

Arch 314

Structures I

Fall 2025 Recitation 004

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Recitation 004

Welcome to session 5!

- Bridge!!
- Quick Recap of this week's lectures
- Homework Review (#8 Three Hinged Arches)
- Lab: 3-Hinged Arches
- Bridge Prelim Report Questions/Review (if you want)



Feel free to ask questions anytime

Bridge

Due Dates:

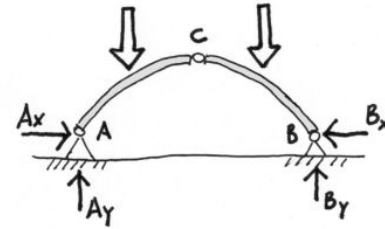
- **Preliminary Report - 10.10 (today!!)**
- Bridge Testing - 11.03
- Final Report - 11.25 (right before Thanksgiving break)

Example reports on “Bridges 1” tab

Lecture:

3-Hinged Arch

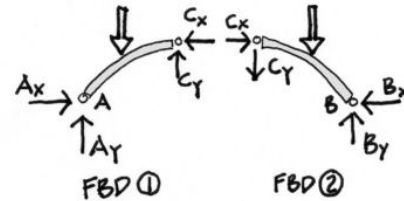
The 3-Hinged Arch has a “hinge” at each pinned support **plus one more internally**. The internal hinge provides one additional statics equation to be written since the moment at C is known ($M_C = 0$). This makes the system **statically determinate**.



3-HINGED ARCH

The solution of the end reactions can usually be obtained in **two steps**.

1. finding the **vertical reactions** by using the diagram of the whole structure and summing moments about each reaction.
2. summing moments at the internal hinge on an FBD of half of the structure to find the **horizontal forces**



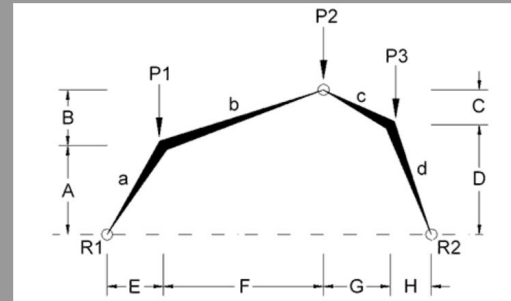
HW #8: Three Hinged Arches

8. Three Hinged Arches

For the three-hinged arch shown, determine horizontal and vertical components of each reaction, and the moments at the knees.

DATASET: 1 -2- -3-

Height A	15 FT
Height B	3 FT
Height C	9 FT
Height D	9 FT
Length E	4 FT
Length F	14 FT
Length G	14 FT
Length H	4 FT
Force P1	6 KIPS
Force P2	19 KIPS
Force P3	10 KIPS

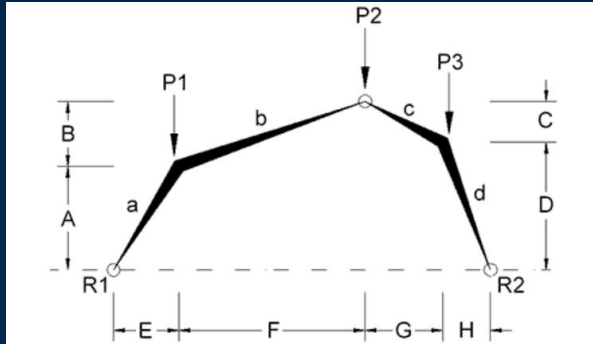


Your answer was correct.
You scored 5 points.

#	Question	Your Response	Correct Answer	Score
1	HORIZONTAL component of R1 (+ = to the right)	11.277 KIPS	11.2778 KIPS	5
2	VERTICAL component of R1 (+ = upward)	15.944 KIPS	15.9444 KIPS	5
3	HORIZONTAL component of R2 (+ = to the right)	-11.277 KIPS	-11.2778 KIPS	5
4	VERTICAL component of R2 (+ = upward)	19.056 KIPS	19.0556 KIPS	5
5	Moment at M1 (+ = tension inside)	105.377 KIP-FT	105.389 KIP-FT	5
6	Moment at M2 (+ = tension inside)	-25.2778 KIP-FT	-25.2778 KIP-FT	5
7	Axial force in member "a" (+ is compression)	19.53 KIPS	19.5298 KIPS	5
8	Axial force in member "b" (+ is compression)	15.036 KIPS	15.036 KIPS	5
9	Axial force in member "c" (+ is compression)	14.466 KIPS	14.4634 KIPS	5
10	Axial force in member "d" (+ is compression)	22.15 KIPS	22.1428 KIPS	5

HW #8: Three Hinged Arches

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Solve for external reactions - vertical

$$\sum M_{@R_2} = 0 = P_3(H) + P_2(G+H) + P_1(F+G+H) - R_{1y}(E+F+G+H)$$

* bring $R_{1y}(E+F+G+H)$ over to left side *

* then divide by $(E+F+G+H)$ to get R_{1y} alone *

$$R_{1y} = \frac{P_3(H) + P_2(G+H) + P_1(F+G+H)}{(E+F+G+H)}$$

$$R_{1y} = \frac{10(4) + 19(14+4) + 6(14+14+4)}{(4+14+14+4)}$$

$$R_{1y} = \boxed{15.944 \text{ kips}} \leftarrow \text{Answer to \#2 (Vertical of } R_1)$$

$$\sum F_y = 0 = R_{1y} - P_1 - P_2 - P_3 + R_{2y}$$

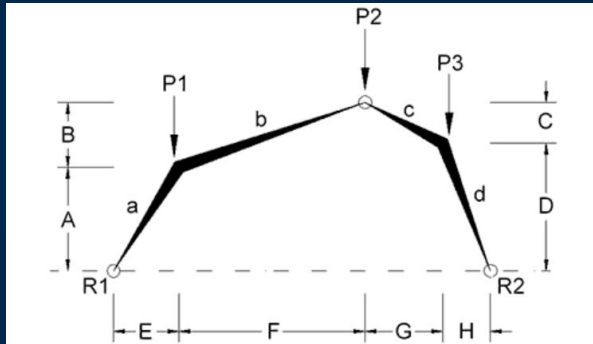
$$0 = 15.944 - 6 - 19 - 10 + R_{2y}$$

$$-R_{2y} = -19.056$$

$$R_{2y} = \boxed{19.056 \text{ kips}} \leftarrow \text{Answer to \#4 (Vertical of } R_2)$$

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Solve for external reactions - horizontal

$$\sum M_k = 0 = R_{1y}(E+F) - R_{1x}(A+B) - P_1(F)$$

* bring $R_{1x}(A+B)$ over *

* divide by $A+B$ to get R_{1x} alone *

$$R_{1x} = \frac{R_{1y}(E+F) - P_1(F)}{(A+B)}$$

#2

$$R_{1x} = \frac{15.944(4+14) - 6(14)}{(15'+3')}$$

#3

$$R_{1x} = \boxed{-11.277 \text{ kips}} \leftarrow \text{Answer to \#1 (horizontal of } R_1)$$

$$\sum F_x = 0 = R_{1x} + R_{2x}$$

$$-R_{2x} = +R_{1x} \quad \#1$$

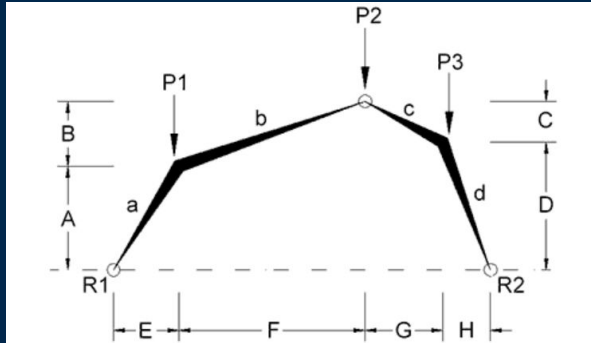
$$-R_{2x} = +11.277 \text{ kips}$$

$$R_{2x} = \boxed{-11.277 \text{ kips}} \leftarrow \text{Answer to \#1 (left = -)}$$

#3

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5. Calculate moment at hinges (M_1)

$$\sum M_{m1} = 0 = R_{1y}(E) - R_{1x}(A) + M_1$$

$$M_1 = R_{1y}(E) - R_{1x}(A) \quad \#1$$

$$\#2 = 15.944(4) - 11.277(15)$$

$$M_1 = \boxed{105.377 \text{ kips}} \leftarrow \text{Answer to \#5}$$

tension = +

6. Moment at M_2

$$\sum M_{m2} = 0 = R_{2y}(H) + R_{2x}(0) - M_2$$

$$M_2 = R_{2y}(H) + R_{2x}(0) \quad \#3$$

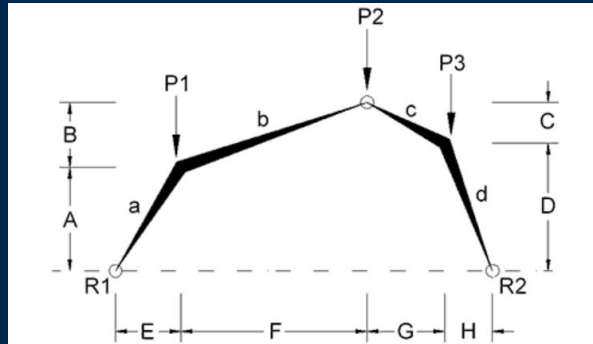
$$\#4 = 19.056(4) + (-11.277)(9)$$

$$M_2 = \boxed{25.2798 \text{ kips}} \leftarrow \text{Answer to \#6}$$

opposite side

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9-10. Determine axial forces

Member a

$$a = \sqrt{R_{ix}^2 + R_{iy}^2}$$

$$= \sqrt{(11.28)^2 + (15.94)^2} \quad \#2$$

$$\#1 \quad a = 19.53 \text{ kips} \quad \text{forces are pointed toward the joint} = \text{compression}$$

Member b

$$\sum F_x = 0 = a_x - b_x \rightarrow \sum F_y = 0 = a_y - P_1 - b_y$$

$$b_x = 11.28 \text{ kips}$$

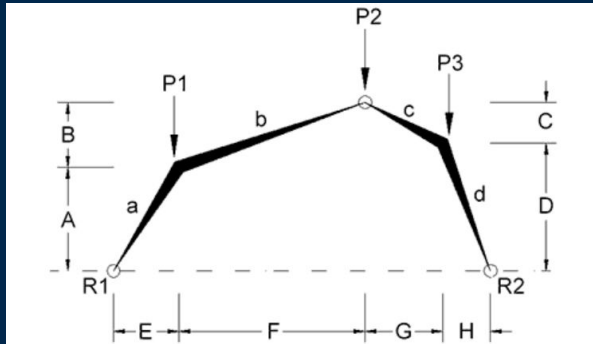
$$b_y = a_y - P_1 = 15.94 - 6 = 9.94 \text{ kips}$$

$$b = \sqrt{b_x^2 + b_y^2} = 15.036 \text{ kips}$$

forward joint = compression

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Member d

$$d = \sqrt{R_{ax}^2 + R_{ay}^2} \quad \#4$$

$$= \sqrt{(11.27)^2 + (9.056)^2} \quad \#3$$

$$d = 22.15 \text{ kips} \leftarrow \text{Answer to \#10}$$

toward joint = compression

Member c

$$\sum F_x = 0 = d_x - c_x$$

$$c_x = d_x - R_{ax} = \#3$$

$$c_x = 11.27$$

$$\sum F_y = 0 = d_y - P_3 - c_y$$

$$= 19.056 - 10 - c_y$$

$$c_y = 9.06 \text{ kips} \quad \#4$$

Answer to #9
toward joint = compression

$$C = \sqrt{c_x^2 + c_y^2}$$

$$C = 14.466 \text{ kips}$$

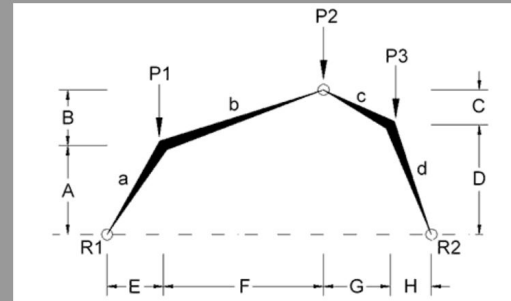
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LAB!

Prelim Report Questions?