CONTROLING)

TOTAL REQ'D FOR SECTION :

$F_{TOTAL} = 132.15\,(60'')\overset{\text{AVERAGE}}{/2} = 3964.5\ \#$

N = NUMBER OF FASTENERS PER SECTION $= 3964.5/200 = 19.82$

SAY 20

(i.e. 20 TOP AND 20 BOTTOM = 40)

SPACING ACROSS SECTION :

AT SUPPORT —— $\frac{5}{5}\,1.51'' = 1.51''$

AT 1' FROM SUPPORT $\frac{5}{4}\,1.51 = 1.888''$

AT 2' FROM SUPPORT $\frac{5}{3}\,1.51 = 2.517''$

AT 3' FROM SUPPORT $\frac{5}{2}\,1.51 = 3.775''$

AT 4' FROM SUPPORT $\frac{5}{1}\,1.51 = 7.55''$

AT 5' FROM SUPPORT $\frac{5}{0}\,1.51 = \infty$

USE :

16 @ 1.5" = 24"

6 @ 2.5" = 15"

3 @ 4" = 12"

1 @ 9" = 9"

= 26 > 20 OK

(THAT MEANS OVER DESIGNED BY 6)

FOR SECTIONS ② + ④

$$f_{V_{BOTTOM}} = \frac{3000\,(107.872)}{1632.6\ (3)} = 66.07 \text{ psi} = 198.22 \ \#/in$$

TOTAL CONNECTORS FOR SECTION:

$F_{TOTAL} = 198.22\,(84) = 16650.5^{\#}$

$N = F/P = 16650.5/200 = 83.25$ SAY 84

SPACING = 1 CONNECTOR / in = 84 PER SIDE

FOR SECTION ③

NO SHEAR - NONE REQUIRED
(BUT YOU WOULD HAVE TO PUT SOME IN AS A MINIMUM - SAY 12" O.C.)

7 SPACES AT 12" O.C. = 7' (5 CONNECTORS SINCE THE FIRST AND LAST ARE COVERED IN SEC. ② + ④)

TOTAL USED

SECTION	TOP	BOTTOM
1	26	26
2	84	84
3	5	5
4	84	84
5	26	26
	225	225

AGG TOTAL = 450

12/13/04

①

CONNECTORS TO RESIST HORIZONTAL SHEAR

THERE ARE BASICALLY TWO METHODS OF DETERMINING THE SPACING OF CONNECTORS, DEPENDANT ON THE SPECIFIC CASE.

METHOD 1: FINDING SPACING AT SPECIFIC LOCATION IN A BEAM.

$$P_{CONN} = sBf_v \Longrightarrow \boxed{s = \frac{P_{CONN}}{Bf_v}}$$

TOTAL SHEARING FORCE OF CONNECTOR - LOOK UP IN TABLES USUALLY.

WIDTH OF CONTACT PLANE

SHEARING UNIT STRESS AT SPECIFIC LOCATION

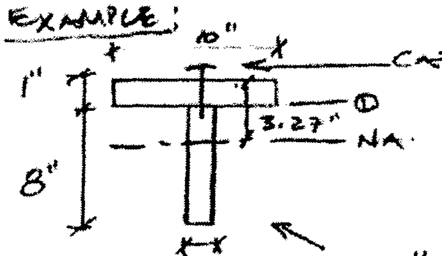
STEP 1: FIND f_v AT SPECIFIC LOCATION.

STEP 2: LOOK UP STRENGTH OF CONNECTOR - i.e. NAIL OR SCREW (USUALLY FORCE IS GIVEN IN PROBLEMS)

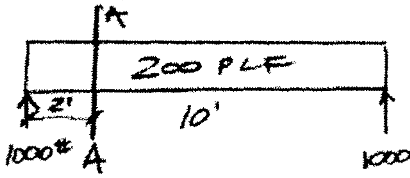
STEP 3: USE EQUATION TO FIND SPACING, S" AND ROUND DOWN FOR REALISTIC SPACING.

i.e. $s = 2.2" \Longrightarrow$ CALL OUT SPACING = 2".

EXAMPLE:



SECTION



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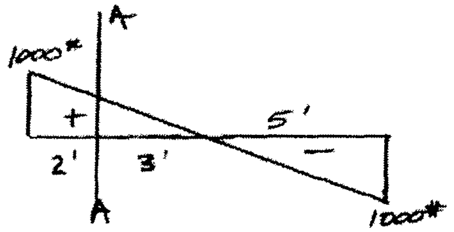


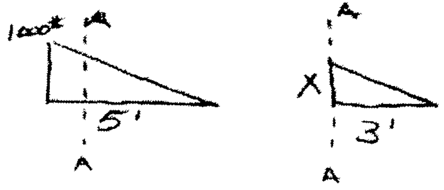
SHEAR

FIND SPACING OF CONNECTORS AT LOCATION A—A (2FT FROM END).

STEP 1: FIND f_v @ A-A IN BEAM AND @ ① IN SECTION - WHERE TWO MAT'LS COMBINED.

$$\boxed{f_v = \frac{VQ}{IB}}$$

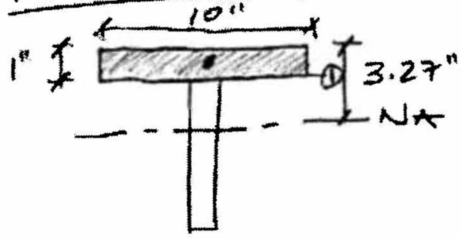
THE SHEAR (V) @ A-A CAN BE FOUND USING SIMILAR TRIANGLES:



$$\frac{X}{1000\#} = \frac{3'}{5'} \Rightarrow X = 600\#$$

SO V AT A—A IS 600# $\Rightarrow$ V = 600#

METHOD 1: (CONT.)



$$Q_{①} = A\bar{x} = 10''(1'')\left(3.27'' - \frac{1''}{2}\right)$$

DIST. FROM CENTROID TO NA.

$$Q = 27.7\ IN^3$$

$I = 211\ IN^4$ (GIVEN)

$B = 2''$ NOTE: B EQUALS BOTH 2" AND 10" AT LOCATION ①. BUT IN DETERMINING CONNECTORS, TAKE THE MOST CRITICAL VALUE → B = 2".

$$f_v = \frac{VQ}{IB} = \frac{600^{\#}(27.7\ IN^3)}{211\ IN^4\ (2\ IN)} = \underline{\underline{39.4\ PSI.}}$$

STEP 2: $P_{CONN} = 250^{\#}$ GIVEN

STEP 3:

$$S = \frac{P_{CONN}}{B f_v} = \frac{250^{\#}}{2''(39.4\ PSI)} = 3.17''$$

MORE CRITICAL VALUE FOR B. (2" VS. 10").

$$\boxed{S = 3.17''}$$

WHEN CALLING OUT SPACING ON DWGS: $\boxed{S = 3''}$ BUT FOR HW OR TEST, USE MORE SPECIFIC VALUE (3.17") UNLESS DIRECTED OTHERWISE.

NOTE: WITH METHOD 1, WE HAVE ONLY DETERMINED VALUE OF SPACING AT A PARTICULAR POINT IN THE BEAM. THIS PROCESS WOULD NEED TO BE REPEATED MULTIPLE TIMES WHEN DETERMINING SPACING OF CONNECTORS FOR THE ENTIRE BEAM.

METHOD 2: FINDING SPACING FOR ENTIRE BEAM

$$\boxed{N = \frac{F}{P_{conn}}}$$

P_{conn} ← STRENGTH OF EA. CONNECTOR.

N ← TOTAL # CONNECTORS NEEDED FOR BEAM

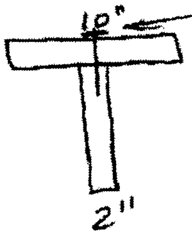
WHERE F = TOTAL SHEARING FORCE
= AREA OF SHEARING UNIT STRESS DIAG * WIDTH OF CONTACT PLANE

STEP 1: DETERMINE TOTAL SHEARING FORCE.

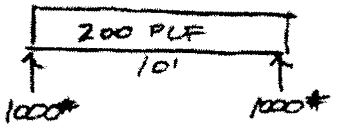
STEP 2: LOOK UP STRENGTH OF CONNECTOR (USUALLY GIVEN IN PROBLEMS)

STEP 3: USE EQUATION TO CALCULATE N. AND ROUND UP FOR REALISTIC VALUE. (i.e. CAN'T HAVE ½ A CONNECTOR)

EXAMPLE:



SECTION



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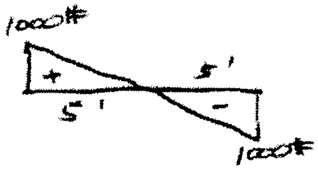
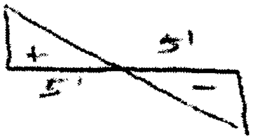


SHEAR



SHEARING UNIT STRESS

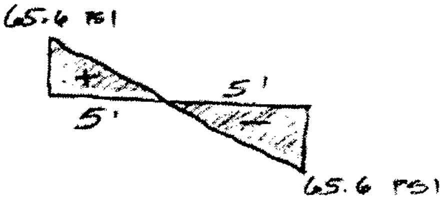
STEP 1: DETERMINE F

FIRST LABEL SHEARING UNIT STRESS DIAG. ↗

$$f_v = \frac{V_{max} Q}{I b} = \frac{1000\#(27.7 \text{ IN}^3)}{211 \text{ IN}^4 (2 \text{ IN})} = 65.6 \text{ PSI}$$

(Q DETERMINED IN METHOD 1.)

SO



$$F = \text{AREA} * \text{WIDTH}$$

$$= \frac{1}{2}(65.6 \text{ PSI})\, 5^{\text{FT}} \underset{\uparrow}{(12"/\text{FT})} \times 2 \; * \; 2" = \boxed{7872\# = F}$$

UNITS!!

METHOD 2: (CONT.)

STEP 2: $P_{CONN} = 250^{\#}$ GIVEN

STEP 3:

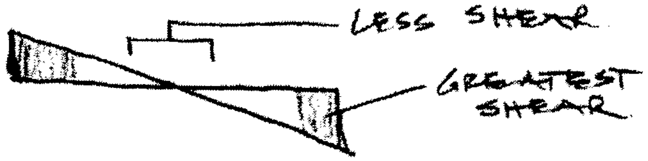
$$N = \frac{F}{P_{CONN}} = \frac{7872^{\#}}{250^{\#}} = 31.5$$

$$\boxed{N = 32}$$ ← A TOTAL OF 32 CONNECTORS IS NEEDED FOR ENTIRE BEAM.

A DESIGNER WOULD USE THE RESULTS FROM BOTH METHOD 1 + METHOD 2 TO DESIGN THE SPECIFIC SPACING OF THE CONNECTORS ALONG THE BEAM.

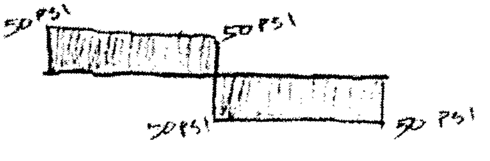
RECOGNIZE THAT, FOR THIS LOADING CASE (A DISTRIBUTED LOAD) THE GREATEST SHEARING STRESSES ARE LOCATED NEAR THE SUPPORTS AND REQUIRES THE MOST CONNECTORS WHEREAS NEAR THE CENTER OF THE BEAM, LESS SHEAR OCCURS, AND THEREFORE LESS CONNECTORS REQ'D.

i.e.



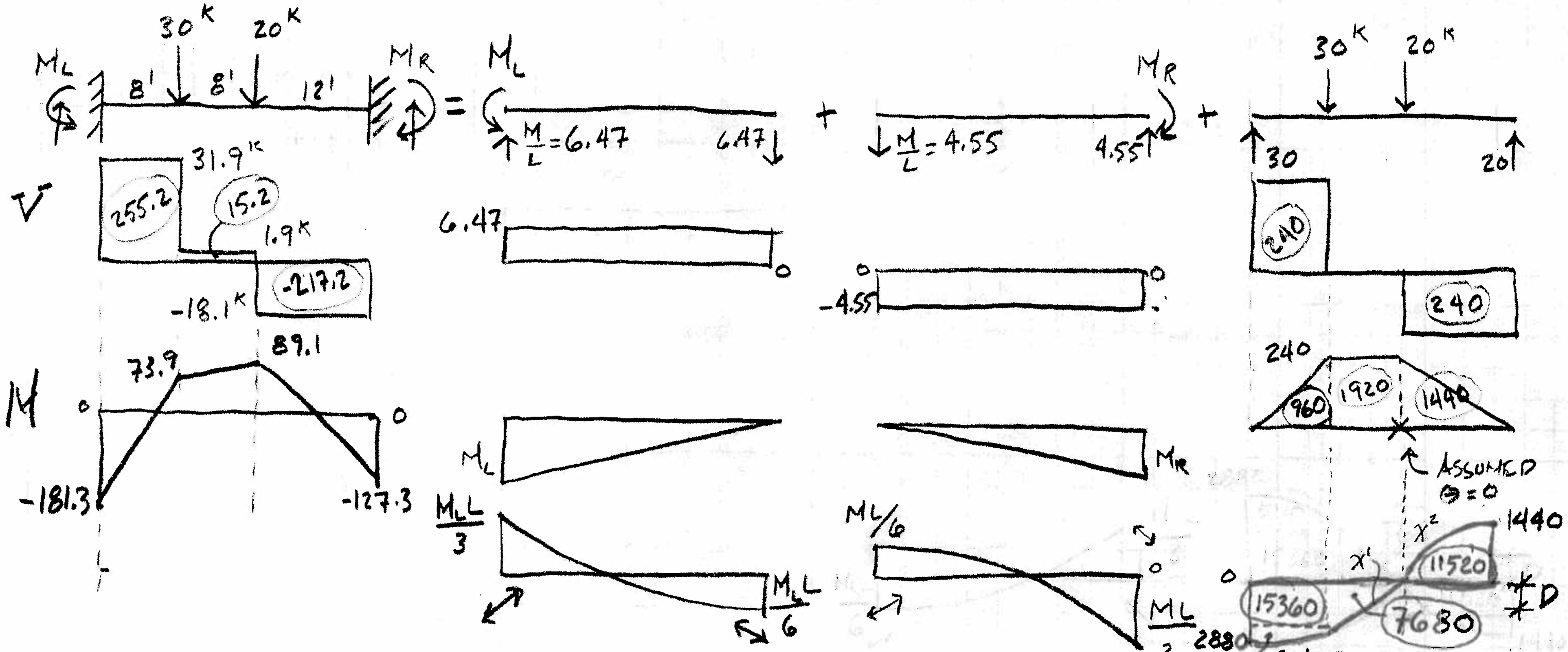
FOR A POINT LOAD, IN THE CENTER OF A BEAM, THE SPACING OF THE CONNECTORS WOULD BE THE SAME THROUGHOUT THE BEAM SINCE THE SHEAR IS CONSTANT.

i.e.



SO $S = \frac{N}{L} = \frac{\text{\# CONNECTS REQ'D}}{\text{LENGTH OF BEAM.}}$

(ROUND "S" DOWN)



$D = \frac{A \cdot B}{L} = \frac{28160 - 11520}{28}$

$D = 594.3$

$\theta_L = 2285.7$ ↻

$\theta_R = 2034.3$ ↔

GET SLOPE DIRECTION BY INSPECTION – LIKE SLOPE ADDS, OPPISITE SLOPE SUBTRACTS

LEFT END:

$\frac{M_L 28}{3} + \frac{M_R 28}{6} - 2285.7 = 0$

$M_L = \frac{2285.7 - M_R(4.667)}{9.33}$

$M_L = 244.90 - M_R(0.50)$

$M_L = \underline{181.3}^{K \cdot 1}$

RIGHT END:

$\frac{M_L 28}{6} + \frac{M_R 28}{3} - 2034.3 = 0$

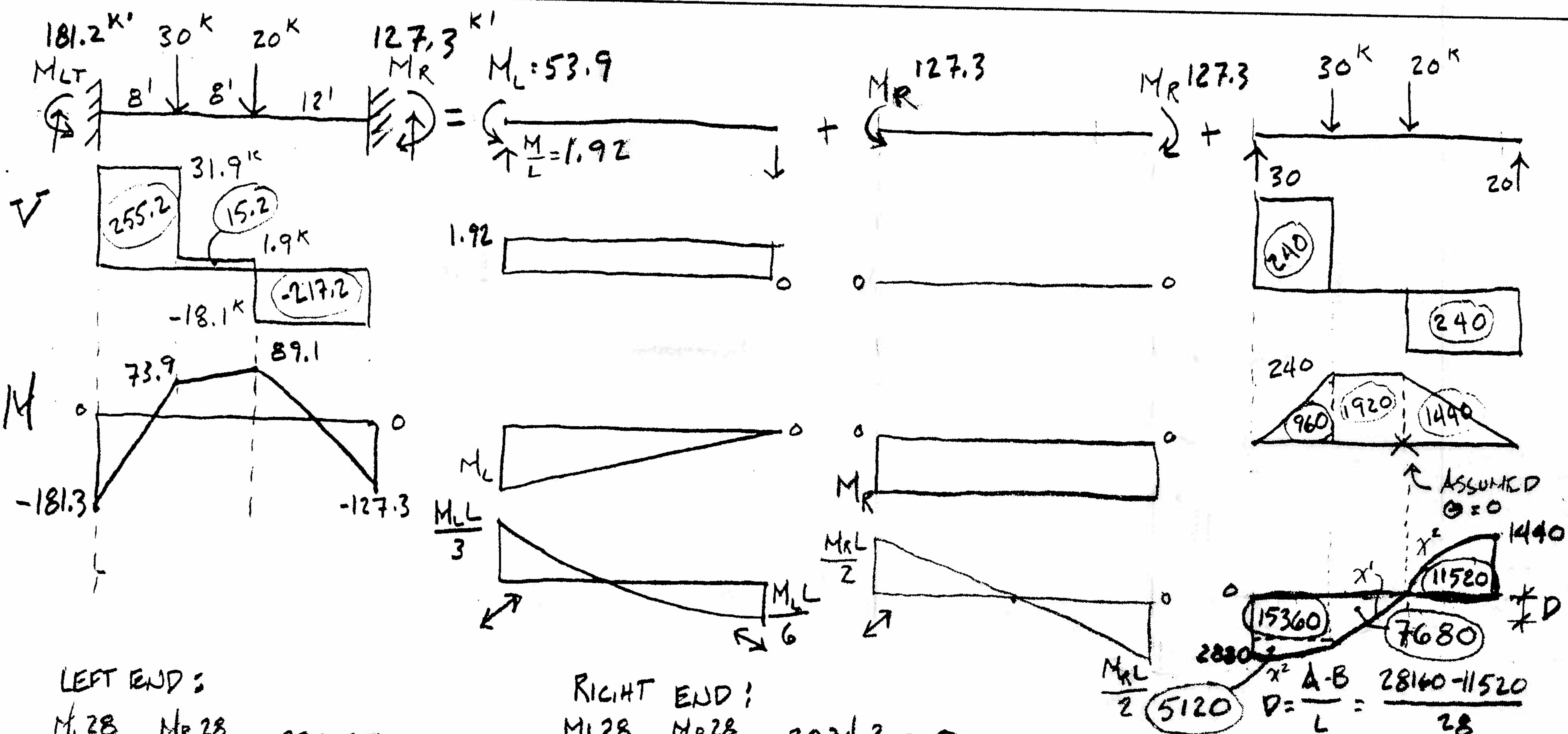
$M_L = 435.89 - M_R(2.0)$

$244.9 - M_R(.5) = 435.89 - M_R(2.0)$

$M_R = \frac{1782}{13.993} = \underline{127.3}^{K-1}$

ARCH 331 | PROBLEM 16-3 E | 11-24-92 PVB

ALTERNATE SOLUTION



$D = \frac{A-B}{L} = \frac{28160 - 11520}{28}$

$D = 594.3$

$\Theta_L = 2285.7$ ↘

$\Theta_R = 2034.3$ ↙↗

GET SLOPE DIRECTION BY INSPECTION – LIKE SLOPE ADDS, OPPISITE SLOPE SUBTRACTS

LEFT END:

$$\frac{M_L 28}{3} + \frac{M_R 28}{2} - 2285.7 = 0$$

$$M_L = \frac{2285.7 - M_R(14)}{9.333}$$

$$M_L = 244.9 - M_R(1.5)$$

$$M_L = 53.9^{K\cdot 1}$$

$$M_{LT} = 127.3 + 53.8 = \underline{181.2}^{K'}$$

RIGHT END:

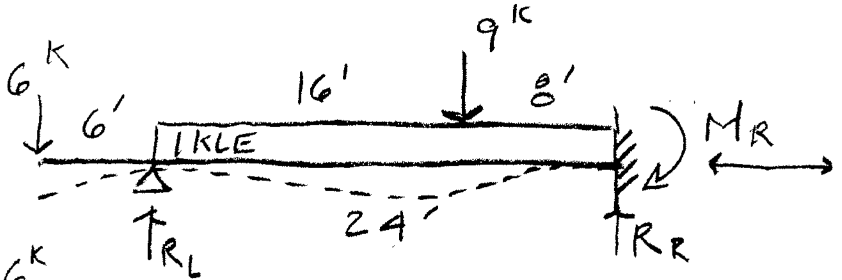
$$\frac{M_L 28}{6} + \frac{M_R 28}{2} - 2034.3 = 0$$

$$M_L = 435.92 - M_R(3)$$

$$244.9 - M_R(1.5) = 435.92 - M_R(3)$$

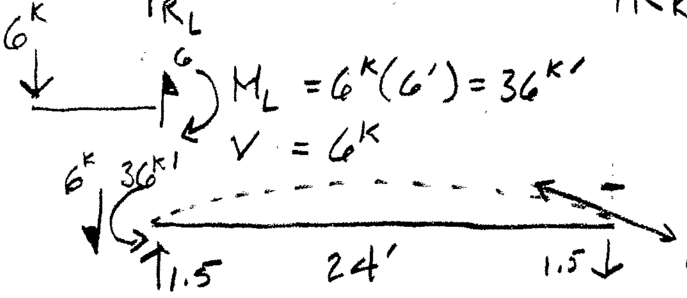
$$M_R = \underline{127.3}^{K'}$$

16-3 Ⓗ



$\longleftrightarrow EI\theta = 0$

FBD I



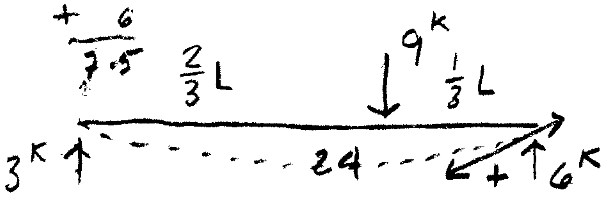
$M_L = 6^K(6') = 36^{K'}$

$V = 6^K$

$R_R = \frac{M}{L} = \frac{36}{24} = 1.5^K$

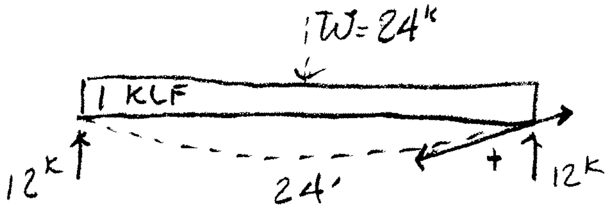
$\theta_R = \frac{ML}{6} = \frac{36(24)}{6} = -144$

FBD II



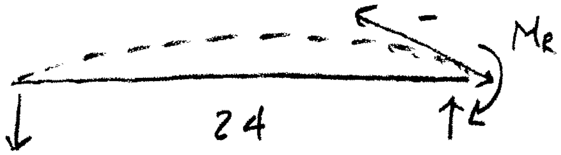
$\theta_R = \frac{5PL^2}{81} = +320$

FBD III



$\theta_R = \frac{WL^2}{24} = +576$

FBD IV



$\theta_R = \frac{ML}{3} = M8$

$R = \frac{M}{L} = \frac{94}{24} = 3.917^K$

$\theta_R = 0 = -144 + 320 + 576 - M8$

$M8 = 752, \quad M = 94^{K \cdot 1}$

$R_R = -1.5 + 6 + 12 + 3.917 = 20.42^K$

$R_L = \overset{7.5}{(1.5 + 6)} + 3 + 12 - 3.917 = 18.58^K$

R TOTAL $\overline{39.00^K}$

APPLIED LOAD

$-6 - 9 - 24 = -39$ ← BALANCED ✓

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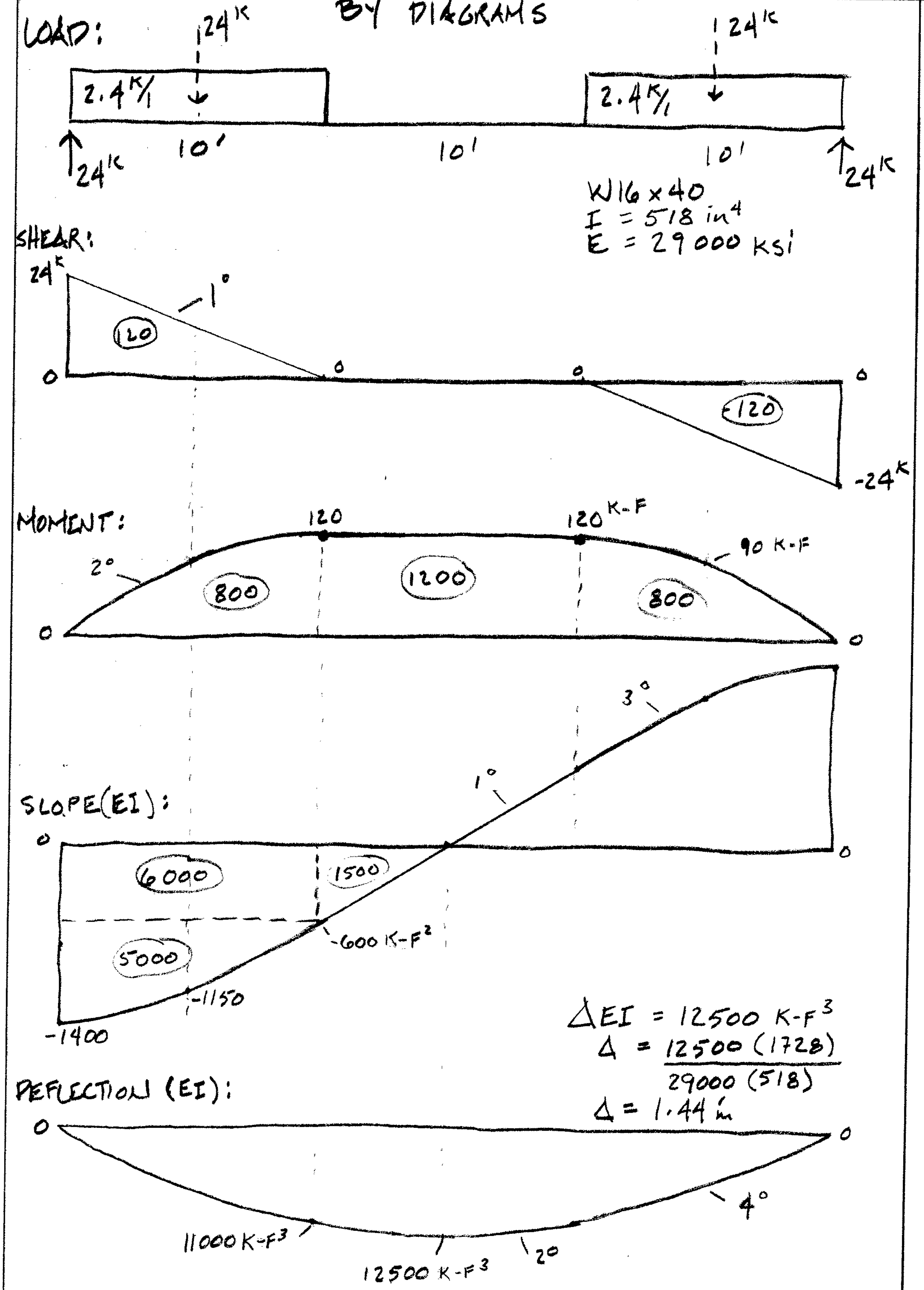
BY DIAGRAMS



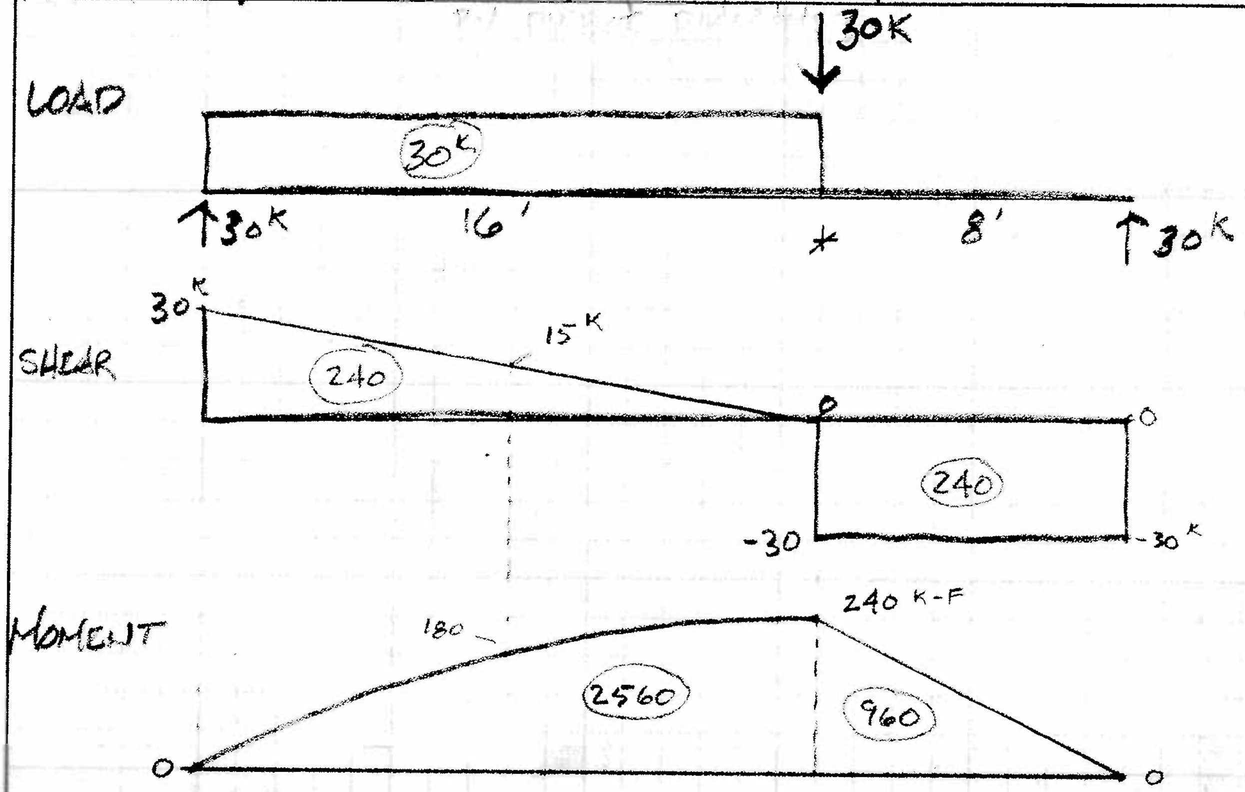
W16 x 40
$I = 518 \text{ in}^4$
$E = 29000$ ksi

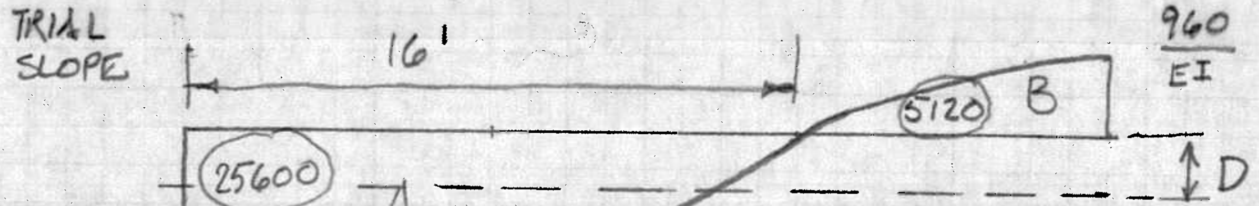
$\Delta EI = 12500 \text{ K-F}^3$

$$\Delta = \frac{12500\,(1728)}{29000\,(518)}$$

$\Delta = 1.44$ in

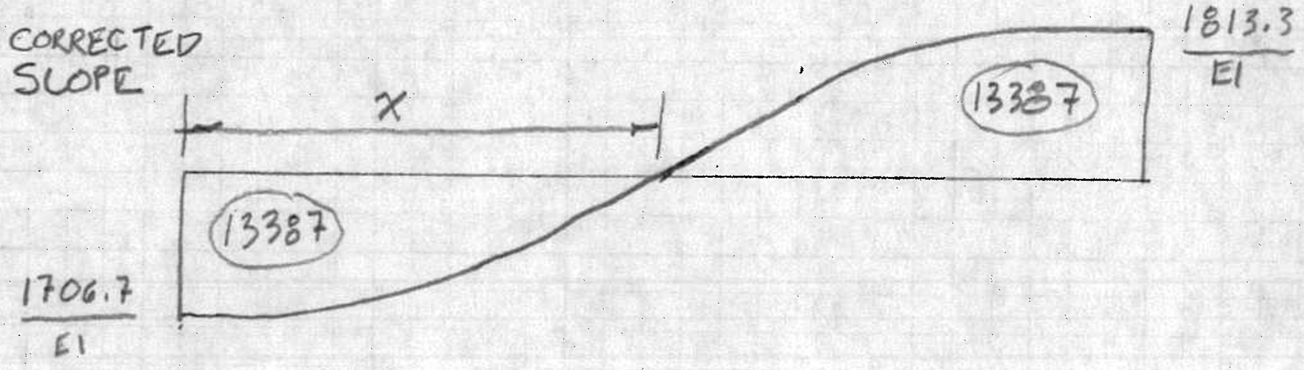
ARCH 331 | PROBLEM 16-1 D | 26 NOV 91 PJB | 1 OF 2





$D = \frac{A - B}{L} = \frac{25600 - 5120}{24}$

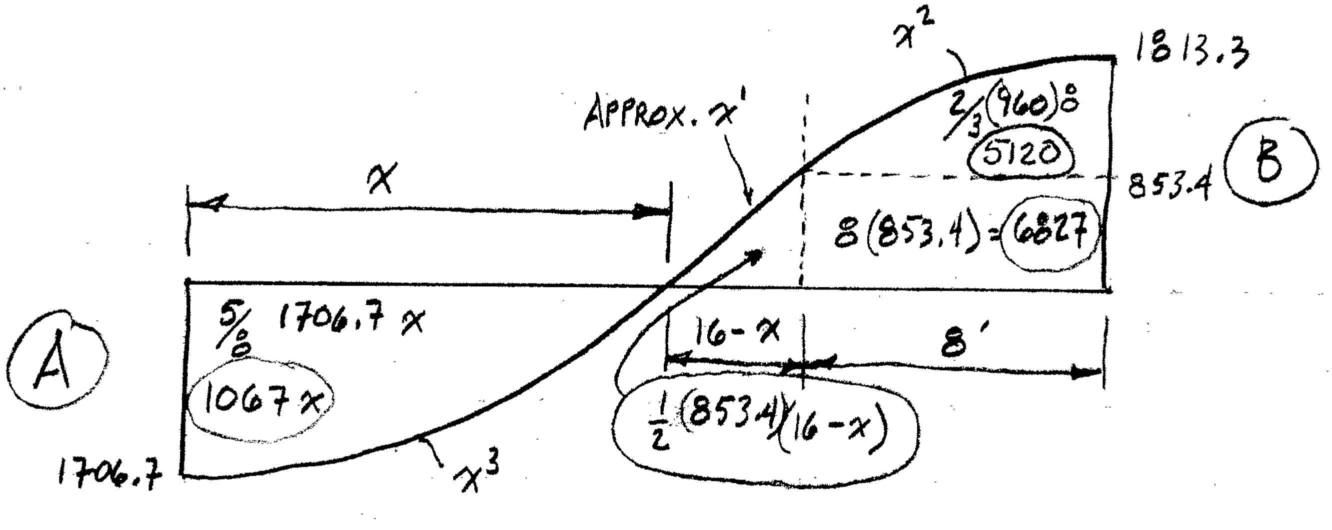
$D = 853.3^k$



FIND x BY TRIAL & ERROR SO THAT AREA A = B

(CONT)

TO SOLVE FOR x, GUESS A TRIAL x AND THEN CALCULATE THE AREAS. ADJUST x SO THAT AREA 'A' BELOW EQUALS AREA 'B' ABOVE THE BASE LINE. APPROXIMATELY TRIANGULARE AREAS NEAR THE CENTER CAN BE ½ bh



TRIAL 1 - $x = 12'$

AREA Ⓐ $1067(12) = 12804$

AREA Ⓑ $5120 + 6827 + \left[\frac{1}{2}\, 853.4(16-12)\right] = 13654$ 6.6% OFF

TRIAL 2 $x = 13'$

AREA Ⓐ $1067(13) = 13871$

AREA Ⓑ $11947 + \left[\frac{1}{2}\, 853.4(16-13)\right] = 13227$ 4.6% OFF

TRIAL 3 $x = 12.6'$

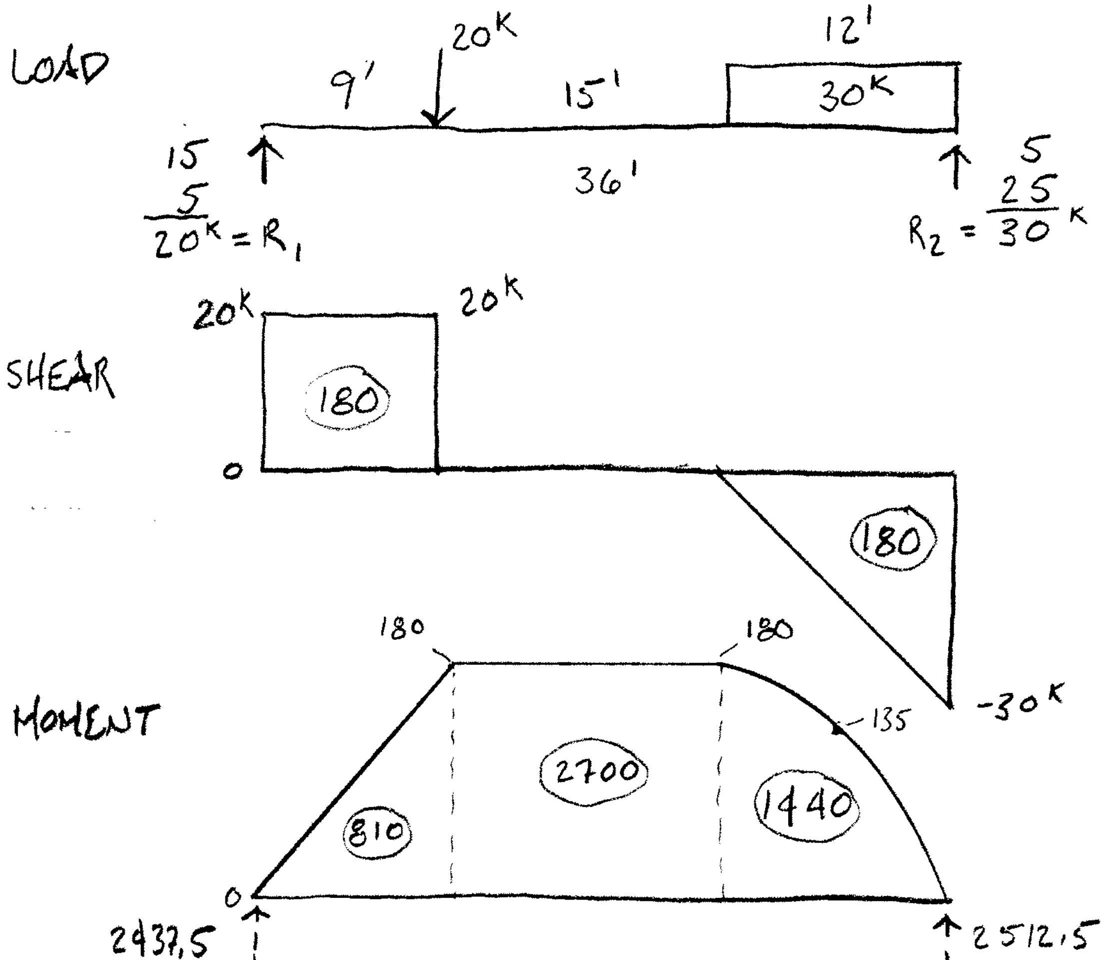
AREA Ⓐ $1067(12.6) = 13444$

AREA Ⓑ $11947 + \left[\frac{1}{2}\, 853.4(16-12.6)\right] = 13398$ 0.3% OFF

AVG AREAS = 13421

$$\text{DEFLECTION} = \frac{\text{AREA}(1728)}{EI} = \frac{13421(1728)}{29000(1330)} = 0.6''$$

ARCH 331 | PROBLEM 16-2 II A | 3 DEC 91 PJB | 1 of 2

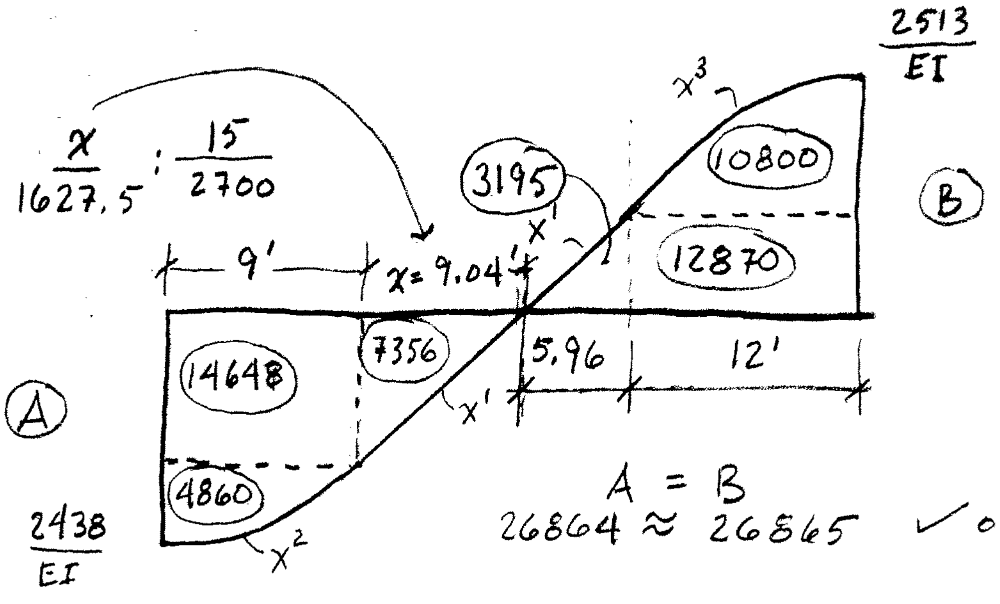


$\Sigma M_{R_1} = 810(\frac{2}{3}9) + 2700(9+\frac{15}{2}) + 1440(\frac{3}{8}12+24') - R_2(36) = 0$

$R_2 = 2512.5$

$\Sigma M_{R_2} = 810(\frac{9}{3}+27) + 2700(12+\frac{15}{2}) + 1440(\frac{5}{8}12) - R_1(36) = 0$

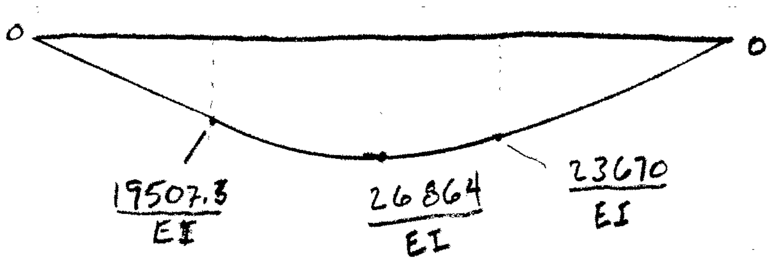
$R_1 = 2437.5$



A = B

26864 ≈ 26865 ✓ ok

DEFLECTION



TO FIND OUT WHAT THE DEFLECTION WOULD EQUAL IN INCHES THE SECTION (I) AND THE MATERIAL (E) MUST BE GIVEN — EG

FOR A W16×67

$$I = 954 \text{ in}^4$$
$$E = 29000 \text{ ksi}$$

$$\Delta_{max} = \frac{26864\,(1728)}{954\,(29000)} = 1.7''$$

WHICH IS $\frac{l}{257} < \frac{l}{360}$ ∴ WITHIN LIMITS FOR SPAN